Impedance Optimization and Prototype Measurement of APS SRF Crab Cavity

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Talk Outlines

• Principle review of wakefield and impedance definitions, cavity design optimization.
• Design and prototype elliptical squashed crab cavity for APS at ANL.
• Warm and cold measurement results.
• Multi-beam MAFIA (CST-PS, GdfidL) wakefield calculation method.
• Comparison of simulation to measurement.
• Future development for the multi-cell and cryomodule structure.
Wake Potentials, Impedances and Loss Factors

Basic Definitions: Wake Potentials

For a point change

\[ \tilde{W}^{\delta}(r_1, r_2, s) = \frac{1}{q_1} \int_{-\infty}^{\infty} dz \left[ \tilde{E}(r_1, r_2, z, t) + \beta c \tilde{e}_z \times \tilde{B}(r_1, r_2, z, t) \right]_{z=(z+s)/(\beta c)} \left[ \frac{V}{C} \right] \]

\( s \) is the distance from \( r_1 \) to \( r_2 \) in opposite beam travel direction (+z) in a moving frame. \( r_2 \) is transverse offset of a witness charge \( q_2 \).

For a beam bunch with a total charge \( q_1 \) line charge shape of \( \lambda(s) \ [1/m] \)

\[ W(r_1, r_2, s) = \int_{-\infty}^{\infty} ds' \lambda(s-s') W^{\delta}(r_1, r_2, s') \left[ \frac{V}{C} \right] \]

MAFIA (GdfidL) can calculate this wake in 3D with a Gaussian bunch \((q_1 \sigma, N)\), for \( \beta \leq 1 \), with multiple beams, multiple monitors in different \( r_1 \) and \( r_2 \) setups and to record this wake in each direction \((x,y,z)\). It can be used for calculating both narrow and broad bands, real and imaginary, short and long range impedances.

Beam Loss Factors

Energy loss \( K_{||} = \int_{-\infty}^{\infty} ds W_{||}(s) \lambda(s) \left[ \frac{V}{C} \right] \)

Transverse kick \( K_{\perp} = \int_{-\infty}^{\infty} ds W_{\perp}(s) \lambda(s) \left[ \frac{V}{C} \right] \)
Wake Potentials, Impedances and Loss Factors

Basic Definitions: Beam (coupling) Impedances

Longitudinal: \[ Z_{\parallel}(x, y, \omega) = \frac{1}{\beta c} \int_{-\infty}^{\infty} ds W_{\parallel}(x, y, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \quad [\Omega] \]

Transverse: \[ Z_{\perp}(x, y, \omega) = \frac{1}{\beta c} \int_{-\infty}^{\infty} ds W_{\perp}(x, y, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \quad [\Omega] \]

This impedance is Fourier transform of bunch change wake potential. \( \omega \) is angular frequency. This beam impedance is beam parameter dependent.

Geometrical Impedances

Longitudinal: \[ Z_{\parallel}(x, y, \omega) = \frac{1}{\beta c} \int_{-\infty}^{\infty} ds W_{\parallel}^{\delta}(x, y, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \quad [\Omega] \]

Transverse: \[ Z_{\perp}(x, y, \omega) = \frac{1}{\beta c} \int_{-\infty}^{\infty} ds W_{\perp}^{\delta}(x, y, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \quad [\Omega] \]

This impedance is Fourier transform of point change wake potential. \( \omega \) is angular frequency. This beam impedance is beam parameter independent.
Wake Potentials, Impedances and Loss Factors

Other impedances definitions (from equivalent circuits or beam dynamics):

\[ \hat{Z}_n(r, \omega) = \frac{\hat{V}_b}{I_b r^n} \left[ \begin{array}{c} \Omega \\ m^n \end{array} \right] \]

Here \( r \) is beam off-axis distance. \( n=0,1,2,\cdots \). \( n=0 \) calls monopole modes, \( n=1 \) calls dipole modes, \( n=2 \) calls quadruple modes, \( \cdots \) etc. \( \wedge \) means a Phasor. \( V_b \) is beam voltage including TTF. \( I_b \) is beam peak current. This impedance should be related to geometrical impedance only.

For example, transverse dipole impedance in a circular accelerator in radius \( R \):

\[ \hat{Z}_y(x, y, \omega) = -i \int_0^{2\pi R} \left[ \hat{E}_y(x, y, \omega) + \beta_z c \hat{B}_x(x, y, \omega) \right] dz \left[ \begin{array}{c} \Omega \\ m \end{array} \right] \]

Here the \( i \) indicates that the driving dipole moment \( I_y \) is \( 90^\circ \) out of phase with the deflecting field \( [E_y(x,y) + \beta_z c B_x(x,y)] \exp(-i\omega t) \). So the beam bunch does not transfer energy through the transverse fields but to longitudinal \( E_z \) field when \( y \) is not constant or zero.
Principle Beam Deflection/Crabbing of a RF cavity

Panofsky–Wenzel Theorem:

\[
P_{\perp} = \left( \frac{e}{\beta c} \right) \int_{0}^{d} \left[ E_{\perp} + \left( \beta c \times B \right)_{\perp} \right] dz = \left( \frac{e}{\omega_0} \right) \int_{0}^{d} (-i) \nabla_{\perp} E_{\perp} dz
\]


\[
\frac{\omega}{\beta c} Z_{\perp}(x, y, \omega) = \nabla_{\perp} Z_{\parallel}(x, y, \omega)
\]


• Panofsky’s theorem implies for any given RF mode, no matter who (E or B) deflecting the beam, there is must an non-zero transverse gradient of longitudinal component of the electric field.
• Transverse Impedance is the transverse gradient of longitudinal impedance divided by the wake number.
• For the beam near axis r=a, transverse impedance can be approximated by \( Z_{\perp} \approx Z_{\parallel} / (ka) \).
• Not only TM110 mode of RF field can do the beam deflecting/crabbing, some TE111 like or TEM modes can do the job also as long as the Panofsky–Wenzel theorem can be applied.
Wake Potentials, Impedances and Loss Factors

Cavity Shunt Impedance:
\[ R_{\parallel}(\omega) = \frac{\hat{V}_{\parallel}(\omega) \cdot V^*_\parallel(\omega)}{P_{\text{wall}}(\omega)} \quad [\Omega, BD] \]
\[ R_{\perp}(\omega) = \frac{2\hat{V}_{\parallel}(\omega) \cdot V^*_\parallel(\omega)}{P_{\text{wall}}(\omega)} \quad [\Omega, EE] \]

Cavity Deflecting Impedance:
\[ R_{\perp}(r, \omega) = \frac{\hat{V}_\perp(r, \omega) \cdot V^*_\perp(r, \omega)}{P_{\text{wall}}(\omega)} \quad [\Omega, BD] \]

Following approximations can be used in the narrowband geometrical impedance (not beam parameter dependent) calculation:

\[ R_{\perp}(r = a, \omega) \approx \frac{R_{\parallel}(r = a, \omega)}{(ka)^2} \quad [\Omega, BD] \]
\[ P_{\text{out}}(\omega) \approx \frac{\hat{I}_b \cdot \hat{V}_b}{2} \quad [W] \]
\[ Z_{\parallel}(r = a, \omega) \equiv \frac{V^2(\omega = a, \omega)}{P_{\text{out}}} \approx \frac{(R_{\parallel}/Q)Q_{\text{ext}}}{2} \quad [\Omega, BD] \]
\[ Z_{\perp}(y = a, \omega) \equiv \frac{V(y = a)}{I_b \alpha} \approx \frac{(R_{\perp}/Q)Q_{\text{ext}}}{2k} \approx \frac{(R_{\perp}/Q)Q_{\text{ext}}}{k} \quad [\frac{\Omega}{m}, BD] \]

Can be calculated by Eigen Solvers like MAFIA E3, MWS, Omega3P, ANSYS
The FFT of the beam bunch induced wake potentials calculated by MAFIA T3 (T2) or GdfidL can be normalized into the geometrical impedances if relative phase got wrap/unwrap properly, so real and reactive parts of impedance including broadband can be referenced to an unit charge.

\[ \hat{Z}_\parallel(r, \omega) = \frac{1}{\beta c} \int ds W_\parallel(r, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \left[ \Omega, BD \right] \]

\[ Z_\parallel(\omega) \approx \frac{(R_\parallel / Q) Q_{ext}}{2} \left[ \Omega, NB \right] \]

\[ \hat{Z}_\perp(r, \omega) = \frac{-i}{\beta c} \int ds W_\perp(r, s) \exp \left( -i \frac{\omega}{\beta c} s \right) \left[ \Omega, BD \right] \]

\[ Z_\perp(\omega) \approx \frac{(R_\perp / Q) Q_{ext}}{2} k \left[ \Omega / m, NB \right] \]

\[ k = \frac{\omega}{\beta c} \]
Deflecting Cavity Shape Optimization Principle

• Normal conducting cavity optimize:
  Transverse shunt impedance $R_t = \frac{V_t^2}{P_{\text{wall}}}$ or $R_t/Q$ for minimum copper wall loss. Because the integrated loss is high comparing to beam power.

• Superconducting cavity optimize:
  $\frac{V_{\text{def}}}{B_{\text{max}}}$ or $(\frac{R_t}{Q})(\frac{\omega U}{\mu_0 H_{\text{max}}})$ due to fundamental limit of critical magnetic field. Because the integrated loss is low comparing to beam power.
Scaling Laws of RF Deflecting Cavities

Cylindrical pillbox

\[
\frac{R_\perp}{Q} = \frac{1920}{\pi[u_1, J_2(u_1)]^2} \left[ \frac{J_1(\alpha)}{\alpha} - J_2(\alpha) \right] \quad [\Omega]
\]

Here \( \alpha = u_{11}r/a, u_{11} = 3.832, \) is root of \( J_1, J_1/J_2 \)
is first/second order of Bessel function.

for \( r \rightarrow 0, \quad R_\perp/Q = 64.16 \ \Omega \)
which is wavelength independent.

\[
\frac{\omega U}{H_{\text{max}}^2} = \left( 7.5\pi u_{11} \right)^2 = 346\lambda^2 \quad [\Omega m^2]
\]

For a 2.8 GHZ cavity,

\[
\left( \frac{R_\perp}{Q} \right) \left( \frac{\omega U}{H_{\text{max}}^2} \right) = 64.16 \times 3.97 = 254.5 \quad [\Omega m^2]
\]

Two-rod transmission line

\[
\frac{R_\perp}{Q} = \frac{960\lambda^2}{\pi^3} \ln \left( \frac{d_c}{d_0} \right) \quad [\Omega]
\]

\[
b = 0.5\sqrt{d_c^2 - d_0^2} \quad a = 0.5(d_c - d_0)
\]

for a 2.8GHz cavity, \( d_0=2\text{cm}, \ d_c=5\text{cm} \) \( R_\perp/Q = 252.3 \ \Omega \) which is wavelength dependent.

\[
\frac{\omega U}{H_{\text{max}}^2} = 30\pi^3 \ln \left( \frac{d_c}{d_0} \right) \left( \frac{1}{d_0} + \frac{1}{d_0 + d_c} \right)^2 \quad [\Omega m^2]
\]

\[
\left( \frac{R_\perp}{Q} \right) \left( \frac{\omega U}{H_{\text{max}}^2} \right) = 252.3 \times 0.206 = 52.04 \quad [\Omega m^2]
\]

Scaling Laws of RF Deflecting Cavities

Cylindrical pillbox

\[ \frac{V_{\text{def}}}{B_{\text{max}}} = \frac{1}{\mu_0} \sqrt{\frac{R}{Q}} \left( \frac{\omega U}{H_{\text{max}}} \right) \]

For a 2.8 GHz cavity with a 50mm beam aperture, two-rod type is only about 45% in efficiency of pillbox type, and even less than the elliptical cavity. But its transverse dimension is 55% or less than the pillbox type.

Two-rod transmission line

The graphs show the comparison between the pillbox and two-rod types of 2.8 GHz cavities. The efficiency and transverse dimensions are plotted against different parameters such as rod gap distance and diameter.
Elliptical Squashed Single-Cell SRF Cavity for APS

optimized squashed dimensions:

- $R_{rc}$: 44 mm
- $R_{rb}$: 25 mm
- $r_{ca}$: 14 mm
- $r_{co}$: 9 mm
- $z_{ca}$: 53.24 mm
- $y_{li}$: 33.66 mm

Single-cell structure with beam pipes

- TM110-y mode frequency: 2815.76 MHz
- $R_t/Q$: 35.27 Ohm
- Geometry factor $G$: 232.29 Ohm
- $\sqrt{R_t/Q}$: 90.51 Ohm
- $B_{max}/V_t$: 157.15 mT/MV
- $E_{max}/V_t$: 75.60 1/m

Transverse Gradient $E_t = V_t/d$

- $B_{max}/E_t$: 8.367 mT/(MV/m)
- $E_{max}/E_t$: 4.025

- cavity effective gap $d$: 53.24 mm
- BCS surface resistance $R_{bc}$ of Nb at 2K: 51.29 nOhm
- Residual resistance $R_0$: 20.00 nOhm
- $Q_0$ at 2K: $3.3E+09$
- BCS surface resistance $R_{bc}$ of Nb at 4.2K: 2498.33 nOhm
- $Q_0$ at 4.2K: $9.2E+07$
Squashed Elliptical Cavity Shape Optimization

MWS, ANSYS, HFSS and Gdfidl simulation by Jiaru and Geoff
Squashed Elliptical Cavity Shape Comparison

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Scaled KEK and JLab–ANL–LBNL’s crab cavity shapes to 800MHz

Figure 4: KEK crab cavity with a model coaxial coupler.
First Cold Single-Cell Crab Cavity Test at 2.1K

First cold test reached design gradient!

Crab Cavity Test #1

RF System unstable
Lorentz Force Detuning on ANL’s Crab Cavity

- Large LDF number caused the RF PLL unstable during the VTA test in high gradient.
- LFD is a few Hz/(MV/m)$^2$ in regular acceleration elliptical cavities.
- The stiffener on this cavity is needed.
LFD Simulated by ANSYS

- ANSYS gives inconsistent LFD with measurement data.
- ANSYS frequency convergence is in first order only.
- Displacement of LFD in 3D structure is too small and comparable to the meshing error.
- Triangle stiffener would reduce LFD by half, to be confirmed by next cold test.
Dumb Bell Measurement and Cell-to-Cell Coupling

- Dispersion curve has backward direction and has an opposite tuning direction to the TM010 mode.
- Double-chain model can explain this abnormal dispersion curve. Cell-to-cell coupling has both TM110 mode magnetic and TE111 mode electric couplings. See Jiaru’s talk this PM.
- Magnetic field enhancement on the iris in pi mode is a constraint going to multi-cell structure. The field direction is crossing the broader wall of the iris when the H field flipping the sign between the adjacent cells.
Waveguide HOM Damped Cavity Structure for APS

$R_{//}/Q$ and $R_{\perp}/Q$
Calculated from MWS eigen solver

Bench $Q_{\text{ext}}$ measurement by using
- RF absorbers on WG ports
- Clamping copper parts (low contact loss)
- Weak coupling to VNA
- Rotatable antennas to suppress the unwanted modes.
Bench Measurement Data Comparing to MAFIA (CST PS) Time Domain Simulation

Monopole Mode Impedance of ANL Single-cell Crab Cavity

- **TM010 (LOM monopole) mode**
- **Monopole and horizontal dipole modes, CST PS, 150 m wake**
- **HOM monopole mode impedance budget**
- **monopole mode, 1-cell, measure**
- **monopole mode, 2-cell, measure**

Frequency (GHz) vs. Monopole Mode Impedance (MΩ)
Bench Measurement Data Comparing to MAFIA (CST PS) Time Domain Simulation

**Dipole Mode Impedance of ANL Single-cell Crab Cavity**

- **Vertical dipole modes, CST PS, 150 m wake**
- **Vertical dipole mode impedance budget**
- **Monopole and horizontal dipole modes, CST PS, 150 m wake**
- **Horizontal dipole mode impedance budget**
- **Vertical dipole modes, 1-cell, measure**
- **Vertical dipole, 2-cell, measure**

**Frequency (GHz)**

**Dipole Mode Impedance (MΩ/m)**
Multi-beam Excitation MAFIA Wakefiled Simulation Method

one-beam excitation scheme for monopole modes

one-beam excitation scheme for horizontally polarized dipole modes (left) and two- and three-beam excitation schemes for vertically polarized dipole modes

three-beam and two-beam excitation scheme for quadrupole modes of different polarizations
MAFIA Multiple Beam Wakefield Simulation Example for JLab’s High Current Cavity

MAFIA T3 impedance spectra (monopole (red), dipole (blue), quadrupole (green))

versus

a) $R/Q_{meas}Q_{ext,meas}$ (black crosses)
b) $R/Q_{calc}Q_{ext,calc}$ (dots (m.), triangles (d.), diamonds (q.))
c) $R/Q_{meas}Q_{ext,calc}$ (brown crosses)
Real and Reactive Parts of Impedance Calculated by Wakefield FFT and Normalization

Monopole modes only:

H dipole modes only:
On-cell LOM/HOM Damping by Dog Bone Wedged Waveguide

V dipole modes only:

Key: Normalized impedance with

\[ Z_{am} = \frac{|W_{zfft}|}{B_{am}} \]
\[ Z_{ph} = W_{zfftph} - B_{ph} \]

With correct phase wrapping within \(-90^\circ\) to \(90^\circ\), the real part of impedance \(Z_{re}\) should be always >0.

\[ B_{neck}/B_{iris} = \approx 0.8 \]

\[ B_{max}/V_{def} = \]

157mT/MV (single-cell) to 240mT/MV (+ end WG)
Multi-cell Structure Options and Optimization

- The best LOM/HOM coupling on crab cavity so far is the on-cell waveguide coupling structure.
- But there is a drawback of magnetic field enhancement on the irises.
- This is due to multi-cell magnetic field in pi mode operation needs flipping direction near the broader wall of the iris. The sum of this surface current enhances at this location. Symmetric structure has less this kind of problem.
- 3-cell pi/2 mode structure is under studied by Geoff.
- A “periodic HOM damping structure” proposed at LINAC2004 is another natural extension this development.

$$\frac{B_{\text{max}}}{V_{\text{def}}} =$$ 157mT/MV (single-cell) to 260mT/MV (two-cell)

Figure 3: Magnitude of the magnetic field on the 3-cell cavity. Note the large field enhancement along the iris.

Ref: G. Wu, R. Rimmer and H. Wang at Linac2004
Summary

• Impedance and wake potential definitions and their relations have been reviewed. This should be help out on numbers of confusion on the impedance budget and calculation.

• MWS eigen solver and MAFIA (GdfidL) wakefield simulation using multi-beam and FFT normalization technique has been discussed and benchmarked. They have been used for the crab cavity design optimization and proofed successful.

• Both cold and warm measurement results on APS’s crab cavity indicated that we have been using good design tool and optimization process.

• Single-cell squashed crab cavity with LOM stub waveguide and HOM “Y” waveguide is workable unit structure for the APS’ impedance budget.

• We are working on further options of cavity prototype and multi-cell cavity design and final cryomodule design integration.

• Please stay tune for J. Shi and R. Rimmer’s talk this PM.