Charge density of light exotic nuclei and $\rho NN$ tensor coupling

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Abstract

We use a relativistic mean field model to study the charge density distributions of exotic oxygen isotopes. Nonlinear isoscalar–isovector terms which are not constrained by the present data are considered and the $\rho NN$ tensor coupling that does not affect the symmetry energy in the mean field model is included. Strong correlations between the neutron radius of $^{208}$Pb and the charge radius of $^{12,13,23,24}$O are found. The $\rho NN$ tensor coupling explicitly enhances the radius correlations between $^{23,24}$O and $^{208}$Pb. This enhancement is due to the neutron occupation of the orbital $2s_{1/2}$. The charge radius is sensitive to the change of the density dependence of the symmetry energy as the isotope goes towards the proton drip line.

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The nuclear equation of state (EOS) is fundamentally important to describe the nuclear phenomenology. The EOS comprises of two components: the saturation property of symmetric matter and the asymmetric energy. There exists a Cooster band for the saturation property of symmetric matter. However, the density dependence of the symmetric energy is much more poorly known to datef[1,2]. Therefore, the understanding of amount of properties of neutron stars, nuclei far from the $\beta$ stability, and radioactive ion collisions exists theoretical uncertainties. Many factors, for instance, the isoscalar–isovector coupling terms [3–7], the isovector–scalar mesons and density dependent coupling constants [8,9], can modify the behavior of the density dependence of the symmetry energy. Recently the density dependence of the symmetry energy has been extensively explored through the inclusion of the isoscalar–isovector coupling terms.

The inclusion of the isoscalar–isovector coupling terms allows one modify the neutron skin of heavy nuclei without changing the saturation property of symmetric matter and a variety of ground-state properties.
that are well constrained experimentally [3]. Theoretical estimates may give an uncertainty of the neutron radius of $^{208}$Pb by about 0.3 fm. Recently, data to data correlations between the neutron radius of $^{208}$Pb and neutron star properties have been established [3–6]

Such correlations are also found in the binding energy of valence neutrons for some neutron rich nuclei [7]. Accordingly, an accurate measurement of the neutron radius for $^{208}$Pb at the Jefferson Laboratory [10] that promises a 1% accuracy is very significant.

With the development of the radioactive beams properties of many exotic nuclei have been explored [11]. Many neutron and proton halo nuclei are identified through the measurement of the reaction cross sections and longitudinal momentum distributions of nucleus breakup. To obtain more definite information of halo nuclei, a pure probe that excludes the strong nucleus breakup. To obtain more definite information of halo nuclei, a pure probe that excludes the strong

conjecture that a relation between the neutron radius of $^{208}$Pb and the charge radius of the proton-rich isotopes for their simple spherical shapes.

The effective Lagrangian density is given as follows

$$\mathcal{L} = \bar{\psi} \left[ i \gamma_\mu \partial_\mu - M_N + g_\sigma \sigma - g_\omega Y_\mu \omega_\mu - g_\rho Y_\mu \tau_3 \rho_0 \right]$$

$$+ \frac{f_\rho}{2M_N} \sigma_{\mu \nu} \partial_\mu \partial_\nu - \frac{1}{2} (1 + \tau_3) Y_\mu A^\mu$$

$$- U(\sigma, \omega, b_0^\mu) + \frac{1}{2} \left( \partial_\mu \sigma \partial_\nu \sigma - m_\sigma^2 \sigma^2 \right)$$

$$- \frac{1}{4} F_{\mu \nu} F^ {\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu - \frac{1}{4} B_{\mu \nu} B^{\mu \nu}$$

$$+ \frac{1}{2} m_\rho^2 b_0^\mu b_0^\nu - \frac{1}{4} A_{\mu \nu} A^{\mu \nu},$$

where $\psi$, $\sigma$, $\omega$, and $b_0$ are the fields of the nucleon, scalar, vector, and charge-neutral isovector–vector mesons, with their masses $M_N$, $m_\sigma$, $m_\omega$, and $m_\rho$, respectively. $A_\mu$ is the field of the photon. $g_i$ ($i = \sigma, \omega, \rho$) and $f_\rho$ are the corresponding meson–nucleon couplings, and $\tau_3$ is the isospin Pauli matrix and its third component, respectively. $F_{\mu \nu}$, $B_{\mu \nu}$, and $A_{\mu \nu}$ are the strength tensors of the $\omega$, $\rho$ mesons and the photon, respectively

$$F_{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad A_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  \hspace{1cm} (2)

The self-interacting terms of $\sigma$, $\omega$-mesons and the isoscalar–isovector ones are in the following

$$U(\sigma, \omega_\mu, b_0^\mu) = \frac{1}{3} g_\sigma \sigma^3 + \frac{1}{4} g_\omega \sigma^4 - \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2$$

$$- 4 g_\rho^2 (A_\lambda g_\rho^2 \sigma^2 + A_\lambda g_\omega^2 \omega_\mu \omega^\mu) b_0^\mu b_0^\nu + \frac{1}{4}.$$  \hspace{1cm} (3)
Since detailed procedures are easily found in references (e.g., see Ref. [16]) we only write out the radial Dirac equations and the tensor potentials from the $\rho NN$ tensor coupling explicitly

$$\frac{dG_\alpha}{dr} = \left(M_\alpha^*(r) + V(r) + E_\alpha + U_1^\rho(r)\right)F_\alpha$$

$$- \left(\frac{\kappa}{r} - U_2^\rho(r) - U_3^\rho(r)\right)G_\alpha,$$

$$\frac{dF_\alpha}{dr} = \left(M_\alpha^*(r) - V(r) - E_\alpha - U_1^\rho(r)\right)G_\alpha$$

$$+ \left(\frac{\kappa}{r} - U_2^\rho(r) - U_3^\rho(r)\right)F_\alpha$$

and

$$U_1^\rho(r) = -\frac{f_\rho}{2M_N} \int_0^\infty dr_1 r_1^2 \rho_\rho(r, r_1) \rho_T(r_1),$$

$$U_2^\rho(r) = -\frac{f_\rho}{2M_N} \int_0^\infty dr_1 r_1^2 \sum_{\alpha} dG_\rho(r, r_1)$$

$$\times \left(\rho_3(r_1) - 8g_\rho(\Lambda_s g_\sigma^2 \sigma^2 + \Lambda_v g_\omega^2 \omega_0^2) b_0\right),$$

$$U_3^\rho(r) = -\frac{f_\rho^2}{2M_N} \int_0^\infty dr_1 r_1^2 \sum_{\alpha} dG_\rho(r, r_1)$$

$$\rho_T(r_1),$$

where

$$\rho_T(r) = \sum_a \frac{2j_a + 1}{4\pi r^2} \int_0^\infty d \left(G_\alpha(r) F_\alpha(r)\right),$$

$$M_\alpha^*(r) = M_N - V^\alpha(r),$$

$$V(r) = g_{\omega\sigma}(r) \omega_0(r) + g_{\rho\sigma}(r) t + e \left(\frac{1 + t}{2}\right) A_0,$$

$$V^\alpha(r) = g_{\alpha\sigma}(r) \sigma(r)$$

with $\rho_3$ the difference of the proton and neutron densities, $t = 1$ for proton and $-1$ for neutron, and $G_\rho(r, r_1)$ the Green function of the $\rho$ meson. The various density definitions follow the standard procedure [16]. The charge density is obtained by folding the proton density and the proton charge distribution [17]

$$\rho_c(r) = \int d^3r' \frac{\mu^3}{8\pi} \exp(-\mu|\mathbf{r} - \mathbf{r}'|) \rho_\rho(r'),$$

$$\mu = (0.71)^{1/2} \text{ GeV}$$

with $\rho_\rho$ the proton density.

The pairing correlation is involved in nonmagic nuclei using Bardeen–Cooper–Schrieffer (BCS) theory. We use constant pairing gaps which are obtained from the prescription of Möller and Nix [18]: $\Delta_\rho = 4.8/N^{1/3}$, $\Delta_\pi = 4.8/Z^{1/3}$ with $N$ and $Z$ the neutron and proton numbers, respectively. For odd nuclei, the Pauli blocking is considered.

We perform calculations with the NL3 parameter set [19] and distinguish two cases with and without the $\rho NN$ tensor coupling for various isoscalar–isovector coupling $\Lambda_v$'s. For simplicity, the isoscalar–isovector coupling $\Lambda_v$ is set as zero. The symmetry energy at saturation density is not well constrained, and hence some average of the symmetry energy at saturation density and the surface energy is constrained by the binding energy of nuclei. For a given coupling $\Lambda_v$, we follow Refs. [3,7] to readjust the $\rho NN$ coupling constant $g_\rho$ so as to keep an average symmetry energy fixed as 25.68 at $k_F = 1.15 \text{ fm}^{-1}$. In doing so, it was found in Ref. [3] that the binding energy of $^{208}$Pb is nearly unchanged for various $\Lambda_v$'s. In our calculation, the total binding energy of light exotic nuclei with different $\Lambda_v$'s is modified by the same order of $^{208}$Pb. The $\rho NN$ tensor coupling $f_\rho$ is taken as $f_\rho/g_\rho = 6.1$ which is used in the Bonn potentials [20] and is within $8.0 \pm 2.0$ obtained from the light cone sum rules by Zhu [21]. Note that this ratio of $f_\rho/g_\rho$, coupled with the relatively large values of $g_\rho$ in many relativistic mean field models that are usually fixed by the empirical values of the symmetry energy, could overestimate the contribution of the rho tensor coupling, and that will be analyzed below.

At the vicinity of the line $N = Z$, the $\rho$ meson together with the isoscalar–isovector term has almost negligible contribution. Consequently, we study the isoscalar–isovector term contribution in nuclei close to the drip lines. Table 1 gives the binding energy, proton radius, neutron thickness for $^{24}$O with and without the $\rho NN$ tensor coupling as $\Lambda_v$ is changed. The binding energy of $^{24}$O shows a small reduction as the $\Lambda_v$ is increased, while for $^{208}$Pb the binding energy shows a weak increase [3]. The neutron thickness is significantly reduced with the increasing $\Lambda_v$. Without the tensor coupling, the proton radius shows a much less modification compared to the neutron radius, which is similar to that for $^{208}$Pb [7]. As the $\rho NN$ tensor coupling is included, an apparent difference appears: the proton radius modification due to the $\Lambda_v$ is largely...
Table 1
Results for $^{24}$O with the NL3 parameter set for $f_\rho = 0$ and $f_\rho = 6.1 g_\rho$, respectively. The total binding energy ($E_b$), the proton radius ($r_p$), and the neutron thickness ($r_n - r_p$) are given for various $\Lambda_v$’s. The experimental binding energy is 168.48 MeV.

<table>
<thead>
<tr>
<th>$\Lambda_v$</th>
<th>$f_\rho = 0$</th>
<th>$f_\rho = 6.1 g_\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_b$ (MeV)</td>
<td>$r_p$ (fm)</td>
<td>$r_n - r_p$ (fm)</td>
</tr>
<tr>
<td>0.000</td>
<td>170.44</td>
<td>2.633</td>
</tr>
<tr>
<td>0.005</td>
<td>170.14</td>
<td>2.635</td>
</tr>
<tr>
<td>0.010</td>
<td>169.74</td>
<td>2.637</td>
</tr>
<tr>
<td>0.015</td>
<td>169.40</td>
<td>2.640</td>
</tr>
<tr>
<td>0.020</td>
<td>168.93</td>
<td>2.643</td>
</tr>
<tr>
<td>0.025</td>
<td>168.32</td>
<td>2.646</td>
</tr>
<tr>
<td>0.030</td>
<td>167.72</td>
<td>2.649</td>
</tr>
</tbody>
</table>

Enhanced, and almost half the contribution to the decrease of the neutron thickness of $^{23,24}$O comes from the increase of the proton radius. We notice that the inclusion of the tensor coupling in $^{208}$Pb just brings out negligible changes due to the quite small tensor potentials.

A data to data correlation between the neutron thickness of $^{208}$Pb and that of $^{23,24}$O is displayed in Fig. 1. The correlation is linear, and the $\rho NN$ tensor coupling increases the correlation slope strongly. The relativistic models with a softer symmetry energy have a larger symmetry energy at low densities. It was pointed out in Ref. [7] that the softer models are the first ones to drip neutrons that show a diffusive distribution with the larger valence neutron radius. A similar result for $^{23,24}$O is obtained that the radius of the valence $2s_1/2$ neutron increases with softening the symmetry energy. On the other hand, the change of the neutron radius is mainly from the modification of core neutron distribution instead of the diffusive neutron distribution that is often called as the neutron skins or halos. Consequently, owing to the coupling between the core neutrons and protons at the neighboring shells in light isotopes the modification of the neutron density distribution leads to a similar modification for the proton (charge) density distribution. In Fig. 1, the charge density for $^{24}$O is displayed. As seen in Fig. 1, the modification of the charge density distribution occurs explicitly in the core region with various $\Lambda_v$’s, and the modification is enhanced due to the inclusion of the $\rho NN$ tensor coupling. Further, we make a brief analysis of the overestimation of the contribution of the $\rho NN$ tensor coupling mentioned above. As we decrease the value of $f_\rho$ by 15%, the slope of the neutron thickness correlation, shown in Fig. 1, is just reduced by 8%. In this sense, the importance of the $\rho NN$ tensor coupling in present results is not much overestimated.

The explicit enhancement of the correlation for $^{23,24}$O as shown in Fig. 1 due to the $\rho NN$ tensor coupling does not occur for isotopes with more neutrons. At the same time, with the reduction of the neutron number the contribution from the $\rho NN$ tensor coupling drops. These phenomena can be explained by the contribution from the orbital $2s_1/2$. The orbital $2s_1/2$ generates large derivatives of the density distribution with respect to the radius near the node at the core region. A large tensor coupling contribution, corre-
sponding to large tensor potentials, may come from the large derivatives appearing in integrals in Eq. (6). For isotopes with $N > 16$, the $2s_{1/2}$ neutron distribution is squeezed due to the deeper binding, and a prominent cancellation, occurring in the integrals over the density derivatives from the regions inside and outside the node, leads to the drop of tensor potentials. The derivatives outside the node are small for $^{24}$O due to the extensive distribution, and the cancellation is slight so that large tensor potentials are produced. For isotopes with $N < 16$, the drop of the contribution from the $\rho NN$ tensor coupling is related to a smaller occupation probability in $2s_{1/2}$. Fig. 3 displays the neutron spin–orbit potential $U_{ls}$ that is obtained in an equation for the big component $G_\alpha$ of the Dirac spinor by eliminating the small component $F_\alpha$ from Eqs. (4), (5). The spin–orbit potential $U_{ls}$ varies sensitively with the different $2s_{1/2}$ occupation. If the Pauli blocking is not considered in $^{23}$O, the $2s_{1/2}$ occupation probability shows a small increment by about 0.1 that may produce an obvious move towards the spin–orbit potential for $^{24}$O. As the occupation probability in $2s_{1/2}$ rises, the difference arisen by the $\rho NN$ tensor coupling goes up, and that corresponds to the correlation enhancement observed in Fig. 1. The difference of the spin–orbit potential exists mainly in the core region as shown in Fig. 3. This is consistent with the charge density distributions shown in Fig. 2.

Now we investigate the role of the isoscalar–isovector term in the proton-rich oxygen isotopes. For the binding energy, the modification due to this term is small. For instance, the binding energy of $^{12}$O changes by 1.5 MeV as the $\Lambda_\nu$ increases from 0 to 0.03. With the decrease of the neutron number, the neutron distribution is little changed by the isoscalar–isovector term. A correlation between the charge radius of proton-rich oxygen isotopes and the neutron thickness of $^{208}$Pb is thus available. The establishment of the data to data correlation is helpful since it may bridge the different kind of experiments: the neutron radius measurement for $^{208}$Pb [10] via parity violating electron scattering and the electron scattering on the exotic nuclei [12,13]. In Fig. 4, the correlation between the charge radius of $^{12,13}$O and the neutron thickness of $^{208}$Pb is shown. The $\rho NN$ tensor coupling has a much less impact on the slope of the correlation relation for $^{12,13}$O than that for $^{23,24}$O. As different from the neutron-rich isotopes, the correlation relation for these proton-rich isotopes shows a nonlinear feature. Contrary to the correlation relation for the neutron-rich isotopes $^{23,24}$O, softer models that produce the small neutron thickness for $^{208}$Pb give rise to larger charge radii for $^{12,13}$O as shown in Fig. 4.
Fig. 4. The correlation of the neutron thickness of $^{208}$Pb and the charge radius of $^{12,13}$O. $r_0$ is the reference radius with $r_0 = 3.0$ fm.

Fig. 5. The charge density distribution for $^{12}$O with various cases.

In Fig. 5, the charge density distributions for $^{12}$O at various cases are shown. The dependence of the isoscalar–isovector term for the charge density distribution is just slightly affected by the $\rho NN$ tensor coupling. Similar to that of the neutron-rich isotopes $^{23,24}$O, the change of charge distribution due to the isoscalar–isovector term is concentrated in the core region. However, comparing to Fig. 2, the rising or dropping trend of the core charge density distribution is different with increasing $A_v$. As shown in Fig. 4, the charge density distribution of the drip line isotope ($^{12}$O) is more sensitive to the change of the density dependence of the symmetry energy. This may be attributed to the explicit increasing mean-field expectation value of the isovector meson for isotopes towards the proton drip line. The charge density distribution can be related to the cross section of the electron scattering on these exotic nuclei. As pointed out in Refs. [14,22], the form factor in the eikonal approximation at high momentum transfer (about 3.0 fm$^{-1}$) is sensitive to the charge distribution at the core region. According to the calculated charge density distributions in some oxygen isotopes, the identification of the isoscalar–isovector terms requires the electron scattering on exotic nuclei with high momentum transfers.

We have not made some extensive discussions on the model dependence of the results based on various RMF parameter sets instead of using the NL3 parameter set, and a detailed discussion on the model dependence may be found elsewhere (for instances, see Refs. [3,7]). For the RMF model the variation of the neutron radius of $^{208}$Pb is accomplished by softening the symmetry energy through introducing the isoscalar–isovector term that does not change a variety of ground-state properties. The different theoretical models may extend the region of the uncertainty for the neutron radius of $^{208}$Pb [3,7] that is about 0.3 fm. From the correlation between the charge radius of some exotic oxygen isotopes and the neutron thickness of $^{208}$Pb, we find there are also considerable uncertainties in the charge radius of exotic oxygen isotopes that awaits experimental identification. For instance, even according to an estimation based on a linear extrapolation, the uncertainty of the $^{12}$O charge radius that is almost from the core region is about 0.3 fm.

In summary, we have studied the charge density distributions of some exotic oxygen isotopes in RMF. The isoscalar–isovector term which is not constrained by the present data is considered to simulate the uncertainty of the neutron thickness of $^{208}$Pb. The $\rho NN$ tensor coupling that does not affect the symmetry energy in the mean field approximation is included. Strong correlations between the charge radius of $^{12,13,23,24}$O and the neutron thickness of $^{208}$Pb are established, showing that charge radii of $^{12,13,23,24}$O, especially $^{12}$O, exist a considerable uncertainty. The neutron thickness for $^{23,24}$O reduces with softening the sym-
metry energy. The strong correlation between the radii of $^{23,24}$O and the neutron radius of $^{208}$Pb can be largely attributed to their special structure and the tensor coupling: the $\rho NN$ tensor coupling explicitly enhances the correlation due to the neutron occupation of the orbital $2s_{1/2}$. The modifications of the charge density distributions for these exotic isotopes due to the isoscalar–isovector term are mainly explicit at the core region. The charge radius is sensitive to the change of the density dependence of the symmetry energy as the isotope goes towards the proton drip line. These new correlations may bridge two kind of experiments, the neutron radius measurement for $^{208}$Pb and the charge distribution measurement for exotic nuclei, to jointly constrain the density dependence of the symmetry energy.

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