Abstract

We study hard photon production in a chemically equilibrating quark–gluon plasma at finite baryon density, and find that the effect of the system evolution on the photon production and large contribution of the bremsstrahlung and Compton $qg \rightarrow \gamma q$ processes make the total photon yield as a strongly increasing function of the initial quark chemical potential.

The thermal production of hard photons in ultrarelativistic heavy ion collisions has been proposed as a possible signature for the formation of a quark–gluon plasma (QGP) [1]. Photons have large mean free paths due to the small cross section for electromagnetic interaction in the plasma, can provide a direct probe of the plasma. Previous authors have studied the production of hard photons in a QGP at finite temperature [2]. Assuming the formation of a QGP already at AGS and SPS energies, however, a finite quark chemical potential $\mu_q$ has to be considered [3], and even at RHIC energies the quark chemical potential $\mu_q$ may not be negligible as indicated by RQMD simulation ($\mu_q \approx 1 - 2T$) [4]. Recently, Hammon, Geiger, and co-workers [5,6] have indicated that the initial QGP system produced at the RHIC energies has finite baryon density, Majumder and Gale [7] have discussed the dileptons...
from QGP produced at the RHIC energies at finite baryon density, and Bass et al. [8] have pointed out that the parton rescattering and fragmentation lead to a substantial increase in the net-baryon density at midrapidity over the density produced by initial primary parton–parton scatterings. These show that one may further study the effect of the quark chemical potential on the signature of the QGP formation. At given energy density estimates based on lowest order perturbation theory indicate a strong suppression of the photon production at non-vanishing quark chemical potential \( \mu_q \) as compared to the case \( \mu_q = 0 \) [9]. Traxler, Vija, and Thoma have computed the photon production rate of a QGP at finite quark chemical potential for a given temperature (also a given energy density) using the Braaten–Pisarski method [10]. In addition, authors of [11–14] have studied the photon production functions of partons. In this work, we describe the evolution and photon production of a chemically equilibrating QGP system at finite baryon density on the basis of the Jüttner distribution function of partons.

We describe the evolution of the system according to [20,21]. For brevity, we do not give the detailed steps of the derivation of the evolution equations of the system. Expanding densities of quarks (anti-quarks)

\[
n_{q(\bar{q})} = \frac{g_{q(\bar{q})}\lambda_{q(\bar{q})}}{2\pi^2} \int \frac{p^2 dp}{\lambda_{q(\bar{q})} + e^{(p^2 + \mu_q)/T}},
\]

over quark chemical potential \( \mu_q \), we can get the baryon density of the system \( n_{b,q} \) and corresponding energy density \( \epsilon_{qgp} \), where \( g_{q(\bar{q})} \) is the degeneracy factor of quarks (anti-quarks). We consider the reactions leading to chemical equilibrium: \( gg \leftrightarrow g\gamma \), \( gg \leftrightarrow q\bar{q} \), \( gg \leftrightarrow s\bar{s} \) and \( q\bar{q} \leftrightarrow s\bar{s} \). Assuming that elastic parton scatterings are sufficiently rapid to maintain local thermal equilibrium, the evolutions of gluon, quark and s quark densities can be given by the master equations, respectively. Combining these master equations together with equations of energy–momentum and baryon number conservation of the system, for the longitudinal scaling expansion of the system, one can get a set of coupled relaxation equations describing evolution of the system on the basis of the thermodynamic relations of the system, as mentioned above. One solve the set of evolution equations under given initial values obtained from Hijing model or self-screened parton cascade model calculation [22] to get the evolutions of temperature \( T \), quark chemical potential \( \mu_q \) and fugacities \( \lambda_q \) for quarks, \( \lambda_g \) for gluons and \( \lambda_s \) for s quarks.

The earlier works [2,11,23] considered the hard photon production due to annihilation \( q\bar{q} \rightarrow g\gamma \) and QCD Compton \( (qg \rightarrow q\gamma \) and \( \bar{q}g \rightarrow \bar{q}\gamma \) scattering processes from a quark matter. Recently, authors of [24–26] have found that the near-collinear bremsstrahlung and a new process called inelastic pair annihilation (IPA), fully including the LPM effect, give also significant contribution to photon produc-
tion. In this work, we shall also compute the photon production rate from these two processes. We first consider photon production from quark annihilation $q\bar{q} \to \gamma g$ and Compton scatterings $gq \to \gamma q$ and $g\bar{g} \to \gamma q\bar{q}$. Their production rates may be calculated by [2]

$$E \frac{dR}{d^3p} = \frac{1}{2(2\pi)^8} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} f_1(E_1)f_2(E_2)$$

$$\times (1 \pm f_3(E_3))(2\pi)^4$$

$$\times \delta(p_1^\mu + p_2^\mu - p_3^\mu - p'^\mu) \sum |M|^2,$$  (2)

where $f(E)$ is the parton distribution function, $\sum |M|^2$ the square of the matrix element for reaction processes summed over spins, colors and flavors, as seen in Ref. [10]. The plus sign is for the annihilation process and the minus for the two Compton processes. Following Refs. [2,10], above equation can be rewritten as

$$E \frac{dR}{d^3p} = \frac{1}{16(2\pi)^7E} \int ds \int dt \sum |M|^2$$

$$\times \int dE_1 dE_2 f_1(E_1)f_2(E_2)$$

$$\times [1 \pm f_3(E_1 + E_2 - E)] \theta(P(E_1, E_2))$$

$$\times (P(E_1, E_2))^{-1/2},$$  (3)

where $\theta$ is the step function, $P(E_1, E_2) = -(tE_1 + (s + t)E_2)^2 + 2E_1s(s + t)E_2 - tE_1 - s^2E_2^2 + s^2t + st^2$. The $s$, $t$ and $u$ are the Mandelstam variables. Inserting Jüttner distribution function of partons and the squared matrix elements $\sum |M|^2$ in (3), we obtain the expressions of the photon production rates. We have for quark annihilation reaction

$$E \frac{dR}{d^3p} |_{\text{ann}} = \frac{5 \alpha \alpha_s \lambda_q \lambda_{\gamma g}}{36 \pi^5 E} \int ds \int dt$$

$$\times \frac{dE_1}{2E_1} \frac{dE_2}{2E_2}$$

$$\times \frac{1 - f_3(E_1 + E_2 - E)}{e^{(E_1+\mu_q)/T} + \lambda_q} \theta(P(E_1, E_2))$$

$$\times (P(E_1, E_2))^{-1/2},$$  (4)

and for Compton scatterings

$$E \frac{dR}{d^3p} |_{\text{com}}$$

$$= -\frac{5 \alpha \alpha_s \lambda_g}{36 \pi^5 E} \int ds \int dt$$

$$\times \frac{dE_1}{2E_1} \frac{dE_2}{2E_2}$$

$$\times \frac{1 - f_3(E_1 + E_2 - E)}{e^{(E_1+\mu_q)/T} + \lambda_q} \theta(P(E_1, E_2))$$

$$\times (P(E_1, E_2))^{-1/2}.$$  (5)

where $\alpha$ is the fine-structure constant, the running coupling constant $\alpha_s = 0.4$, the plus sign is for the Compton process $g\bar{q} \to \gamma q$ and the minus for the Compton process $gq \to \gamma q$.

Arnold, Moore, and their co-workers [24,25] have studied the hard photon production from near-collinear bremsstrahlung and IPA processes in an equilibrated QGP at zero quark chemical potential. They have pointed out that a correct treatment of the near-collinear bremsstrahlung and IPA which contribute to the leading-order emission rate requires a summation of all ladder graphs of the form shown in Fig. 3 of [25]. The total photon emission rate is given by an integral over all particle momenta of the emission rate for that particle momentum. Factoring out the same coefficient $A(E)$ which appears in the leading logarithmic result (1,2) of [25], we obtain the contribution of bremsstrahlung and IPA processes to the leading-order emission rate, as shown by (2.1) of [25]. Similar to the treatment in the preceding part, we extend these expressions to the chemically equilibrating QGP system at finite baryon density to explore the effect of the quark chemical potential on the photon production from the bremsstrahlung and IPA processes. Determining dimensionless transverse momenta $\tilde{p}_T \equiv p_T/m_D$ and $\tilde{q}_T \equiv q_T/m_D$, taking the dimensionless ratio of thermal masses $\kappa \equiv m_\gamma/m_D$, and noting that the argument of the $p_1$ integral is invariant under $p_1 \to -E - p_1$, the equation of photon production rate is rewritten as [25]
Thus, we get the integral value \( (P_\parallel, E) = 2Q_{\text{extr}} \). Finally, inserting Jüttner distributions in (6), from (6) we gain the photon production rates of the bremsstrahlung and IPA processes, respectively.

We integrate above photon production rates over the space-time volume of the reaction according to Bjorken’s model. The volume element \( d^4x = d^2x_T d\tau dy d\tau \), where \( \tau \) is the evolution time of the system and \( y \) the rapidity of the fluid element. We consider only central collisions so the integration over transverse coordinates just yields a factor of \( d^2x_T = \pi R_A^2 \), where \( R_A \) is the nuclear radius. Eqs. (4), (5), and also (6) can be related to the rapidity via \( d\tau / p/E = dp_T dY = d^2p_T dY \). We finally obtain the photon spectra

\[
\frac{dn}{d^2p_T} = \pi R_A^2 \int \tau d\tau \int dy \int dY \left( E - \frac{dn}{d^2p d^4x} \right),
\]

where \( Y \) is the rapidity of photons.

In this work, we focus on discussing Au\(^{197} + Au^{197} \) central collisions at the RHIC energies. Authors of [5] have calculated non-equilibrium initial conditions from perturbative QCD within Glauber multiple scattering theory for \( \sqrt{s} = 200 \) A GeV. Taking into account higher order contribution by a factor \( K = 2.5 \) from comparison with experiment at the RHIC, they have obtained the energy density and number densities of gluons, quarks, anti-quarks as well as the initial temperature \( T_0 = 0.552 \) GeV. From these densities we have also obtained initial temperature \( T_0 = 0.566 \) GeV and initial quark chemical potential \( q_0 = 0.285 \) GeV based on thermodynamic relations of the chemically equilibrating QGP system at finite baryon density, as mentioned in the preceding part, at \( \lambda_{q0} = 0.09, \lambda_{q0} = 0.02 \) and \( \lambda_{s0} = 0.01 \). Obviously, these two initial temperatures are near the one from HiJing model calculation, as shown in Ref. [15]. With the help of [15] we take initial values: \( T_0 = 0.70 \) fm, \( T_0 = 0.57 \) GeV, \( \lambda_{q0} = 0.09, \lambda_{q0} = 0.02 \) and \( \lambda_{s0} = 0.01 \). To further understand the effect of finite baryon density on the photon production, we have solved the set of coupled relaxation equations for initial quark chemical potentials \( q_0 = 0.000, 0.285, 0.570 \) and 0.855 GeV, and obtained the evolutions of the temperature \( T \), quark chemical potential \( q_0 \) and fugacities \( \lambda_0, \lambda_0 \) and \( \lambda_0 \) of the system. The calculated evolution paths of the system in the phase diagram have been shown in
The calculated evolution paths of the system in the phase diagram for initial values \( \tau_0 = 0.70 \), \( T_0 = 0.57 \text{ GeV} \), \( \lambda_{g0} = 0.09 \), \( \lambda_{q0} = 0.02 \) and \( \lambda_{s0} = 0.01 \), where the solid, dotted, dashed and dash-dotted lines are, in turn, the evolution paths for initial quark chemical potentials \( \mu_{q0} = 0.000, 0.285, 0.570 \) and 0.855 GeV, and the dot-dot-dashed line is the phase boundary. The time interval between small circles is 0.6 fm (i.e., \( 30 \times \) calculation-step 0.02 fm). The phase diagram is calculated at \( B^{1/4} = 0.25 \text{ GeV} \).

The calculated spectra for the \( q\bar{q} \rightarrow \gamma g \), \( gq \rightarrow \gamma q \) and \( g\bar{q} \rightarrow \gamma\bar{q} \) processes are, in turn, shown in panels (a), (b) and (c) of Fig. 3, for bremsstrahlung and IPA processes in panels (a) and (b) of Fig. 4. With the increase of the initial quark chemical potential the anti-quark density goes down, thus the photon production for the \( q\bar{q} \rightarrow \gamma g \) process calculated from Eq. (4) will go down. One has well known that the baryon-free QGP converts into the hadronic matter only with decreasing the temperature along the temperature axis of the phase diagram, and the phase transition occurs at a certain critical temperature \( T_c \). However, in this work, both the quark chemical potential and the temperature of the system are functions of time, compared with the baryon-free QGP it necessarily takes a long time for value \((\mu_q, T)\) of the system to reach a certain point of the phase boundary to make the phase transition. Furthermore, in the calculation we have found that with increasing the initial quark chemical potential the production rate of gluons goes up, and thus their equilibration rate goes down, leading to the little energy consumption of the system, i.e., slow cooling of the system. Since gluons are much more than quarks in the system, with increasing the initial quark chemical potential the cooling of the system further slows down. These cause the quark phase lifetime to further increase, for example the calculated lifetimes of the quark phase for initial quark chemical potentials \( \mu_{q0} = 0.000, 0.285, 0.570 \) and 0.855 GeV are, in turn, about 3.42, 3.58, 3.82 and 4.22 fm. These evolution features of the system can be clearly seen in Fig. 1. With the help of Eq. (11) one can note that these factors obviously heighten the contribution of the integration over the evolution of the system to the photon production. Thus the calculated spectra for the...
Fig. 3. The calculated photon spectra for processes $q\bar{q} \rightarrow g\gamma$, $gq \rightarrow \gamma q$ and $g\bar{q} \rightarrow \gamma \bar{q}$ are, in turn, shown in (a), (b) and (c) panel. The solid, dotted, dashed and dot-dashed lines are, respectively, the calculated spectra for initial quark chemical potentials $\mu_q = 0.000$, $0.285$, $0.570$ and $0.855$ GeV based on the evolution described in Fig. 1.

$q\bar{q} \rightarrow \gamma g$ from Eqs. (4) and (11) still go somewhat up with increasing the initial quark chemical potential, as seen in panel (a) of Fig. 3. One also knows that since with increase of the initial quark chemical potential the quark density goes up and anti-quark density goes down, the photon production calculated by Eq. (5) for the process $g\bar{q} \rightarrow \gamma \bar{q}$ will go down, and for the process $gq \rightarrow \gamma q$ will go up. The calculations of Eq. (11) have shown that due to the effect of the evolution of the system, with increasing the initial quark chemical potential the increase of the photon production for the process $gq \rightarrow \gamma q$ becomes even faster, and the decrease of the photon production for the process $g\bar{q} \rightarrow \gamma \bar{q}$ obviously slows down, as shown in panels (b) and (c) of Fig. 3. Inserting Jüttner distributions in (6), we note that in (6) the part relating to the quark chemical potential is only the factor $1/[e^{(p_{t}\parallel + E - \mu_q)/T} + 1]e^{(p_{t} - \mu_q)/T}/[e^{(p_{t} - \mu_q)/T} + 1]$. For the bremsstrahlung process, the dependence of the photon production on the quark chemical potential is mainly given by the factor $1/[e^{(p_{t} + E - \mu_q)/T} + 1]e^{(p_{t} - \mu_q)/T} / [e^{(p_{t} - \mu_q)/T} + 1]$. This factor is the product of
the two quark distribution functions. With increasing the quark chemical potential the quark density of the system goes up, thus photon production of the bremsstrahlung process goes rapidly up, too, as seen in panel (a) of Fig. 4. Compared with the process $gq \rightarrow \gamma q$, we have found that the contribution of the bremsstrahlung process is quite near the one of the $gq \rightarrow \gamma q$ process at the lower $p_T$ region, and larger than the one of the $gq \rightarrow \gamma q$ process at the higher $p_T$ region. For the IPA process, the factor $e^{(p_\| - \mu_q)/T}$ becomes important. Due to the integration from $-E/2$ to zero, the factor $e^{(p_\| - \mu_q)/T}$ is indeed an anti-quark distribution function. With increasing the quark chemical potential the anti-quark density goes down, thus the photon production of the IPA process goes down, as shown in panel (b) of Fig. 4. We find that the bremsstrahlung and $gq \rightarrow \gamma q$ processes make the total photon production as a strongly increasing function of the initial quark chemical potential. Overall we can see that the total photon production increases with the initial quark chemical potential, as shown in Fig. 5.

In conclusion, in order to reveal the effect of finite baryon density on the photon production in a chemically equilibrating QGP system which is produced from Au$^{197} + $ Au$^{197}$ central collisions at the RHIC energies, we have first extended those expressions of the photon production for the Compton ($gq \rightarrow q\gamma$ and $g\bar{q} \rightarrow \bar{q}\gamma$), annihilation $q\bar{q} \rightarrow g\gamma$, bremsstrahlung and IPA processes in [2,11,25] to the chemically equilibrating system at finite baryon density. Subsequently, based on the evolution of the chemically equilibrating QGP system at finite baryon density, we have computed the photon production of the system. We have found that the increase of the initial quark chemical potential will change the hydrodynamic behavior of the system to cause the quark phase lifetime to increase. This effect is to heighten the photon yield of the Compton ($gq \rightarrow q\gamma$ and $g\bar{q} \rightarrow \bar{q}\gamma$), annihilation $q\bar{q} \rightarrow g\gamma$, bremsstrahlung and IPA processes. On the other hand, the increase of the initial quark chemical potential will directly heighten the photon yield of the Compton $gq \rightarrow q\gamma$ and bremsstrahlung processes. It is shown that the bremsstrahlung and Compton $gq \rightarrow q\gamma$ are two main processes of the photon production. They not only compensate the photon suppression of processes $g\bar{q} \rightarrow \bar{q}\gamma$, $q\bar{q} \rightarrow g\gamma$, and IPA caused by the increase of the initial quark chemical potential, but also make the total photon yield as a strongly increasing function of the initial quark chemical potential. Since photons are produced in an evolving QGP system, obviously for comparison with the experiment it is necessary that one study the photon production in the evolving QGP system. In this work, our results has demonstrated the importance of the system evolution for the photon production.

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