Role of $\rho$NN tensor coupling and $2s_{1/2}$ occupation in light exotic nuclei

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We study the properties of light neutron-rich exotic nuclei with relativistic-mean-field models. The nonlinear isoscalar-isovector term that is not constrained by the present data is considered. The $\rho$NN tensor coupling is included. The data-to-data correlation between the neutron radius of $^{208}$Pb and that of light exotic oxygen, fluorine, and neon isotopes is enhanced by the inclusion of the $\rho$NN tensor coupling subject to the $2s_{1/2}$ occupation of the outlayer neutrons. This enhancement is determined by the single-particle property rather than by the bulk-matter property. The role of the $\rho$NN tensor coupling and the pairing correlation in reducing the model dependence of the neutron thickness in light exotic nuclei is investigated. The influence of the isoscalar-isovector term on halo neutrons, characteristic of the $2s_{1/2}$ occupation, is discussed.

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1. INTRODUCTION

Experimental and theoretical studies of exotic nuclei with extreme isospins are hot points in current nuclear research [1]. The available and future radioactive beams provide opportunities for studying the structure properties of exotic nuclei and the underlined basic interactions at low densities. The density dependence of isospin-dependent interactions is important in describing the nuclear equation of state (EOS) and nuclear phenomenology.

The EOS comprises two components: the saturation property of symmetric matter and the symmetric energy. The saturation property of symmetric matter is better constrained, whereas the density-dependence of the symmetry energy is much more poorly known to date [2,3]. Many factors, such as the isoscalar-isovector coupling terms [4], the isovector-scalar mesons, and the density-dependent coupling constants [5,6], can modify the behavior of the density dependence of the symmetry energy. Recently the density dependence of the symmetry energy has been extensively explored through the inclusion of isoscalar-isovector coupling terms [4,7–9]. These new terms enable one to modify the neutron skin of heavy nuclei without changing the charge radius and a variety of ground-state properties that are well constrained experimentally.

Horowitz and colleagues studied the data-to-data relationships between the neutron-rich skin of $^{208}$Pb and the properties of neutron stars [4,7–9]. It is surprisingly interesting that the neutron radius of $^{208}$Pb is around 5.5 fm whereas the radius of a neutron star is about 10 km. The underlying physics is the EOS. For instance, the pressure of neutron-rich matter that supports the neutron star against the gravitation balances the surface tension of a heavy nucleus to give the neutron skin thickness [4]. Theoretical estimates may give an uncertainty of the neutron radius of $^{208}$Pb by about 0.3 fm [10]. An accurate measurement of the neutron radius of $^{208}$Pb is important to give information of the density dependence of the symmetry energy and hence the properties of neutron stars. In Refs. [11,12], it was pointed out that the measurement of the neutron thickness of $^{208}$Pb, aimed at constraining the density dependence of the symmetry energy, would be also instrumental in determining the compression modulus of nuclear matter.

Besides the bulk nuclear properties studied through the inclusion of the isoscalar-isovector coupling terms, in Ref. [13] the data-to-data relationship between the binding energy of valence neutrons for some neutron-rich nuclei and the neutron thickness of $^{208}$Pb was established. In our previous study [14], we found that the dependence of the charge distribution of $^{23,24}$O on the isoscalar-isovector coupling can be sensitive to the neutron occupancy of the outlayer $2s_{1/2}$ orbital. The $2s_{1/2}$ orbital plays an important role in the formation of the halo structures in neutron-rich fluorine isotopes such as $^{23,24}$F [15,16]. We will look into the modification, caused by the inclusion of the isoscalar-isovector coupling term, to the nucleon distributions, halos, and single-particle properties for those exotic isotopes with the outlayer neutron occupation in $2s_{1/2}$. The light exotic nuclei such as $^{23,24}$O have a high neutron fraction. As the relationships established for the properties of neutron stars and the neutron thickness of $^{208}$Pb show a clear model dependence (e.g., see Ref. [8]), it is interesting to see whether it exists the model dependence for the relationship established between the heavy nucleus and light exotic nuclei.

The neutron and proton halos are usually identified through the measurement of the reaction cross sections and longitudinal momentum distributions of nucleus breakup. To obtain more definite information of halo nuclei, a pure probe that excludes the influence of the strong interaction is preferred. Therefore, an accurate measurement of the neutron radius for $^{208}$Pb at the Jefferson Laboratory [17] that promises a 1% accuracy is of significance for the light halo nuclei in the sense that the relationship between the nucleon radius of halo nuclei and the neutron thickness of $^{208}$Pb is well established. Actually, the significance of the establishment of such relationships becomes clear that the properties of light nuclei far from $\beta$ stability are also related to the poorly known density dependence of the symmetry energy.

We take into account the isoscalar-isovector terms that modify the density dependence of the symmetry energy in
the framework of the relativistic-mean-field (RMF) models. In a RMF model, the ρNN tensor coupling that does not change the EOS of nuclear matter plays its role in finite nuclei [14]. The ρNN tensor coupling is included in this study.

The paper is organized as follows. Section II presents the brief RMF formalism that includes the ρNN tensor coupling. The modification to the spin-orbit potential that is due to the ρNN tensor coupling is stressed. In Sec. III, we present calculated results for $^{208}$Pb and light exotic isotopes with the 2$s_{1/2}$ occupation. A brief summary is given in Sec. IV.

II. FORMALISM

The effective Lagrangian density is given as

$$\mathcal{L} = \frac{1}{2} \left[ \dot{\psi}^\dagger \dot{\psi} - M_N^2 + g_\sigma \psi^\dagger \sigma \psi - g_\omega \psi^\dagger \omega \psi - g_\rho \psi^\dagger \rho \psi \right] + \frac{f_\rho}{2 M_N} \partial_\mu \psi^\dagger \partial^\mu \psi - \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\dagger \omega + \frac{1}{2} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^\dagger \rho + \frac{1}{4} A_{\mu\nu} A^{\mu\nu},$$

where $\psi$, $\sigma$, $\omega$, and $\rho$ are the fields of the nucleon, scalar, vector, and charge-neutral isovector-vector mesons, with their masses $M_N$, $m_\sigma$, $m_\omega$, and $m_\rho$, respectively; $A_\mu$ is the field of the photon; $g_i$ ($i = \sigma, \omega, \rho$) and $f_\rho$ are the corresponding meson-nucleon couplings; $\sigma$ and $\tau$ are the isospin Pauli matrix and its third component, respectively; $F_{\mu\nu}$, $B_{\mu\nu}$, and $A_{\mu\nu}$ are the strength tensors of the $\omega$, $\rho$, and photons, respectively, and

$$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad B_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \quad A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  

The self-interacting $\sigma$, $\omega$-meson terms and the isoscalar- isovector coupling terms are

$$U(\sigma, \omega^\dagger, \rho^\dagger) = 2 g_2 \sigma^2 + \frac{1}{2} g_3 \sigma^4 - \frac{1}{2} c_3 (\omega^\dagger \omega)^2 - 4 g_\rho^2 (g_8^2 \Lambda_\sigma \sigma^2 + g_8^2 \Lambda_\rho \rho^\dagger \rho) b_0^\dagger b_0^\dagger.$$  

In the following we present the field equations in RMF. The Dirac spinor may be denoted by a quantum number collection $\{a\} = \{n, \kappa, j, t, m\}$:

$$\psi_a(r) = \begin{cases} \frac{i}{2} [G_a(r)/r] \Phi_{+m} \chi_t, & \text{for } \sigma, \\ \{-F_a(r)/r\} \Phi_{-m} \chi_t, & \text{for } \omega, \rho, \text{ and } \text{photon}. \end{cases}$$

where $G$ and $F$ are the radial spinors, $\Phi$ is the spherical harmonics, $\chi$ is the isospinor with $t = 1$ for proton and $t = -1$ for neutron, $n$, $j$, and $m$ are the principal, total angular momentum, and magnetic quantum numbers, respectively, and $\kappa$ is subsequently defined. The following relations are useful for deriving the radial Dirac equations:

$$\alpha \cdot \textbf{p} = \frac{\alpha r}{r^2} (\alpha \cdot \textbf{r}) (\alpha \cdot \textbf{p}) = \frac{\alpha r}{r} (p_r + i \gamma^0 \textbf{K}),$$

$$\sigma_r \Phi_{+m} = -\Phi_{-m}. $$

with

$$p_r = -i \left( \frac{\partial}{\partial r} + \frac{1}{r} \right), \quad \alpha_r = \frac{1}{r} \alpha \cdot \textbf{r},$$

where $\sigma_r$ is the projection of the Pauli matrix to the $\textbf{r}$ direction. The operator $\textbf{K}$, defined as

$$\textbf{K} = \gamma^0 (\textbf{\Sigma} \cdot \textbf{L} + 1),$$

is another eigenoperator of the Hamiltonian with its eigenvalue

$$\kappa = \begin{cases} j + 1/2, & \text{for } j = l - 1/2, \\ -(j + 1/2), & \text{for } j = l + 1/2. \end{cases}$$

Performing the variation for the Hamiltonian, one obtains the radial Dirac equations as

$$\frac{d G_a}{dr} = [M_a^n(r) - V(r) + E_a] F_a - \frac{\kappa}{r} U_a^\dagger(r) \Phi_{+m},$$

$$\frac{d F_a}{dr} = [M_a^n(r) + V(r) - E_a] G_a + \frac{\kappa}{r} U_a^\dagger(r) \Phi_{-m}.$$  

where

$$M_a^n(r) = M_N - V^\sigma(r), \quad V(r) = g_\omega(r) b_0^\dagger b_0 + g_\rho b_0^\dagger b_0 + e \left( \frac{1 + r}{2} \right) A_0 - U_a^\dagger(r).$$  

The boson fields $\phi = \sigma, \omega, \rho,$ and photons obey the following equations in the spherical nuclei:

$$\left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - m_\phi^2 \right) \phi(r) = -s_\phi(r).$$

where for the photon, $m_\phi = 0$, and

$$s_\phi(r) = \begin{cases} g_\sigma \rho_\sigma + g_\omega \rho_\omega + g_\rho \rho_\rho, & \text{for } \sigma, \\ g_\omega \rho_\omega - c_3 \rho_\omega - 8 g_8^2 \Lambda_\sigma \omega_\sigma b_0, & \text{for } \omega, \\ g_\rho \rho_\rho - 8 g_8^2 \omega_\rho b_0 + 4 g_8^2 \rho_\rho, & \text{for } \rho, \\ e \rho_\rho = e \left( \frac{2}{4\pi} + \frac{2}{3} \right) (G^2_a(r) + F^2_a(r))(1 + t)/2, & \text{photon}. \end{cases}$$
In the RMF approximation for spherical nuclei, only the temporal component of the vector meson survives. The boson fields can be solved through the Green’s functions:

\[ \phi(r) = \int_0^\infty dr_1 r_1^2 G_\phi(r, r_1)s_\phi(r_1). \]  

(14)

For the massive fields, one has,

\[ G_\phi(r, r_1) = m_\phi j_0(i m_\phi r_<) h_0^+(i m_\phi r_>). \]  

(15)

with

\[ j_0(i m r) = \frac{1}{2 m r}(e^{m r} - e^{-m r}), \quad h_0^+(i m r) = -\frac{1}{m r}e^{-m r} \]  

and

\[ r_< = \min[r, r_1], \quad r_> = \max[r, r_1]. \]

For the photon, the Green’s function is

\[ G_\phi(r, r_1) = \begin{cases} 1/r, & \text{as } r > r_1 \\ 1/r_1, & \text{as } r < r_1. \end{cases} \]

The tensor potentials related to the \( \rho NN \) tensor coupling are given by

\[ U_1^\rho(r) = -\frac{f_\rho}{M_N^2} g_\rho \int_0^\infty dr_1 r_1^2 G_\rho(r, r_1)\rho_T(r_1), \]

\[ U_2^\rho(r) = -\frac{f_\rho^2}{2M_N^2} g_\rho \int_0^\infty dr_1 r_1^2 \frac{dG_\rho(r, r_1)}{dr} \times \left[ \rho_\rho(r_1) - 8 g_\rho \left( g_\rho^2 \Lambda_\sigma^2 + g_\rho^2 \Lambda_\omega^2 \right) h_0 \right], \]

\[ U_3^\rho(r) = -\frac{f_\rho^2}{2M_N^2} \int_0^\infty dr_1 r_1^2 \frac{dG_\rho(r, r_1)}{dr} - \rho_T(r_1), \]

\[ \rho_T(r) = \sum_\alpha \frac{2 j_\alpha}{4 \pi \tau^2} \frac{d}{dr} [G_\alpha(r)F_\alpha(r)]. \]

The spin-orbit potential can be obtained from the equation for the big component \( G_\alpha \) by elimination of the small component \( F_\alpha \) from Eqs. (9) and (10). Without the tensor coupling, the spin-orbit potential is given as

\[ U_{ls} = \frac{1}{2M_N^2} \frac{d}{dr} [V_\sigma(r) + V(r)] \hat{\mathbf{L}} \cdot \mathbf{S}, \]

(20)

with \( M_N = M^* + E_\alpha - V(r) \approx 2M - [\sigma^2 + V(r)]. \) As the \( \rho NN \) tensor coupling is included, the \( U_{ls} \) is modified as

\[ U_{ls} = \left\{ \frac{1}{2M_N^2} \frac{d}{dr} [V_\sigma(r) + V(r)] + \frac{U_1^\rho + U_2^\rho}{M_N r} \right\} \hat{\mathbf{L}} \cdot \mathbf{S}, \]

(21)

The modification to the spin-orbit potential can be expected to be considerable for the large tensor potentials, and the latter may possibly appear as the single-particle motions cause the large shift in density distributions together with the high neutron-to-proton asymmetry, and the isoscalar-isovector coupling terms are involved. Besides the modification to the spin-orbit potential, the tensor potentials may also modify the orbit-orbit potential.

The pairing correlation is involved in nonmagic nuclei by use of Bardeen-Cooper-Schrieffer (BCS) theory. For the procedure for involving BCS pairing correlations, the reader is referred to the literature, such as Ref. [18]. We take constant pairing gaps that are obtained from the prescription of Möller and Nix [19]: \( \Delta_n = 4.8/N^{1/3}, \Delta_p = 4.8/Z^{1/3}, \) where \( N \) and \( Z \) are the neutron and proton numbers, respectively. For odd nuclei, Pauli blocking is considered. The solution of the coupled Dirac and meson equations can be obtained in an iterative procedure that may easily found in the literatures, and it is not reiterated here.

### III. RESULTS AND DISCUSSIONS

We perform calculations with the NL3 [20] and S271 [4] parameter sets. These two models share the same binding energy per nucleon (16.24 MeV) and incompressibility (271 MeV) at the same Fermi momentum \( k_F = 1.30 \text{ fm}^{-1}. \) The difference of the two models is in the density dependence of the symmetry energy. The symmetry energy is given by [7]

\[ a_s = \frac{k_F^2}{6E_F^*} + \frac{g_\rho^2}{3\pi^2} \frac{k_F^4}{m_\rho^2}, \]

with \( E_F^* = \sqrt{k_F^2 + M^*} \) and \( m_\rho^2 = m_\sigma^2 + 8g_\rho^2(g_\sigma^2 \Lambda_\sigma^2 + g_\rho^2 \Lambda_\omega^2). \) The density dependence of the symmetry energy is dominated by the second term of Eq. (22). However, at low densities, the first term becomes important and the different effective nucleon mass \( M^* \) may bring about effect in the symmetry energy. For the NL3 model \( M^* = 0.59 \text{MeV} \) at saturation density, and \( M^* = 0.7 \text{MeV} \) for the S271 model. We look into the difference of the properties of light exotic nuclei described by these two models.

For simplicity, the isoscalar-isovector coupling \( \Lambda \) is set as zero, unless otherwise mentioned. For various isoscalar-isovector coupling \( \Lambda \)’s, we distinguish two cases, with \( \rho NN \) tensor coupling and without. The symmetry energy at saturation density is not well constrained, and hence some average of the symmetry energy at saturation density and the surface energy is constrained by the binding energy of nuclei. For a given coupling \( \Lambda \), we follow Refs. [4,13] to readjust the \( \rho NN \) constant \( g_\rho \) so as to keep an average symmetry energy fixed as 25.68 at \( k_F = 1.15 \text{ fm}^{-1}. \) In doing so, it was found in Ref. [4] that the binding energy of \( ^{208}\text{Pb} \) is nearly unchanged for various \( \Lambda \)’s. The \( \rho NN \) tensor coupled \( f_\rho \) is taken as \( f_\rho/g_\rho = 6.1, \) which is used in the Bonn potentials [21] and is within 8.0 ± 2.0 obtained from the light cone sum rules by Zhu [22]. A large \( \rho NN \) tensor coupling may be also obtained from the dispersion-theoretical analysis of the nucleon electromagnetic form factors [23,24] and the analysis of the NN scattering [25]. In principle, the NL3 parameters should be refitted because of the inclusion of the \( \rho NN \) tensor coupling. However, we may reasonably expect that the refitting will not bring about much change to the parameters that determine the major RMF features such as the large scalar and vector potentials and their density dependence, because the \( \rho NN \) tensor coupling just modifies the total binding energy up to a few mega-electron-volts and because the \( \rho NN \) tensor potentials are small compared with the nuclear potential. We will again touch on this point. The ratio of \( f_\rho \) to \( g_\rho \) \((f_\rho/g_\rho = 6.1, \) coupled with the relatively
large values of $g_\rho$ in many RMF models, might overestimate the contribution of the $\rho$ tensor coupling, and that will be discussed later.

The isoscalar-isovector term modifies the neutron thickness of $^{208}$Pb without compromising its success in reproducing the charge radius and the binding energy. This was verified by the NL3 and S271 parameter sets in Refs. [4,13]. We start the investigation from the inclusion of the $\rho NN$ tensor coupling. In $^{208}$Pb, it brings about negligible modifications due to the quite small tensor potentials. In Fig.1, we display the radius correlation of $^{138}$Ba and $^{208}$Pb. Without pairing correlations, it is the same as that shown in Ref. [13]. As the pairing correlation is on, a small model dependence appears. The tensor coupling plays a favorable role in removing the small model dependence.

In Fig. 2, we plot nucleon density distributions for $^{24}$O in NL3. Not only the sensitivity of the neutron density distribution but also that of the proton to the isoscalar-isovector coupling $\Lambda_\nu$ is clearly enhanced by the $\rho NN$ tensor coupling. The tensor potentials are modified by changing the $\Lambda_\nu$ as seen in Eq. (18), affecting the nucleon density distributions. For heavy nuclei like $^{208}$Pb, the tensor coupling has a negligible influence on the neutron and proton density distributions. In either case, whether the tensor coupling is included or not, the proton density distribution in $^{208}$Pb almost remains fixed compared with the modification to the neutron radius. As the tensor coupling does not change the EOS in RMF approximations, it is helpful to constrain the density dependence of the symmetry energy through the enhanced sensitivity of the nucleon density distributions to the isoscalar-isovector coupling that influences the density dependence of the symmetry energy.

In Fig. 3, the data-to-data correlation between the neutron thickness of $^{23,24}$O and that of $^{208}$Pb is displayed. It is shown that the sensitivity to the $\Lambda_\nu$ is enhanced because of the inclusion of the $\rho NN$ coupling, and the correlation between the neutron thickness of $^{24}$O and $^{208}$Pb is more sensitive to the $\Lambda_\nu$ than that established for $^{23}$O and $^{208}$Pb. The enhanced sensitivity that is due to the inclusion of the $\rho NN$ tensor coupling is related to the neutron occupation in the $2s_1/2$ orbital, as pointed out in Ref. [14]. In Table I, we present the occupancy of the $2s_1/2$ neutrons in $^{24}$O as a function of $\Lambda_\nu$ and $f_\rho$. It shows that the $2s_1/2$ occupancy is just slightly increased by the inclusion of the isoscalar-isovector and $\rho NN$ tensor couplings. The change rate of the tensor potentials against the $\Lambda_\nu$, related to the slope shown in Fig. 3, may roughly be obtained from an integral from Eq. (18) that shows an explicit and linear dependence on $\Lambda_\nu$. Considering the value of such an integral is related to the $2s_1/2$ occupancy, the different $2s_1/2$ 

In Fig. 1, the correlation of the neutron thickness of $^{208}$Pb and that of $^{138}$Ba for models NL3 and S271.

FIG. 1. The correlation of the neutron thickness of $^{208}$Pb and that of $^{138}$Ba for models NL3 and S271.

FIG. 2. Neutron and proton density distributions for $^{24}$O in NL3 with the various isoscalar-isovector coupling $\Lambda_\nu$. The lower panel does not include the $\rho NN$ tensor coupling.

FIG. 3. The correlation of the neutron thickness of $^{208}$Pb and that of $^{23,24}$O.

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TABLE I. The occupancy of the $^{2s}_{1/2}$ neutrons in $^{23,24}$O as a function of $\Lambda_s$ and $f_\rho$ in NL3. In $^{23}$O, it is 0.5.

<table>
<thead>
<tr>
<th>$\Lambda_s$</th>
<th>0</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\rho = 0$</td>
<td>0.832</td>
<td>0.838</td>
<td>0.841</td>
<td>0.842</td>
</tr>
<tr>
<td>6.1$g_\rho$</td>
<td>0.838</td>
<td>0.844</td>
<td>0.848</td>
<td>0.851</td>
</tr>
</tbody>
</table>

occupancies for $^{23,24}$O thus lead to the different slopes shown in Fig. 3.

We make a brief analysis of the overestimation of the contribution of the $\rho NN$ tensor coupling previously mentioned. As the value of $f_\rho$ is decreased by 20%, the slope of the neutron thickness correlation, shown in Fig. 3, is just slightly reduced. In this sense, the importance of the $\rho NN$ tensor coupling in present results is not much underestimated. It is mainly the special nuclear structure that brings out the large contribution of the $\rho NN$ tensor coupling. In Fig. 5, it shows that the wave function of $^{2s}_{1/2}$ state, together with its long tail, in $^{24}$O is little affected by the $\rho NN$ tensor and isoscalar-isovector couplings. However, because of the existence of the node at the interior of the core (at $r \approx 2$ fm), the density of the $^{2s}_{1/2}$ neutrons changes rapidly against the radius within the node and in the vicinity, which produces a large contribution to tensor potentials and the corresponding spin-orbit potential at the core region. This causes a consistent modification to the nucleon density distributions shown in Fig. 2.

As seen in Eq. (21), the single particle energies can be modified due to the inclusion of the $\rho NN$ tensor coupling. In Fig. 5, we display the single neutron energies and the spin-orbit splitting of $1p$ state as a function of the neutron thickness of $^{208}$Pb. The modification that is due to the tensor coupling, shown in the upper panel of Fig. 5, seems important not only for the $^{2s}_{1/2}$ nucleons but also for nucleons inside the Fermi core. It is interesting to find that the $1d_{5/2}$ orbital, closest to the $^{2s}_{1/2}$,

![FIG. 4. Big and small components of the wave function of the $2s_{1/2}$ neutron state in $^{24}$O with the NL3. The inset is for the density of the $2s_{1/2}$ neutrons.](image1)

![FIG. 5. Single-particle energies for $^{24}$O with respect to the neutron thickness of $^{208}$Pb in NL3. The upper panel presents the energy difference $E_n(f_\rho/g_\rho = 6.1) - E_n(f_\rho/g_\rho = 0)$ for various neutron levels. The nucleon spin-orbit splittings of $1p$ orbitals are shown in the middle and bottom panels, with the inclusion of the $\rho NN$ tensor coupling (solid line) and without (dashed line).](image2)

![FIG. 6. The spin-orbit splitting of $1d$ orbitals in $^{48}$Ca with respect to the neutron thickness of $^{208}$Pb in NL3, with the inclusion of the $\rho NN$ tensor coupling (solid lines) and without (dashed lines).](image3)

Because we do not refit the NL3 parameters after introducing the $\rho NN$ tensor coupling, we would like to see the influence of the $\rho NN$ tensor coupling on the $1d_{5/2} - 1d_{4/2}$ splitting in $^{48}$Ca. In Fig. 6, the $1d_{5/2} - 1d_{4/2}$ splitting is displayed as a function of the neutron thickness of $^{208}$Pb. As shown in
In Fig. 6, the influence of the tensor coupling on the splitting is small for $\Lambda_v = 0$. Although the influence of the tensor coupling enlarges with the inclusion of $\Lambda_v$, it is much less than that for $^{24}\text{O}$ (see Fig. 5). It is interesting to point out that the proton $1d_{5/2} - 1d_{3/2}$ splitting in $^{48}\text{Ca}$ is partly related to the proton occupation of the $2s_{1/2}$ orbital. The calculated proton $1d_{5/2} - 1d_{3/2}$ splitting in $^{48}\text{Ca}$ with the NL3 is $\approx 1$–1.3 MeV larger than the experimental ones [26]. The inclusion of the tensor coupling is intended to reduce the deviation of the proton $1d_{5/2} - 1d_{3/2}$ splitting in $^{48}\text{Ca}$ from the experimental one unless the model is readjusted to give a smaller $1d_{5/2} - 1d_{3/2}$ splitting. Despite the fact that the relation between the neutron $1d_{5/2} - 1d_{3/2}$ splitting and the $\Lambda_v$ is modified because of the inclusion of the tensor coupling, the relative influence of the proton $1d_{5/2} - 1d_{3/2}$ splitting is quite small. The uncertainties of the charge radii in $^{48}\text{Ca}$ are about 1/5 and 1/7 of those of the neutron ones with and without the inclusion of the proton tensor coupling, respectively. The uncertainty of the charge radius is just slightly increased by the tensor coupling, and the charge radius of $^{48}\text{Ca}$ seems still to be almost a constant against $\Lambda_v$.

As pointed out in Refs. [15,16], the fluorine isotopes in which the outlayer neutrons occupy the $2s_{1/2}$ orbital are possible candidates for halo nuclei. The $2s_{1/2}$ orbital plays an important role in the formation of the halo. For this aspect, F isotopes such as $^{24,25}\text{F}$ are similar to $^{23,24}\text{O}$. For the experimental aspect, the fluorine isotopes may be more suitable since the fluorine chain is stable up to $^{31}\text{F}$ [27], whereas, for the oxygen chain, $^{24}\text{O}$ seems to be the heaviest stable isotope. The result of our calculations is not consistent with this. However, it is interesting to find that the total binding energy of $^{28}\text{O}$ relative to $^{24}\text{O}$ decreases with the increasing $\Lambda_v$ (namely, with the decreasing $r_n - r_p$ for $^{208}\text{Pb}$) for both cases with and without $f_p$, as shown in Fig. 7. It implies that a correct description for the energy difference of the two isotopes would be relevant to the appropriate inclusion of isospin-dependent interactions. In Fig. 7, we also present the total binding energy of $^{28}\text{O}$ relative to $^{28}\text{O}$ with respect to the $\Lambda_v$, and it shows that the decreasing trend seems more rapid compared with that against the $\Lambda_v$.

In the following, we look into the modification that is due to the isoscalar-isovector coupling to $^{24,25}\text{F}$. The nucleon density distributions for $^{24,25}\text{F}$ are displayed in Fig. 8. The larger $2s_{1/2}$ neutron occupancy in $^{25}\text{F}$ than that in $^{24}\text{F}$ produces a visible difference in the nucleon density distributions for various $\Lambda_v$, which is similar to what occurred in oxygen isotopes.

We show the relationship between the neutron thickness of $^{208}\text{Pb}$ and that of fluorine isotopes in Fig. 9. Corresponding to the increase of the $2s_{1/2}$ occupancy near $N = 16$, the...
sensitivity on the isoscalar-isovector coupling is enhanced because of the inclusion of the $\rho NN$ tensor coupling, similar to what happened in oxygen isotopes. As the isospin goes on increasing, the enhancement that is due to the $\rho NN$ tensor coupling is largely weakened. In this case, we even find from the right-hand bottom panel that the neutron skin of isotopes with extremely high isospin shows a quite weak dependence on the isoscalar-isovector coupling, which is different from an empirical understanding. Empirically, the higher isospin gives rise to a larger source term for the isovector meson and correspondingly a larger contribution of isoscalar-isovector coupling. The abnormality exhibited in fluorine isotopes with $N > 17$ (e.g., $^{31}\text{F}$) is attributed to the diffusive neutron distribution at low densities, where the EOS is little changed by the isoscalar-isovector term [13]. A similar abnormality also occurs in oxygen isotopes [14]. The slowly varying neutron density distribution with respect to the radius leads to quite small tensor potentials. The sharp variation of the density distribution of the $2s_{1/2}$ orbital on both sides of the node in heavier isotopes may produce a cancellation in the tensor potentials [14]. On the other hand, the density distribution from the $2s_{1/2}$ neutrons may be smeared out by the filling of outlayer neutrons.

The results in Fig. 9 show an explicit model dependence, and this model dependence is also shown in Fig. 3 for $^{23,24}\text{O}$. In these figures, it is shown that the role of the $\rho NN$ tensor coupling seems favorable in reducing the model dependence. In the upper left-hand and lower right-hand panels of Fig. 9 and in the upper panel of Fig. 3, the slopes obtained from the NL3 and S271 models are explicitly shifted to approximately parallel by inclusion of the tensor coupling. The importance of this shift can be understood in the following way. One may obtain an approximately linear (data-to-data) correlation between the excitation energy of the isoscalar giant monopole resonance and the neutron skin (see Ref. [12]). This means that models (e.g., NL3 and S271) with the same incompressibility that produce different correlation slopes are difficult to be scaled to have the same incompressibility. For $^{24}\text{O}$ and $^{25,26}\text{F}$, the inclusion of the tensor coupling explicitly reduces the model dependence of the neutron thickness correlation, as shown in Figs. 3 and 9. The role of the tensor coupling observed in Figs. 1, 3, and 9 seems to improve the zero-order $\rho NN$ Yukawa coupling.

In heavy nuclei such as $^{138}\text{Ba}$, shown in Fig. 1, the pairing correlation can cause a small inconsistency between two different models. In Fig. 10, we show that the model dependence is considerably shifted when the pairing correlation is turned off, as compared with Fig. 9. For light exotic nuclei with high isospins, the consideration of the pairing correlation is important because the filling near the Fermi surface is close to the continuum and because the density of the energy level becomes large. A consideration of using the state-dependent pairing gap may be necessary. Appropriate consideration of the pairing correlation together with the $\rho NN$ tensor coupling would reduce the model dependence of the neutron thickness correlation. Another factor that affects the model dependence shown in Figs. 3 and 9 is the various density dependences of the symmetry energy provided by the two models. Following Eq. (22), we discussed the influence of the effective nucleon mass on the density dependence of the symmetry energy. On the other hand, the S271 model does not provide a prediction for the total binding energy that is as accurate as the NL3 model. These factors also influence the model dependence observed in Figs. 3 and 9. Nevertheless, the model dependence for $^{24,25}\text{F}$ shown in Fig. 9 and for $^{23,24}\text{O}$ shown in Fig. 3 is considerably reduced with the inclusion of the $\rho NN$ tensor coupling and the pairing correlation. The results for these nuclei are useful for providing the constraint on the density dependence of the symmetry energy as combined with the future experiments.

The oxygen and fluorine isotopes with the same neutron number usually have different shapes for ground states. The oxygen isotopes are spherical and the fluorine isotopes are deformed. Except for the limited difference observed in Figs. 3 and 9, the results for $^{24,25}\text{F}$ and for $^{23,24}\text{O}$ are similar without considering the deformation for fluorine isotopes. Deformation may have impact on the neutron thickness of fluorine isotopes. However, in a picture in which the variation of neutron thickness, as the $\rho NN$ tensor coupling is included, is tightly related to the outlayer $2s_{1/2}$ neutron occupation, the deformation influence is small because the $2s_{1/2}$ orbital is geometrically spherical.

We discuss another nucleus $^{26}\text{Ne}$ that also possesses the outlayer $2s_{1/2}$ neutrons. Figure 11 shows the nucleon density distributions for various $\Lambda_\alpha$ and the correlation of neutron thickness between $^{26}\text{Ne}$ and $^{208}\text{Pb}$. Although the inclusion of the $\rho NN$ tensor coupling enhances the sensitivity of the density distribution and neutron thickness of $^{26}\text{Ne}$ to the isoscalar-isovector coupling, a careful comparison of the results for $^{24}\text{O}$ and $^{26}\text{Ne}$ indicates that the role of the $\rho NN$ tensor coupling is reduced in $^{26}\text{Ne}$ because its neutron-proton asymmetry is comparably lowered. The importance of the $\rho NN$ tensor coupling relies on two factors: outlayer filling of the $2s_{1/2}$ and the ratio $N/Z$.  

![FIG. 10. The same as shown in Fig. 9, but without the inclusion of the pairing correlation.](image-url)
neutron thickness of $^{208}$Pb with the NL3 and S271 models in which the data-to-data correlation of neutron thickness between $^{26}$Ne and $\rho$ is satisfied. The extended distribution of the $^{2}$ neutron for $^{208}$Pb predicted weak binding halo nuclei [15,16]. In Ref. [13], it was pointed out that models binding energy of the $^{2}$ orbital in neutron-rich nuclei.

To estimate the influence of the isoscalar-isovector term on the extended distribution of the $^{2}S_{1/2}$ orbital, we present the binding energy of the $^{2}S_{1/2}$ neutron for $^{23}$O, $^{24}$F and, $^{25}$Ne in Fig. 12. $^{23}$O, $^{24}$F, and $^{25}$Ne are characteristic of one neutron occupying the $^{2}S_{1/2}$ orbital, and they are possible candidates of halo nuclei [15,16]. In Ref. [13], it was pointed out that models with small neutron skin in $^{208}$Pb predicted weak binding energies for the valence neutron orbitals in neutron-rich nuclei.

![Graph showing the nucleon density distributions for various cases and the data-to-data correlation of neutron thickness between $^{28}$Ne and $^{208}$Pb. The results are provided by the NL3 model.](image)

**FIG. 11.** The nucleon density distributions for various cases and the data-to-data correlation of neutron thickness between $^{28}$Ne and $^{208}$Pb. The results are provided by the NL3 model.

It is also the case here, though the modification to the $^{2}S_{1/2}$ binding energy is relatively slight, as shown in Fig. 12. The $^{2}S_{1/2}$ neutron density distribution is almost unchanged against $\Lambda_v$ (for instance, see Fig. 4). The modification produced by the isoscalar-isovector term is mainly concentrated in the core region because the isoscalar-isovector term that modifies the density dependence of the symmetry energy has a quite limited role in the very low density region. For these possible candidates of halo nuclei, the uncertainty of the nucleon radius is dominated by the core region contribution, and therefore the halo distribution is little modified by the isoscalar-isovector term.

A notable point is that the isoscalar-isovector term modifies not only the neutron radius but also charge (proton) radius explicitly in light exotic nuclei [14]. For instance, the modifications of the proton and neutron radii by the $\Lambda_v$ are almost equally important in $^{24}$O. The establishment of the data-to-data correlation may therefore bridge the different kind of experiments: the neutron radius measurement for $^{208}$Pb [17] by means of parity-violating electron scattering and future electron scattering on the exotic nuclei [28,29].

**IV. SUMMARY**

In this study, the additional isoscalar-isovector term is considered in the RMF models to simulate the uncertainty of the neutron thickness of $^{208}$Pb, and the $\rho NN$ tensor coupling that does not affect the symmetry energy in the mean-field approximation is included. Light exotic nuclei of oxygen, fluorine, and neon isotopes that are characteristic of the outlayer filling of the $^{2}S_{1/2}$ neutrons are studied to establish data-to-data correlations between the neutron thickness of $^{208}$Pb and the ground-state observables of these light exotic nuclei.

Both the dependence of the neutron density distribution and that of the proton in light exotic nuclei on the isoscalar-isovector term are enhanced by the inclusion of the $\rho NN$ tensor coupling. The data-to-data correlation of the neutron thickness between $^{208}$Pb and the light exotic nuclei is enhanced by the $\rho NN$ tensor coupling. The enhanced sensitivity to the isoscalar-isovector term that is due to the inclusion of the $\rho NN$ tensor coupling is related to the structural factor, the outlayer neutron occupancy in the $^{2}S_{1/2}$ orbital. For a certain occupancy in $^{2}S_{1/2}$, this sensitivity will be reduced with the rise of the proton number. For heavy nuclei like $^{208}$Pb, the tensor coupling has a negligible influence on both the neutron and the proton density distribution. It is helpful to constrain the density dependence of the symmetry energy through the enhanced sensitivity of the nucleon density distributions to the isoscalar-isovector term that influences the density dependence of the symmetry energy. For experimental choice, $^{25}$F may be favored considering there is a suitable compromise between its stability and the enhanced degree of such a sensitivity.

The role of the $\rho NN$ tensor coupling is favorable for reducing the model dependence of the neutron thickness in light exotic nuclei. The pairing correlation is important for the description of the light exotic nuclei. The consideration of the pairing correlation together with the $\rho NN$ tensor coupling

![Graph showing the binding energy of the $^{2}S_{1/2}$ orbital against the neutron thickness of $^{208}$Pb with the NL3 and S271 models in which the $\rho NN$ tensor coupling is included.](image)

**FIG. 12.** The binding energy of the $^{2}S_{1/2}$ orbital against the neutron thickness of $^{208}$Pb with the NL3 and S271 models in which the $\rho NN$ tensor coupling is included.
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reduces the model dependence of the neutron thickness in light exotic nuclei. However, because the Fermi surface of these exotic nuclei is close to continuum, it would be better to consider the state-dependent pairing gap, and the work is in progress. For possible candidates of halo nuclei with $2s_{1/2}$ halo neutrons, the uncertainty of the nucleon radius induced by the isoscalar-isovector term is dominated by the core region contribution and the halo distribution is little modified by the isoscalar-isovector term.

The sense of the establishment of the data-to-data correlation for the neutron thickness of heavy nuclei and light exotic nuclei lies in two aspects. On the one hand, the uncertainty that occurs in the neutron radius of heavy nuclei exists not only in the neutron radius but also in the proton radius of light exotic nuclei. On the other hand, these correlation relations may bridge two kinds of experiments, the neutron radius measurement for $^{208}$Pb and the charge distribution measurement for light exotic nuclei, to jointly constrain the density dependence of the symmetry energy.

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