Development of the Edge-Focus Wiggler for FEL and SASE

Goro ISOYAMA
Institute of Scientific and Industrial Research, Osaka University
FEL Experiments at ISIR, Osaka University

- 40 MeV, L-band (1.3 GHz) electron linac
  - Basic study on Self-Amplified Spontaneous Emission (SASE)
  - Development of an Free-Electron Laser (FEL) for users experiments

Main components of FEL
- Electron accelerator
- Wiggler or undulator
- Optical resonator
Focusing Property of Planar Wiggler

- Simple model of planar wiggler
  - Series of alternating dipoles with drift space in between.
  - Electron goes in and out of a dipole with an entrance angle

- Natural focusing
  - Horizontally defocused and vertically defocused at edges of dipoles.
  - Horizontal focusing due to centrifugal force in dipoles cancels the edge defocusing.
  - Vertical focusing force remains in wiggler.

- Diagram showing electron beam path through wiggler and focusing properties.
Wiggler

- Focusing force in a planar wiggler
  - in vertical direction
  - not in horizontal one

\[ k_0 = \frac{8 - \pi}{3\pi} \left( \frac{e}{m_0 c} \right)^2 \left( \frac{B_0}{\gamma} \right)^2 \]

- SASE gain proportional to \( \lambda^{3/2} \); gain length larger in the short wavelength regions.

- Wiggler for X-ray SASE is longer than 100 m.

**Separated**
(quadrupoles between wiggler units)

**Focusing force necessary to the beam**
Development of Edge Focusing Wiggler

The edge focusing (EF) wiggler is a combined function wiggler in which the transverse field gradient is imposed to the wiggler field.

- Quantitative study on performance of the edge focusing wiggler with numerical calculation
- Experimental model
  - New fabrication method to cancel magnetic errors
  - Measurement of magnetic field
    - Precise measurement of magnetic field with measurement error corrections
    - Magnet gap dependency of field gradient
- Practical wiggler
  - Strong focusing wiggler with EF wiggler
  - Measurement of magnetic field (strength and field gradient)

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<th>Basic study</th>
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<td>Experimental model</td>
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<tr>
<td>- Period: 60 mm</td>
</tr>
<tr>
<td>- Period number: 5</td>
</tr>
<tr>
<td>- Edge angle: 2°</td>
</tr>
<tr>
<td>- Weak focusing</td>
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<th>Demonstration</th>
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<td>FEL and SASE experiments</td>
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<table>
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<tr>
<th>Practical wiggler</th>
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<tr>
<td>- Period: 60 mm</td>
</tr>
<tr>
<td>- Period number: 32</td>
</tr>
<tr>
<td>- Edge angle: 5°</td>
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<tr>
<td>- Strong focusing</td>
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EF Wiggler

Began developing the EF wiggler, which is shorter than the separated function type and easy to fabricate.

- Simple structure
- High field gradient, proportional to $\phi$
- No obstacles in the gap space

Magnet structure of EF wiggler

Field gradient along beam axis

$$k_x = \frac{e}{\gamma m_0 c} \frac{4 B_0}{\lambda_w} \phi$$

$$k_y = k_0 - k_x$$

Applicable in all spectral regions
- Infrared FEL
- X-ray FEL
- SR storage ring
Experimental Model of EF Wiggler

Parameters of experimental model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Period number, period length</td>
<td>5 periods, 60 mm</td>
</tr>
<tr>
<td>Total length</td>
<td>300 mm</td>
</tr>
<tr>
<td>Peak magnetic field</td>
<td>0.42 T (gap 30 mm)</td>
</tr>
<tr>
<td>Edge angle</td>
<td>2.0 deg</td>
</tr>
<tr>
<td>Magnet gap</td>
<td>30, 40, 50, 60 mm</td>
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</table>

Magnet structure

Experimental model
Magnetic Field Measurement

- Measured at KEK (High Energy Accelerator Organization)
  - Two independent, temperature controlled Hole probes
  - Longitudinal position $z = -300 \sim 300$ mm in 1 mm steps for $g = 30 \sim 60$ mm in 10 mm steps.

Measured along beam axis at various transverse positions $(x, y)$ for 3D mapping of magnetic field.
**Measured Magnetic Field**

- EF wiggler should be a good wiggler, but experimental model too short.
- 35 period pseudo-wiggler based on magnetic field of model.
  - calculated by adding seven sets of the measured magnetic field after longitudinally and progressively shifting them by 30 cm (= 6 [cm/period] ÷ 5 [periods]) with respect to every contiguous unit.

![Graph of measured magnetic field with a peak value of 4206.9 Gauss.](geometry)

![Graph showing the vertical magnetic field and its second integral.](geometry)
Study on effects of angular errors of hole probes and linear stages on measured magnetic fields using a model magnetic field.

**Model magnetic field (Planar wiggler+ Gradient)**

\[
\begin{align*}
B_x^\text{model} &= G_y \\
B_y^\text{model} &= B_0 \cosh(ky)\cos(kz) + G_x \\
B_z^\text{model} &= -B_0 \sinh(ky)\sin(kz)
\end{align*}
\]

**Gradient**

\[
\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = G
\]
Angular Errors of Hole Probes and Stages

Angular error of Hole probe

While circle: ideal without error
Grey circle: realistic with error

Angular Error of linear stage

Black vector: ideal without error
Red vector: realistic with error

\[ B_{x}^{mes} = B_{y} \sin \theta_{x} \cos \phi_{x} + B_{z} \sin \theta_{x} \sin \phi_{x} + B_{x} \cos \theta_{x} \]

Intrusions of the other field components

Magnetic field measured at different positions
Effects of Linear Stage on Field Gradient

Angular error of the linear stage in x-direction for Hole probe measuring $B_y$

Based on model magnetic field and initial setting conditions of the linear stage

Field gradient of $B_y$

$$\frac{\partial B_y(0,0,z)}{\partial x} \approx \frac{\partial B_y^{mes}(0,0,c)}{\partial a} + B_0 k \sin(kc)\psi_x$$

$\psi_x$ determined by symmetric property of wiggler field to the center at $z = 0$

$\psi_x = 0.9 \text{ mrad}$

Small alignment error in the linear stage, which inevitably remains in the ordinal set-up, results in a large error in the measured field gradient in the EF wiggler, but the angular error may be derived from asymmetry in the measured field gradient and it can be corrected.
Field gradients measured and calculated along the beam axis at magnet gaps from 30~60 mm.

Angular error corrected

Field gradient decreases with increasing magnet gap.
Field gradient of ~1 T/m averaged over the beam axis is realized at gap = 30 mm.
Magnet Gap Dependency of Field Gradient

- Peak magnetic field and field gradient measured as a function of magnet gap.
- Gap dependency of field gradient more gradual than that of peak magnetic field, so that strong focusing force can be provided even at a larger gap.

**Derived from dipole model**

\[
\frac{\partial B_y}{\partial x} = 4 \frac{B_0}{\lambda_w} \phi
\]
Modeling of EF Wiggler for Field Gradient

**EF wiggler**

- Two pairs of magnet arrays extending along z direction
- One-period wiggler with two direction of magnetization in x direction

**Focusing field (EF – Planar wiggler)**
Analytical Formula of $B_y(x)$

Analytical formula for $B_y(x)$ derived from the formula for $B_y(z)$

$$B_y(x) = 2B_{rm} \cdot \exp\left(-\pi g / \lambda_T\right) \cdot \left(\frac{\sin(\pi / M)}{\pi / M}\right) \cdot \left\{1 - \exp\left(-2\pi h / \lambda_T\right)\right\} \cdot \sin\left(\frac{2\pi}{\lambda_T}\cdot x\right)$$

$$\frac{\partial B_y}{\partial x} \approx 3.5 \frac{B_{T0}}{\lambda_W} \phi$$

$$B_{T0} = 2B_r \cdot \exp\left(-\pi g / \lambda_T\right) \cdot \left(\frac{\sin(\pi/2)}{\pi/2}\right) \cdot \left\{1 - \exp\left(-2\pi h / \lambda_T\right)\right\}$$

Field gradients for EF wigglers with different period lengths and magnet widths.
**Strong Focusing Wiggler**

<table>
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<tr>
<th>FEL and SASE at ISIR, Osaka University</th>
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<tr>
<td>• Electron beam energy: 10~30 MeV</td>
</tr>
<tr>
<td>• Wavelengths: 30~220 µm</td>
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</table>

- Small average beam size required for the wide wavelength range
  - Strong focusing scheme

<table>
<thead>
<tr>
<th>Magnet blocks</th>
<th>90・20・15 mm³</th>
</tr>
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<tbody>
<tr>
<td>Permanent magnet and coating</td>
<td>Nd-Fe-B・TiN</td>
</tr>
<tr>
<td>Period length and period number</td>
<td>60 mm・32 periods</td>
</tr>
<tr>
<td>Total length</td>
<td>1.938 m</td>
</tr>
<tr>
<td>Peak magnetic field at g=30 mm</td>
<td>0.39 T・K=2.18</td>
</tr>
<tr>
<td>Number of FODO cells and cell length</td>
<td>4 cells・0.48 m</td>
</tr>
<tr>
<td>Field gradient and edge angle</td>
<td>3.2 T/m・5 deg.</td>
</tr>
</tbody>
</table>

**Strong Focusing**: double focusing scheme with alternating field gradients for strong focusing
Calculated Beam Sizes in Wiggler

\[ E = 16 \text{ M eV} \]
\[ \varepsilon_n = 150 \text{ mm mrad} \]

Usual operation conditions for FEL experiment at Osaka University

**Planar wiggler**

\[ \sigma_x \text{ (ave)} = 2.294\text{mm} \]
\[ \sigma_y \text{ (ave)} = 0.989\text{mm} \]

**Edge Focus wiggler**

\[ \sigma_x \text{ (ave)} = 1.976\text{mm} \]
\[ \sigma_y \text{ (ave)} = 1.055\text{mm} \]

Average beam sizes are smaller in EF wiggler by 8% 

→ advantage greater for longer wiggler
Magnet Array of SF Wiggler

Half Length of EF wiggler

Edge magnet unit

Magnet Blocks are arranged so that strength and angular errors of magnetization cancel each other.
Magnetic Field Measurement

- Vertical and horizontal components ($B_y, B_x$)
- $z = -1300~+1300$ mm in 1 mm steps
- Temperature controlled Hole probes
- Magnet gap 30~50 mm
Magnetic Field and Field Gradient

**Magnetic field distribution**

**Field gradient**

![Graph showing magnetic field distribution and field gradient vs. z in mm.](image)
Electron Orbit

$E_b = 11\text{MeV}$

**Gap=30mm**

- $X$ orbit ($\Delta B_{\text{ENT}} = 0\text{Gcm}$)
- $Y$ orbit ($\Delta B_{\text{ENT}} = -20\text{Gcm}$)

**Gap=40mm**

- $X$ orbit ($\Delta B_{\text{ENT}} = -10\text{Gcm}$)
- $Y$ orbit ($\Delta B_{\text{ENT}} = -20\text{Gcm}$)
SF Wiggler at ISIR, Osaka University
SASE and FEL with SF Wiggler

**SASE spectrum**

- $E_b = 10 \text{MeV}$

  - 3rd harmonic ($74 \mu\text{m}$)
  - 2nd harmonic ($\sim 110 \mu\text{m}$)
  - Fundamental ($218 \mu\text{m}$)

- Detector output [mV]
- Wavelength [$\mu\text{m}$]

**FEL signal (GeGa detector)**

- $E_b = 16 \text{MeV}$

- Beam current at Acc. out
- Wiggler entrance
- Wiggler exit
- FEL signal ($93 \mu\text{m}$)

- 2 $\mu\text{s}$

- NBHFPG

**Points**

- **SASE**: fundamental peak at $\lambda = 220 \mu\text{m}$ and higher harmonic peaks.
- **FEL**: Lasing in 25$\sim$150 $\mu\text{m}$
- **Successfully demonstration of SF-EF wiggler used for SASE and FEL in wide range of electron energy**
Thank you for your attention