AdS/CFT correspondence and the Quark-Gluon Plasma

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Plan of the talk

- The AdS/CFT correspondence
- Viscosity as gravity absorption
- Universality of viscosity/entropy ratio in AdS/CFT

Not in the talk:

- Spectral functions; photon and dilepton rates Teaney; Kovtun, Starinets
- Energy loss, jet quenching parameter (Liu's talk)

Problem with strong coupling

- QCD is asymptotically free theory, α_s is small at large momentum scale
- Perturbative expansion of QCD thermodynamics is not well behaved for realistic temperatures.
- For thermodynamics we can use lattice, resummation techniques (Blaizot's talk).
- Kinetic coefficients are almost impossible to extract from lattice data

String theory as a tool

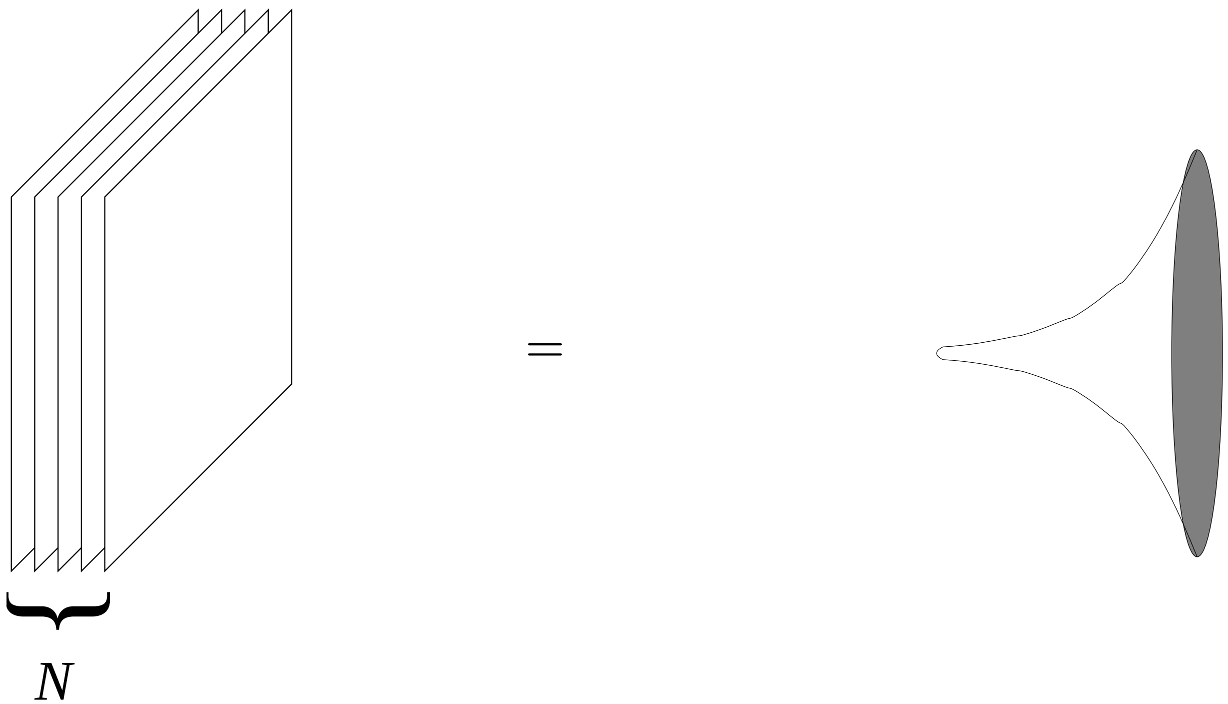
- From the work in the late 90's, a remarkable connection has appeared between a certain string theory in certain curved spacetimes and certain gauge theories in flat (3+1) dimensional space
- This is originally called the anti de-Sitter / Conformal field theory correspondence (AdS/CFT), Maldacena's conjecture.
- Extensions of this idea shed fresh new light on the problem of quark confinement

The Gauge/Gravity Duality

Stack of N D3-branes in type IIB string theory: described in two different pictures:

As a quantum field theory of degrees of freedom on the branes: $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

As string theory on a the curved spacetime (induced by the matter density on the branes)



The limit of infinitely strong coupling in gauge theory is the limit when string theory becomes Einstein's general relativity

The two sides of the correspondence

- The $\mathcal{N}=4$ SYM theory:
  - contains gauge field, 4 Weyl fermions, 6 real scalars.
  - Zero beta function: g is a parameter of the theory (compared to QCD: $\Lambda_{\rm QCD}$ is a parameter). Also N_c.
- Type II B string theory:
  - contains a finite number of massless fields (graviton, dilatons, forms), infinite number of massive string excitations
  - two parameters: inverse string tension $\alpha' = l_{\rm st}^2$, and string coupling $g_{\rm st}$
  - on AdS$_5\times$ S^5 background

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} d\Omega_5^2$$

Mapping of parameters

The AdS/CFT correspondence between parameters of two theories:

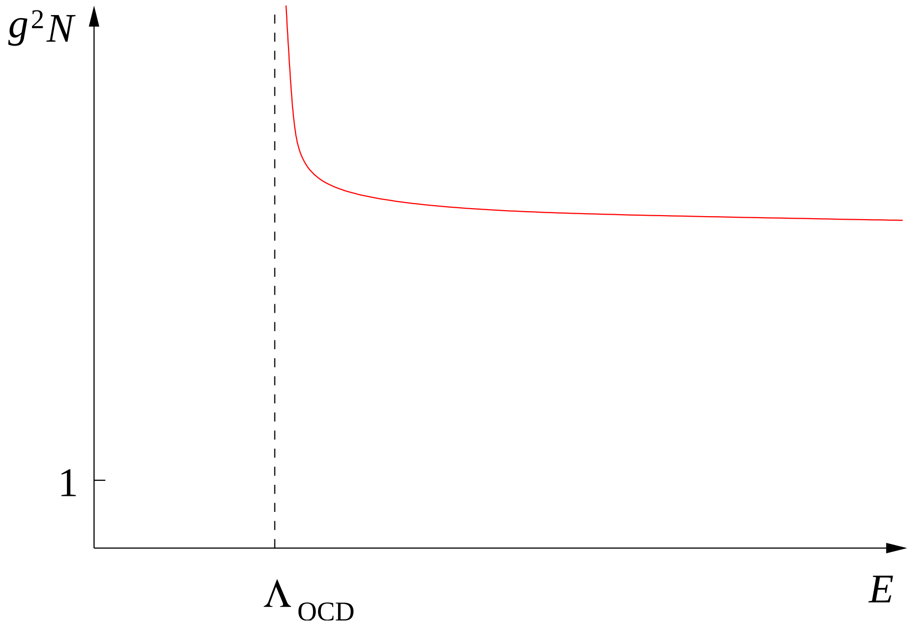
$$g^2 = 4\pi g_{\mathrm{st}}$$

$$g^2 N_c = \left(\frac{R}{l_{\mathrm{st}}}\right)^4$$

AdS/CFT is useful in the limit $g \to 0$, $g^2 N_c \to \infty$: string theory becomes classical supergravity. Allows analytical treatment.

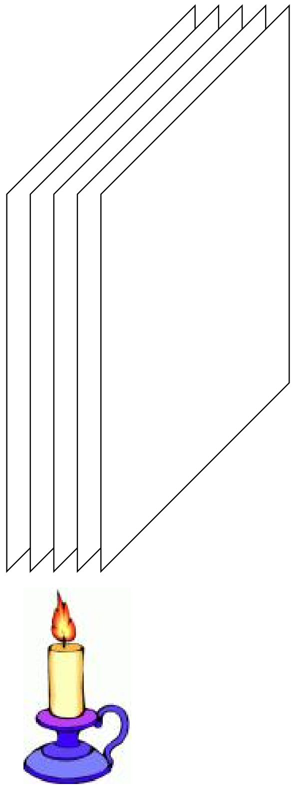
Extensions

Many more examples of gauge/gravity duality have ben constructed
including theories with confinement and chiral symmetry breaking
To be able to compute using string theory, the gauge coupling needs to be large at all scales



This is unlike asymptotically free QCD

Finite temperature AdS/CFT correspondence



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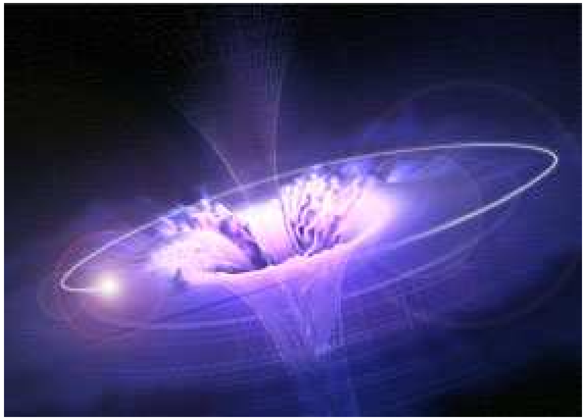


Thermal gauge theory = black hole in anti de-Sitter space

Entropy at strong coupling

Black 3-brane background:

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\vec{x}^2] + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

Hawking temperature;

$$T_H = \frac{r_0}{\pi R^2}$$

Entropy is computed from area of the horizon, and the result is $S = \pi^6 R^8 T^3 V_{\rm 3D}$. Using AdS/CFT mapping:

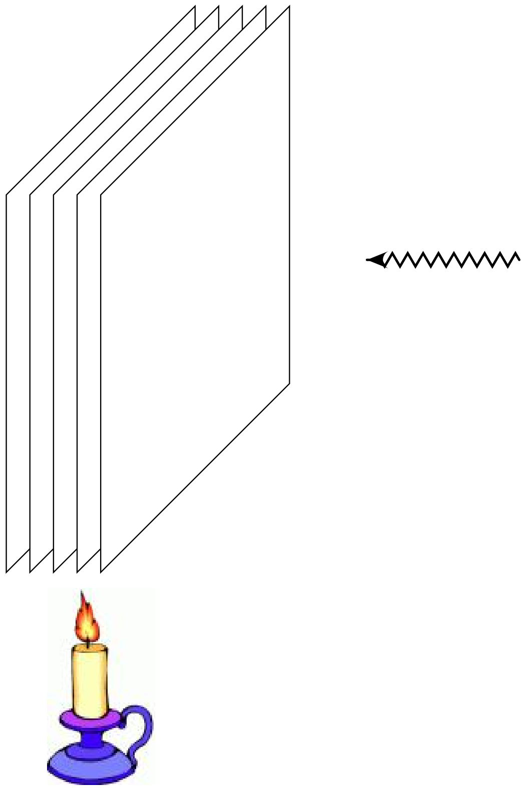
$$s = \frac{S}{V_{\rm 3D}} = \frac{\pi^2}{2} N_c^2 T^3$$

At zero 't Hooft coupling: $s = \frac{2\pi^2}{3} N_c^2 T^3$

$$s(g^2 N_c = \infty) = \frac{3}{4} s(g^2 N_c = 0)$$

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



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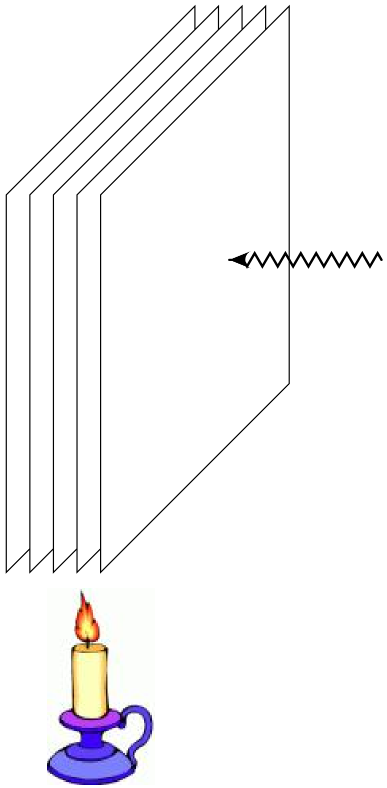

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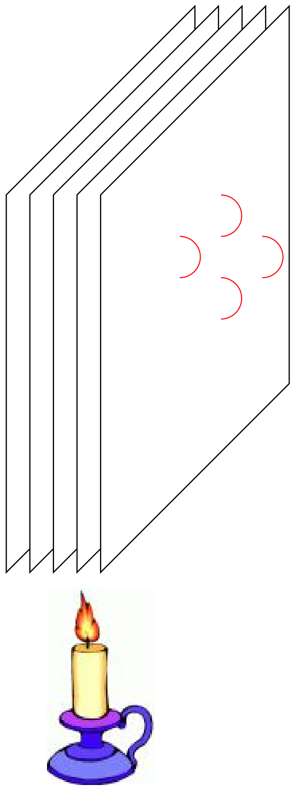

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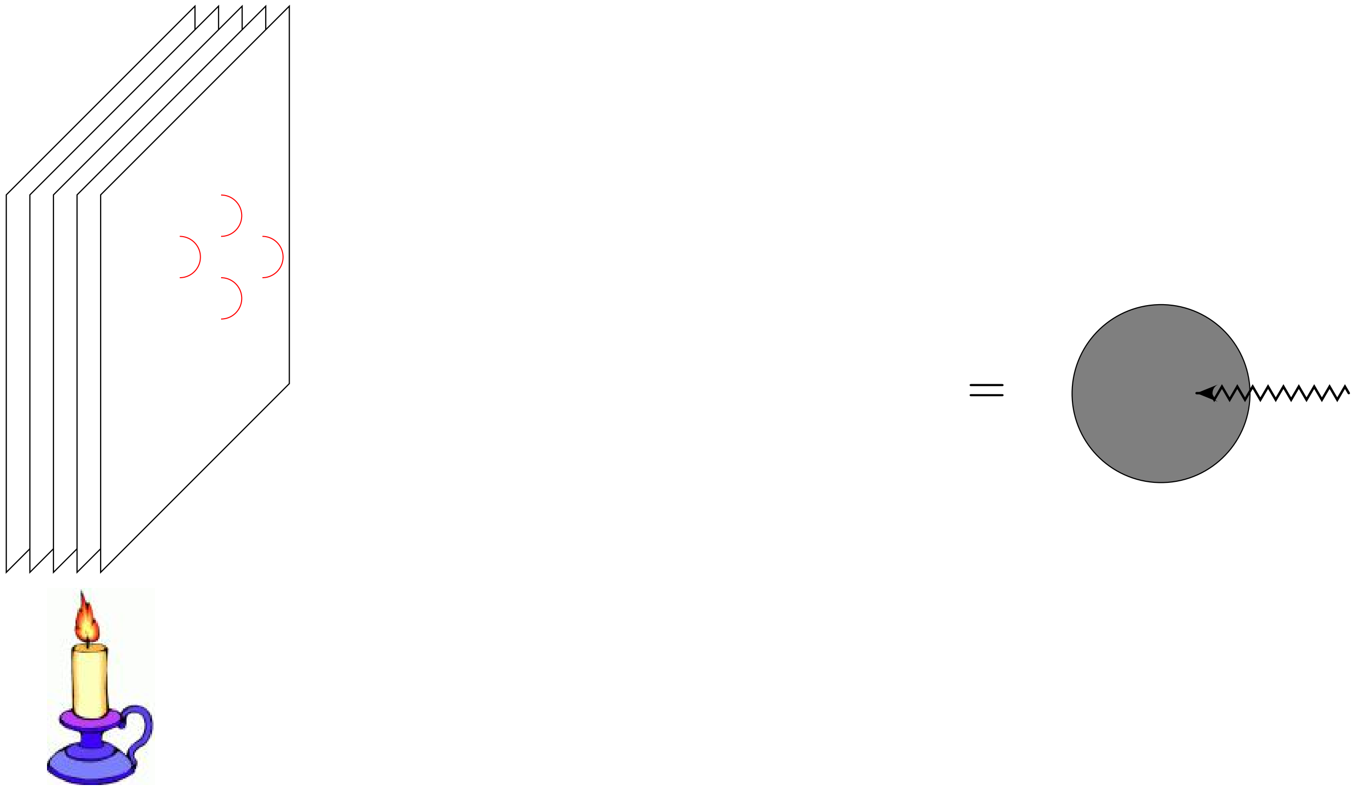

Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

More formally

- Viscosity is given by Kubo's formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t,\vec{x}),\, T_{xy}(0,0)] \rangle$$
$$= \lim_{\omega\to 0} \lim_{\vec{q}\to 0} \mathrm{Im}\, G^R_{xy,xy}(\omega,\vec{q})$$

- Via AdS/CFT correspondence, the imaginary part of the retarded Green's function is mapped to the graviton absorption cross section.

$$\sigma_{\mathrm{abs}} = -\frac{16\pi G}{\omega} \mathrm{Im}\, G^R(\omega)$$

- viscosity $\sim$ absorption cross section for low-energy gravitons

$$\eta = \frac{\sigma_{\mathrm{abs}}(0)}{16\pi G}$$

Universality of viscosity/entropy density ratio

- Absorption cross section $=$ area of horizon (follows from a couple of theorems in general relativity)
- Entropy is also proportional to area of horizon: $S = A/(4G)$

$\Rightarrow$ in *all* theories with gravity duals:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

where η is the shear viscosity, s is the entropy per unit volume.

This is valid in a large, but restricted, class of strongly coupled quantum field theories, which are in a sense infinitely strongly coupled

Boltzmann equation is never used

Viscosity/entropy ratio and uncertainty principle

Estimate of viscosity from kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

Quasiparticles: de Broglie wavelength $\lesssim$ mean free path

Therefore $\eta/s \gtrsim \hbar$

- Weakly interacting systems have $\eta/s \gg \hbar$.
- Systems with gravity duals are strongly coupled, so it is not suprising that $\eta/s \sim 1$
- What is surprising is *universality*: $\eta/s = \hbar/(4\pi)$ in all theories with gravity duals
- We don't know how to derive the constancy of η/s without AdS/CFT.

Moreover, $\hbar/(4\pi)$ is smaller than η/s achieved in any laboratory liquids: plasmas with gravity duals as the most ideal fluids?

Quark gluon plasma

What is the consequence for the QGP:

- η/s is large both for $T \ll T_c$ (pion gas) and $T \gg T_c$ (weakly coupled QGP)
- minimum near T_c: related to transition from hadrons to quarks? (Csernai, Kapusta, McLerran)
- However: in the weak coupling limit η/s in QCD is larger than η/s in $\mathcal{N} = 4$ SYM at the same 't Hooft coupling by a factor of 7 (Huot, Jeon, Moore)
- So for QGP most likely η/s is above $1/(4\pi)$, in accordance with our hypothesis
- but what is the minimal η/s and how close it is to $1/(4\pi)$ is not clear at this moment
- AdS/CFT suggests η/s as the measure of perfectness of QGP fluid

Conclusion

- Gauge/gravity duality provides unexpected tools to compute the viscosity of some strongly coupled theories
- The class of theories with gravity dual description is limited, but contains very interesting theories with infinite coupling
- The calculation of the viscosity is easy: viscosity $\propto$ absorption cross section of low-energy gravitons by the black hole.
- In this class, the ratio η/s is equal to a universal number $\hbar/4\pi$, much smaller than in any other system in Nature
- The ratio η/s is the measure of perfectness of the QGP