Experimental signatures of perfect fluidity at RHIC

Derek Teaney

Arkansas State University

Extracting Transport from the Heavy Ion Data

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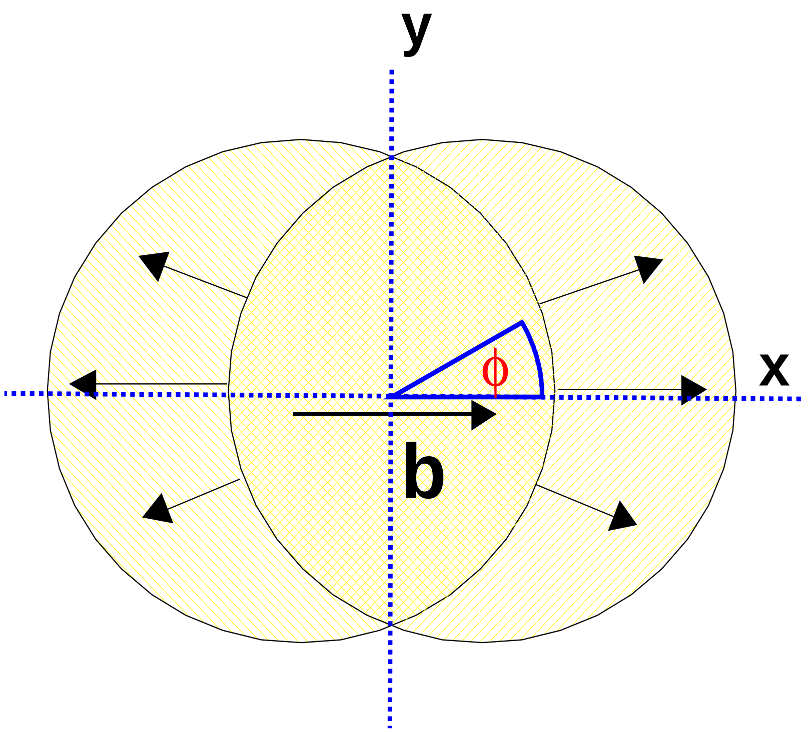
Arkansas State University

Observation:



There is a large momentum anisotropy:

$$v_2 \equiv \frac{\langle p_x \rangle^2 - \langle p_y \rangle^2}{\langle p_x \rangle^2 + \langle p_y \rangle^2} \approx 20\%$$

Interpretation

- The medium responds as a fluid to differences in X and Y pressure gradients
- Hydrodynamic models work well enough.

Is the system *Large* enough? Does it live *Long* enough for hydro?

How *Long* and *Large* is *Long/Large* Enough ?

- What is the mean free path? $\ell_{\rm mfp} \equiv \frac{\eta}{e+p}$
- The mean free path should be less than the expansion rate $\frac{1}{\tau}$:

$$\underbrace{\frac{\eta}{e+p}}_{\ell_{\rm mfp}} \frac{1}{\tau} \quad \ll \quad 1$$

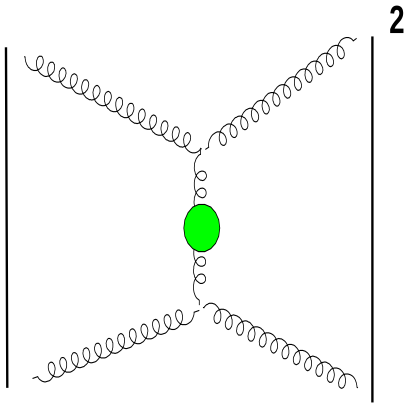
- Then using the relation: $(e+p) = sT$.

$$\underbrace{\frac{\eta}{s}}_{\text{Liquid parameter}} \qquad \times \qquad \underbrace{\frac{1}{\tau T}}_{\text{Experimental parameter: } \sim 1} \qquad \ll 1$$

1. η/s needs to be small to have interacting QGP at RHIC.
2. Even if η/s is small, dissipative effects are significant!

Perturbative estimate of η/s:

$$\frac{\eta}{e+p} \sim \frac{1}{n\sigma} = \frac{1}{\underbrace{n}_{\sim T^3} \times \underbrace{\sigma}_{\alpha_s^2/T^2}} \sim \frac{1}{\alpha_s^2 T}$$



- So the Figure of Merit:

$$\overbrace{\eta/(e+p)}^{1/\alpha_s^2 T} \quad \times \quad \overbrace{\text{expansion rate}}^{1/\tau} \quad \ll \quad 1$$

$$\underbrace{\frac{1}{\alpha_s^2}}_{\eta/s \text{ Liquid Parameter}} \quad \times \quad \underbrace{\frac{1}{\tau T}}_{\text{Experimental Parameter}} \quad \ll \quad 1$$

- Arnold, Moore, Yaffe found $\eta \approx 150\, T^3/g^4$, and taking $\alpha_s \to 1/2$

$$\frac{\eta}{s} \approx 0.3 \left(\frac{0.5}{\alpha_s}\right)^2$$

Estimates of η/s

1. Perturbative QCD – Kinetic Theory Arnold, Moore, Yaffe.

   $\eta \approx 150\, T^3 \frac{1}{g^4}$. Set $\alpha_s \rightarrow 1/2$ and $m_D \rightarrow$ a reasonable value

$$\frac{\eta}{e+p}\,\frac{1}{\tau} \approx \underbrace{0.3}_{\eta/s}\,\underbrace{\frac{1}{\tau T}}_{\sim 1}$$

2. Strongly Coupled conformal N=4 SYM – AdS/CFT Son, Starinets, Policastro

   No kinetic theory exists.

$$\frac{\eta}{e+p}\,\frac{1}{\tau} = \underbrace{\frac{1}{4\pi}}_{\eta/s}\,\underbrace{\frac{1}{\tau T}}_{\sim 1}$$

$\mathcal{N}=4$ calculation was important. It showed that η/s can be small enough to have hydro at RHIC, at least theory in some field theories

Outline – Where does hydro break down?

1. Small Systems
   - Implications of CuCu Data on Viscosity
2. Large Viscosity and “large” p_T
   - Efforts to simulate RHIC collisions with viscous hydro
3. Heavy Particles
   - Heavy Quarks and implications for Hydro

Observing the mean free path in the Cu-Cu Run:

Based on Bhalero, *et al.*, Phys. Lett. B. ; see also Heiselberg; Voloshin and Poskanzer

- For peripheral AuAu they argue

$$\frac{\eta}{e+p}\frac{1}{\tau} \sim 1$$

- Inverse Knudsen number $\equiv$ the typical number of scatterings

$$\begin{aligned} K^{-1} &= \frac{R}{\ell_{mfp}} \sim \sigma \int_{\tau_o}^{\tau_f} \overbrace{n(\tau')}^{\sim\left(\frac{1}{S}\frac{dN}{dy}\right)} \, d\tau' \\ &= \sigma \left(\frac{1}{S}\frac{dN}{dy}\right) \log\left(\frac{\tau_f}{\tau_o}\right) \end{aligned}$$

- In the low density limit $K^{-1} \ll 1$ then expect $v_2 \propto K^{-1}$

$$v_2 \propto \frac{1}{S}\frac{dN}{dy}$$

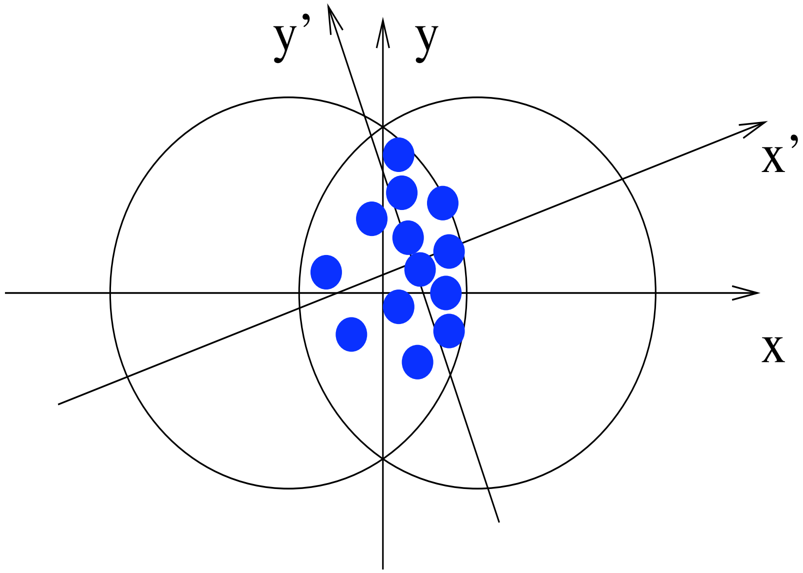
- Comparing $\frac{1}{S}\frac{dN}{dy}$ in CuCu and AuAu

$$\left(\frac{v_2}{\epsilon}\right)_{\rm CuCu}\Big|_{b=5.5\,{\rm fm}} \approx 0.5 \times \left(\frac{v_2}{\epsilon}\right)_{\rm AuAu}\Big|_{b=8.0\,{\rm fm}}$$

Low density limit prediction!

What is ϵ in small systems?

- Dynamically ϵ should be understood as ϵ_{part}. (Phobos)



Two particle correlations yield

$$v_2\{2\} = \sqrt{\langle v_2^2 \rangle}$$

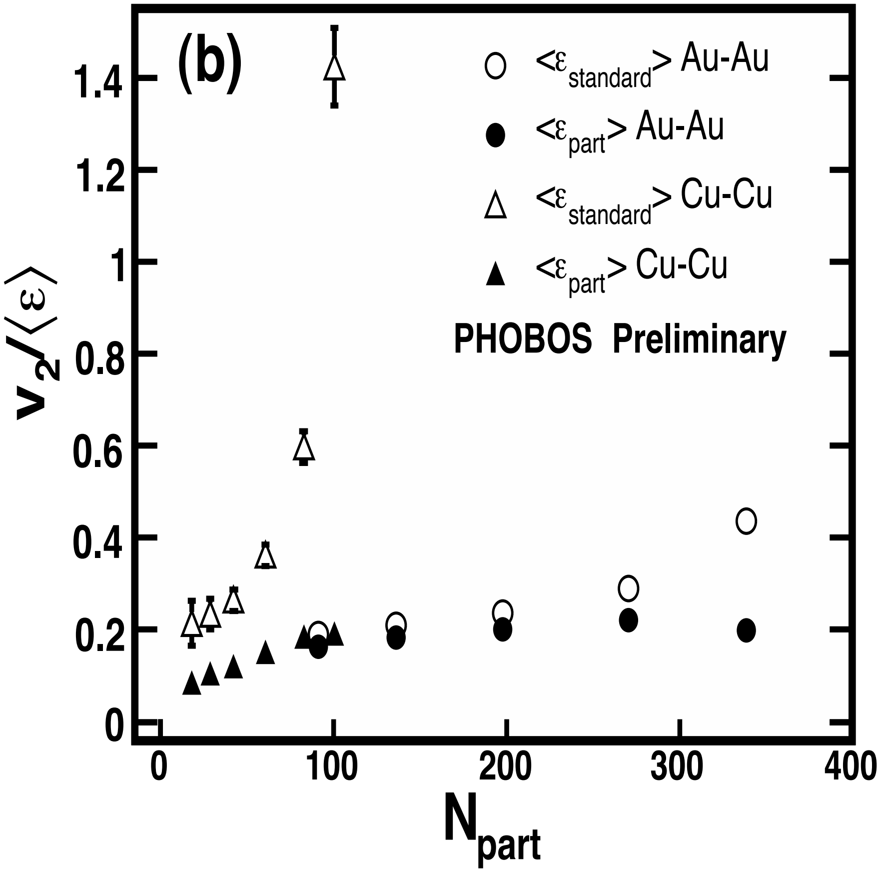
- To estimate then $v_2/\epsilon_{\mathrm{part}}$ we should estimate (Bhalero and Olltirault)

$$\frac{v_2}{\epsilon_{\mathrm{part}}} \approx \frac{v_2\{2\}}{\epsilon\{2\}} \qquad \text{with} \qquad \epsilon\{2\} \equiv \sqrt{\left\langle \epsilon_{\mathrm{part}}^2 \right\rangle}$$

- Fortunately

$$\epsilon_{\mathrm{part}} \approx \sqrt{\left\langle \epsilon_{\mathrm{part}}^2 \right\rangle}$$

PHOBOS



- Low density prediction:

$$
\begin{aligned}
\mathrm{CuCu} &\approx 0.5 \times \mathrm{AuAu} \\
\left(\frac{v_2}{\epsilon}\right)\Bigg|_{N_p=50} &\approx 0.5 \times \left(\frac{v_2}{\epsilon}\right)\Bigg|_{N_p=100} \\
0.1 &\approx 0.2
\end{aligned}
$$

Deviations from low density limit?

CuCu Conclusions

- Estimate Knudsen number from this data

$$K^{-1} \equiv \frac{R}{\ell_{\rm mfp}} \lesssim 3$$

- Estimate the η/s from the Knudsen number

$$\frac{R}{K^{-1}} = \ell_{\rm mfp} = \frac{\eta}{sT}$$

- Substitute $R = 1.10\,\mathrm{fm}$ and $T = 200$ MeV

$$\boxed{\frac{\eta}{s} \gtrsim 0.33}$$

* I Initially lost interest in CuCu because $v_2\{2\} \neq v_2\{4\}$.

* $v_2\{2\} \neq v_2\{4\}$ is understood as flucts in $\epsilon_{\rm part}$. I need to see that

$$\frac{v_2\{2\}}{\epsilon\{2\}} \approx \frac{v_2\{4\}}{\epsilon\{4\}}$$

Viscous Simulations of the Heavy Ion Reaction

- Viscous simulations are starting to appear

    1. A. Muronga, D. Rischke; D. Teaney; Chaudhuri, Heinz; R. Baier, P. Romatschke; Koikidee

- Normally in viscous hydrodynamics the stress tensor is written

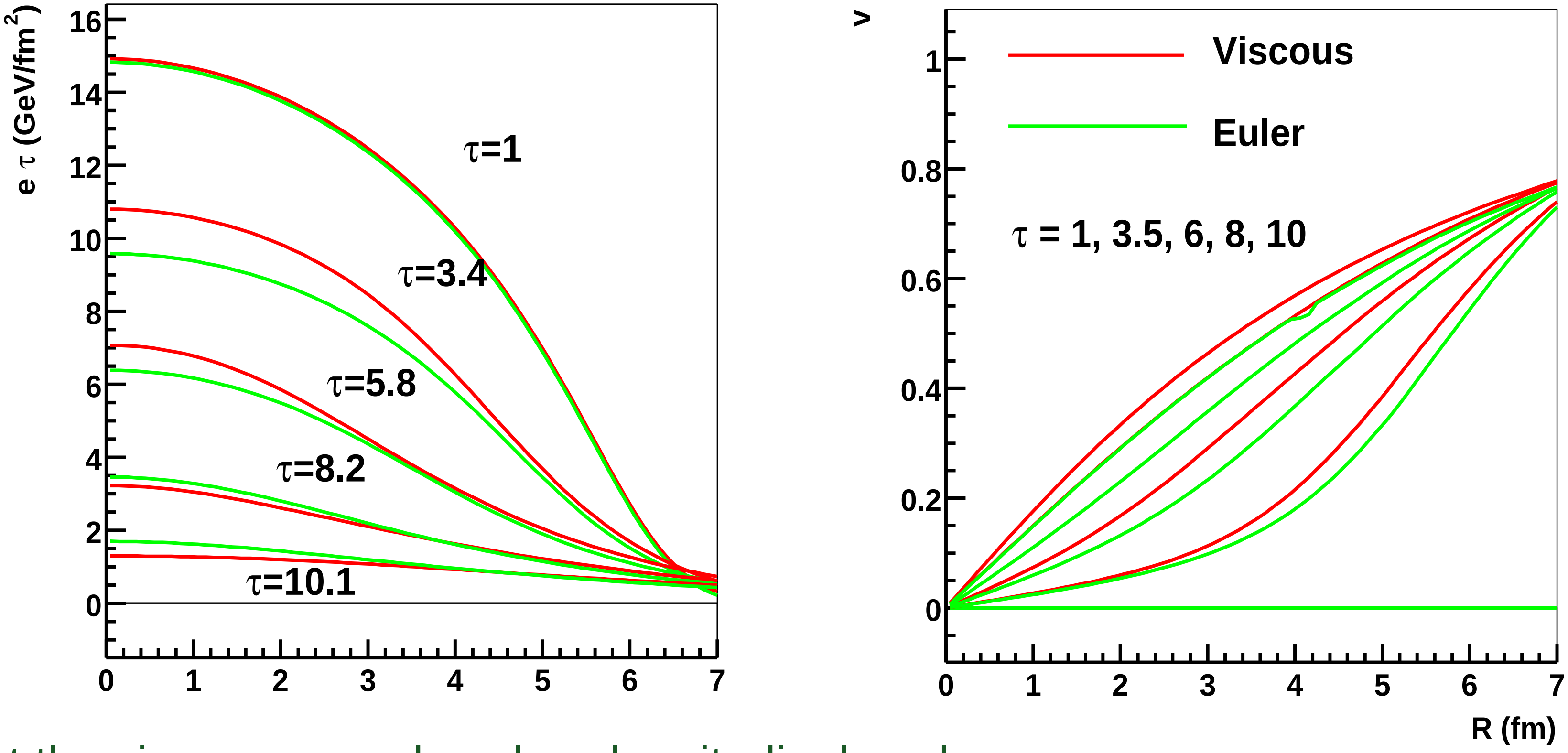
$$T_{ij} = p\delta_{ij} - \eta \langle \partial_i u_j \rangle \qquad \langle \partial_i u_j \rangle \equiv \partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial \cdot u$$

- All schemes to solve viscous hydro involve a relaxation time approximation

$$T_{ij} = p\,\delta_{ij} + \pi_{ij} \qquad \partial_t \pi_{ij} = -\frac{\pi_{ij} + \eta \langle \partial_i u_j \rangle}{\tau_R}$$

How different is the viscous solution from the ideal solution?

Bjorken Solution with transverse expansion $\eta/s = 0.2$



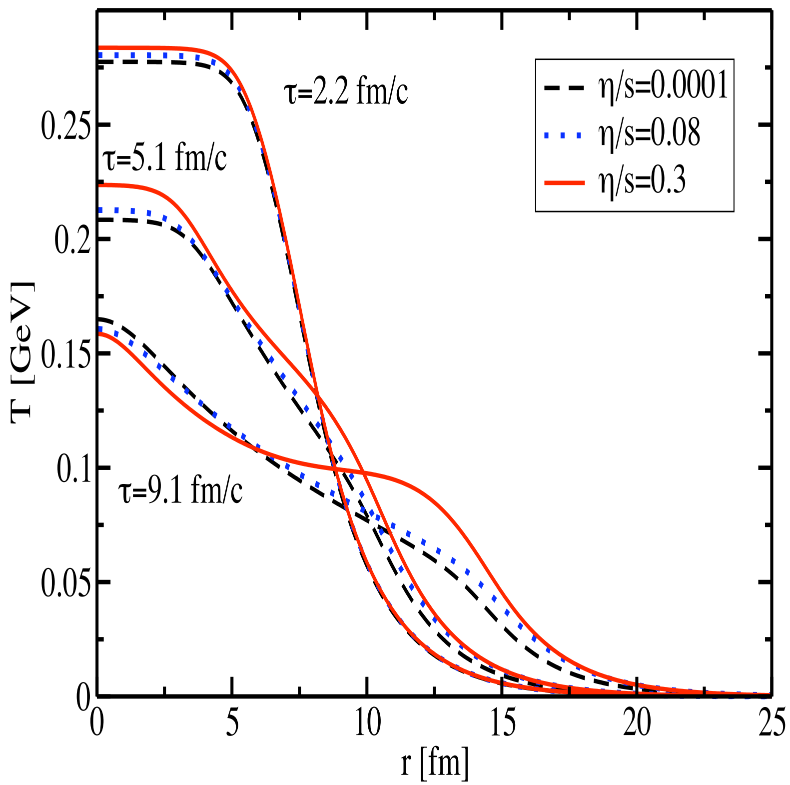
- First the viscous case does less longitudinal work.
- Then the transverse velocity grows more rapidly because the transverse pressure is larger.
- The larger transverse velocity then reduces the energy density more quickly than ideal hydro.

Viscous corrections do NOT integrate to give an O(1) change to the flow.

Similar results by other groups:

R. Baier, P. Romatschke, nucl-th/0610108

- Temp. vs. Rad. for different τ



1. Viscosity doesn’t change the solution particularly much.
2. There is a consensus on this result.

Freezeout & Viscous Corrections to Spectra (1)

- The solution is the “same” as ideal case

- Viscosity also changes the thermal distribution function

$$f \longrightarrow f_o + \delta f \qquad \delta f \sim \frac{p^i p^j}{T^2}\,\pi_{ij}$$

- The viscous correction grows with momentum

$$\delta f \sim \frac{\ell_{\rm mfp}}{L}\left(\frac{p_T}{T}\right)^2$$

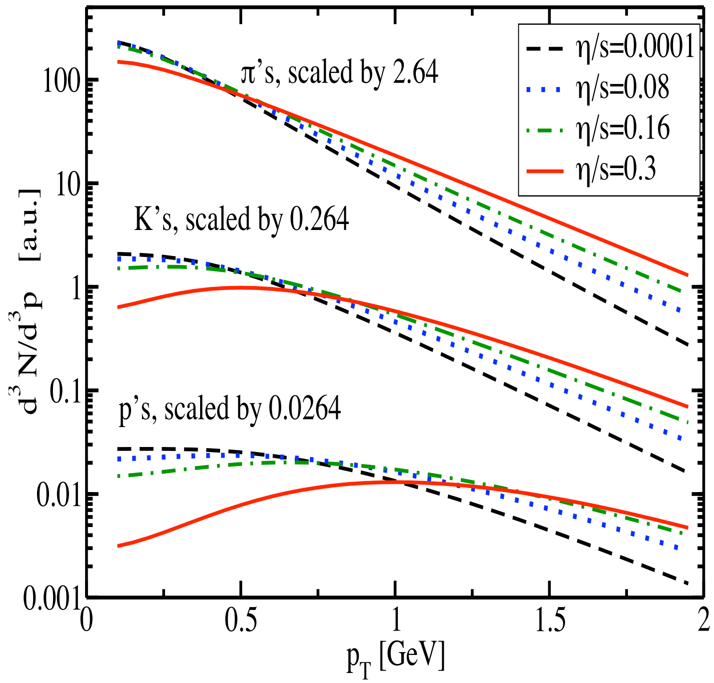
- Substitute this viscous correction when performing freezeout integrals for spectra

$$E\frac{dN}{d^3p} \sim \int p^\mu d\Sigma_\mu \, (f_o + \delta f)$$

Freezeout & Viscous Corrections to Spectra (2)

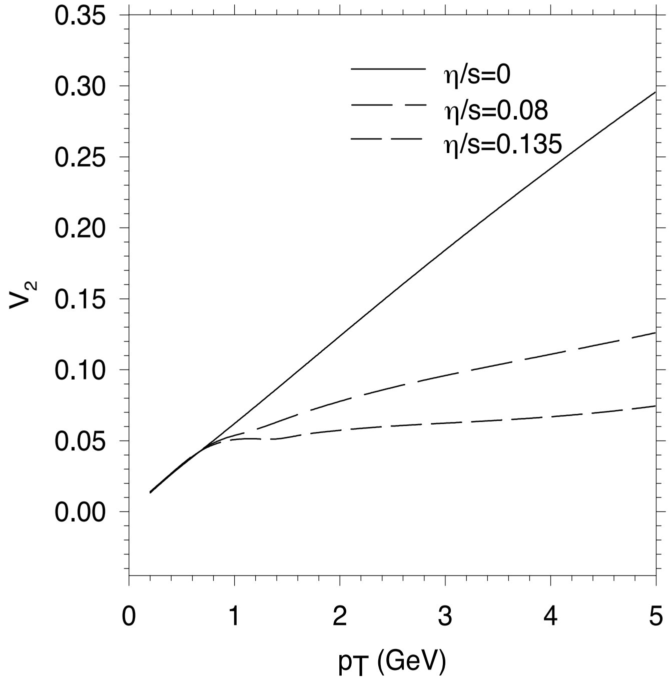
R. Baier, P. Romatschke nucl-th/0610108

- Freezeout $T = 0.135$



Chaudhuri, Phys. Rev. C. (2006)

- Freezeout $T = T_C - \text{tiny}$



- The correction comes entirely from the viscous correction to the distribution function – Yuck!

When viscosity becomes large hydro becomes unusable.

Viscous corrections to the spectra seen in the Data ?

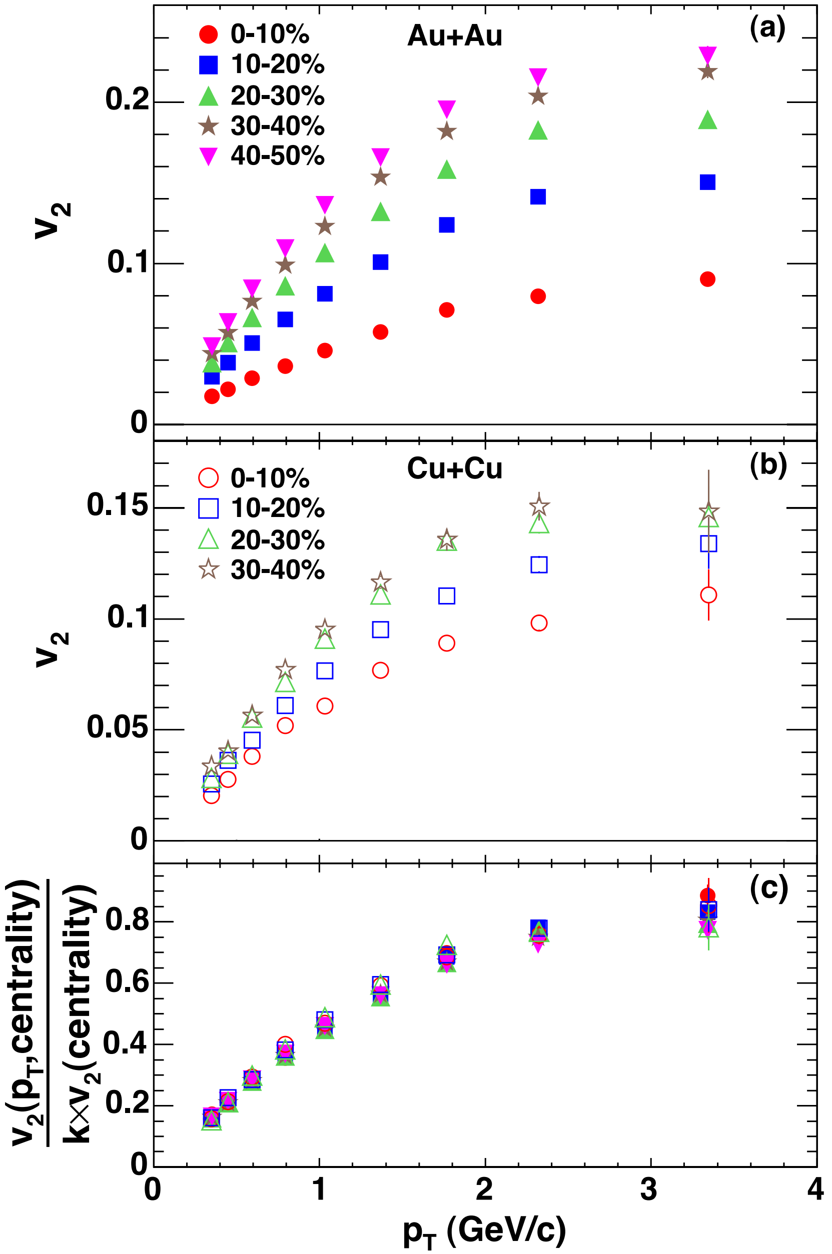
- Expect the spectrum to change as a function of mean free path to system size

$$f \rightarrow f_o + \delta f \qquad \delta f \sim \frac{\ell_{\rm mfp}}{L} \left(\frac{p_T}{T}\right)^2$$

- This leads to the prediction that $v_2(p_T)$ is not a universal function of p_T

$$\left.\frac{v_2(p_T)}{v_2}\right|_{\rm CuCu} \neq \left.\frac{v_2(p_T)}{v_2}\right|_{\rm AuAu}$$

- PHENIX, nucl-ex/0608033 plots



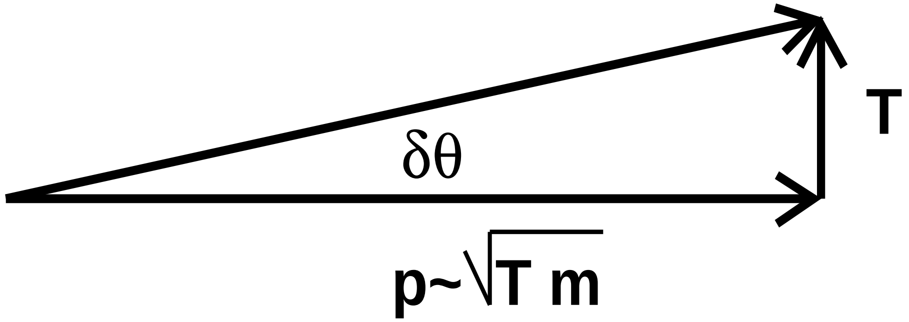
I find this universal p_T dependence difficult to reconcile with viscous hydro

Viscous Hydro Summary

- Viscous hydro is growing up.
- There are still tough issues to be resolved.
- Viscosity will place boundaries on where to believe ideal hydro.
  - Low p_T – How low?
  - Not too early – How early is too early?
  - Not too late – How late is too late?

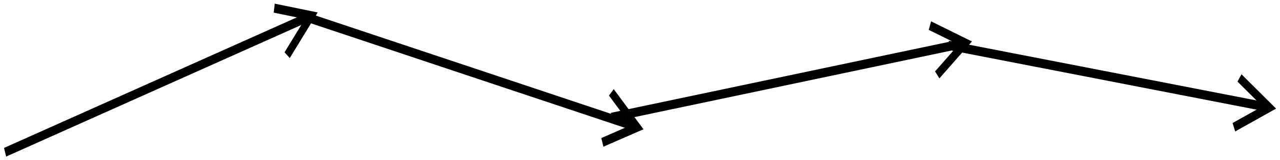
Will heavy quarks thermalize?

- Collisions scarcely change the direction of the heavy quark



$$\delta\theta \sim \sqrt{\frac{T}{m}}$$

- Thus for heavy quarks random walk



$$(\Delta\theta)^2 \sim N_{kick}\,(\delta\theta)^2 \sim N_{kick}\,\frac{T}{m}$$

- The equilibration time is then

$$\tau_R^{\text{heavy}} \sim \frac{M}{T}\tau_R^{\text{light}} \sim \frac{M}{T}\frac{\eta}{e+p}$$

Can we learn something about the mean free path from heavy quarks?

Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

$$\begin{aligned} \frac{dx}{dt} &= -\frac{p}{M} \\ \frac{dp}{dt} &= -\underbrace{\eta_D p}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}} \end{aligned}$$

- The drag and the random force are related

$$\langle \xi_i(t)\xi_j(t')\rangle = \frac{\kappa}{3}\delta_{ij}\,\delta(t-t') \qquad\qquad \eta_D = \frac{\kappa}{2MT}$$

- η_D is rate of momentum loss. $1/\eta_D$ is what we intuitively called τ_R^{charm}.

- Einstein relation between the drag and diffusion coefficients.

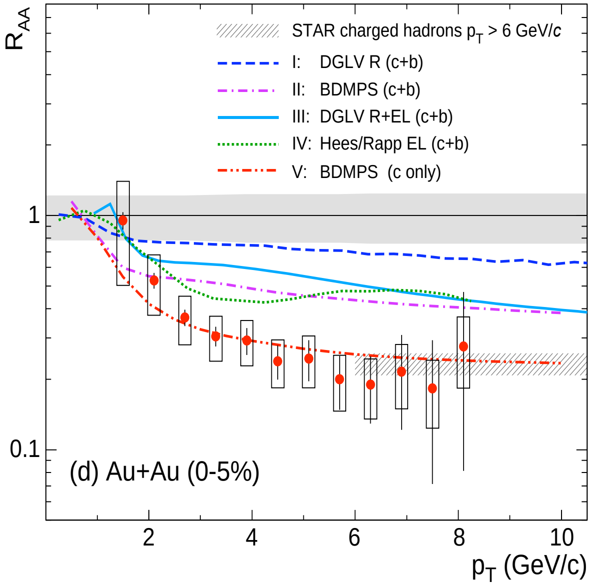
$$\frac{1}{\eta_D} = \frac{M}{T}\,D$$

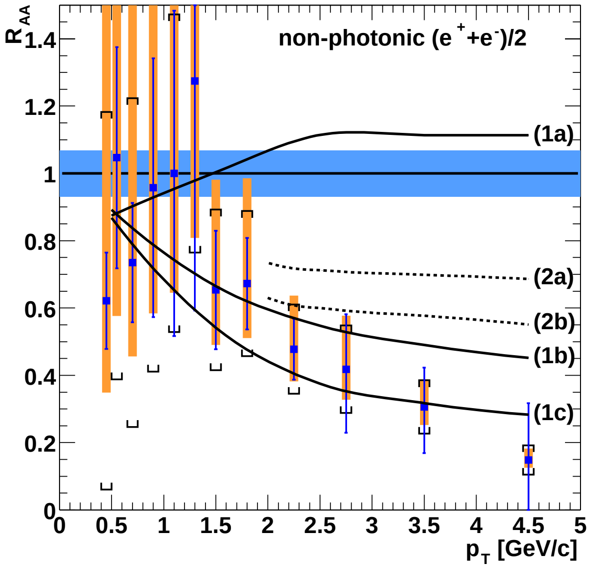
All parameters are related to the heavy quark diffusion coefficient

Want to describe heavy quark energy loss and flow

STAR: nucl-ex/0607012

Phenix: Phys. Rev. Lett. 96 (2006).





- These data are at high momentum.
- The Langevin description and Diffusion is strictly speaking non-relativistic.

Need to extrapolate to learn about transport coefficients

Extrapolating Langevin results to the experimentally relevant momentum range

- Assumes lots and lots of small changes rather than a few hard collisions.
- The drag and gaussian fluctuations becomes a function of momentum.

$$\frac{dp}{dt} \propto p \Leftrightarrow \text{Extreme Model}$$

- For a given diffusion D, this model will have the smallest R_{AA}

  1. Drag increases with momentum – Relaxation time independent of momentum!
  2. Fluctuations are minimal at large p_T

Can safely use this model to give an upper bound on D

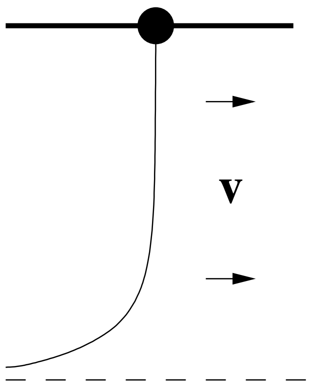
Connections with strongly coupled $\mathcal{N} = 4$ Super Yang Mills

- An example of a field theory where this model is realized

$$\frac{dP}{dt} = \frac{\sqrt{\lambda}\pi T^2}{2} \frac{P}{M}$$

with $\lambda = g^2 N_c$ is the coupling constant

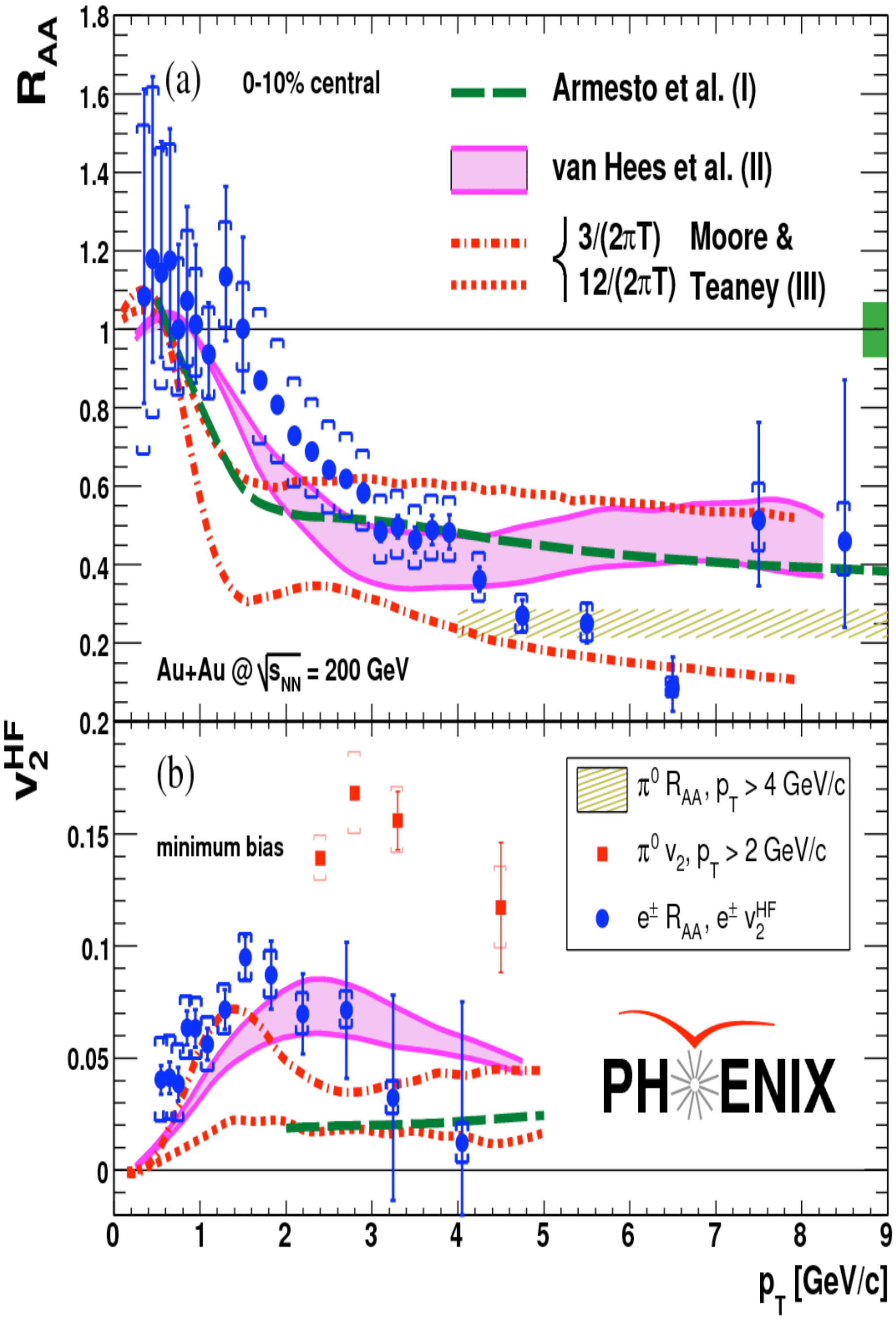
- Langevin approach appears relatively naturally



  - C. Herzog *et al.*; DT and J. Casalderrey; S. Gubser

A Langevin model for the heavy quarks in the medium

- Hydro + Heavy Quarks
- All parameters are related to the diffusion coefficient of the heavy quark, $\tau_R^{\text{charm}} = \frac{M}{T} D$
- Initial spectrum of heavy quarks from Caciarri et. al.
- Fragmentation and Decay of hadrons to get electrons
- Similar Langevin approach followed by Rapp & van Hees
  - Different Momentum dependence



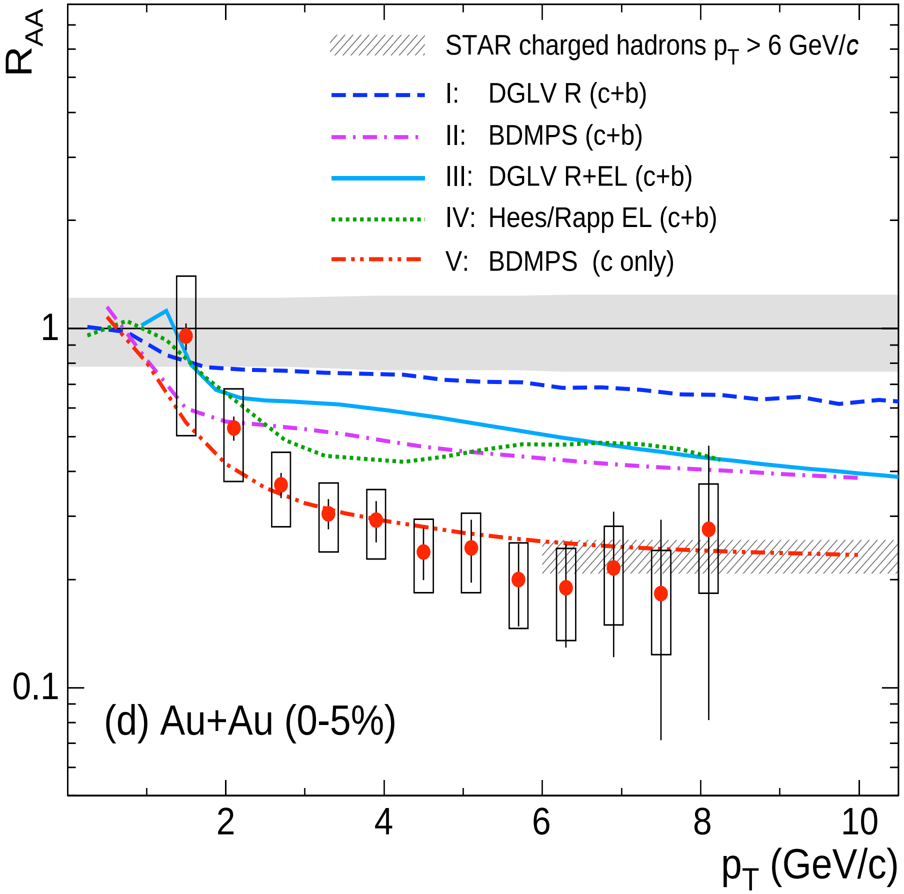
Summary

1. Suppression and Elliptic Flow are intimately related.
2. From the suppression pattern, we estimate that

$$D \lesssim \frac{12}{2\pi T}$$

3. With this diffusion coefficient, I can't produce enough elliptic flow.
4. Rapp and van Hees come closest to data with coalescence
   - Increases R_{AA}
   - Increases v_2

STAR Data



Much more to life than Langevin; apologies to many theorists

Wicks, Horowitz, Djordjevic, Armesto, Mustafa, Zhang, Molnar . . .

Summary – Where does hydro breakdown?

1. Small Systems
   - Implications of CuCu Data on Viscosity
     - Does not support a viscosity which is too small

$$\frac{\eta}{s} \gtrsim 0.33$$

     - More experimental and theoretical work is needed

2. Large Viscosity and “large” p_T
   - Efforts to simulate RHIC collisions with viscous hydro
     - Need η/s small (say $\eta/s = 0.3$) in order for hydro to be self-consistent.
     - Work is in progress to settle remaining issues

3. Heavy Particles

- Heavy Quarks and implications for Hydro
    - The data are interesting but confusing
    - Taken at face value

$$D \lesssim \frac{12}{2\pi T}$$

    - Difficult to accommodate the large elliptic flow of heavy flavors
    - Same problem in the light quark case

Looking forward to LHC