Thermalization via instabilities

Michael Strickland

Frankfurt Institute for Advanced Studies

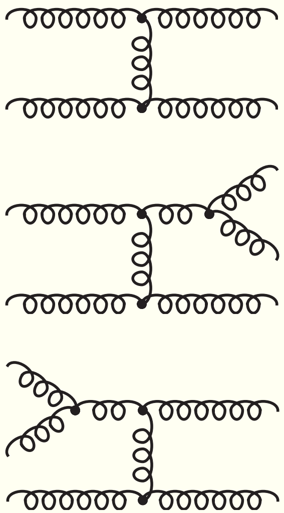
Quark Matter 2006, Shanghai, China
16 November 2006

FIAS
Frankfurt Institute for Advanced Studies

Motivation – Isotropization/Thermalization

- Need to understand mechanisms and time scales necessary for the isotropization and equilibration of a QGP at weak coupling.

- Consider pure glue. Processes include:
    - $2 \leftrightarrow 2$ elastic scattering (super slow)
    - Inelastic processes, e.g. $2 \rightarrow 3$ and $3 \rightarrow 2$ processes
    - Effect of *soft background fields* : expansion is in $g\,A$ not g; CGC $A \sim 1/g$



- Equilibrium: background fields screen the interaction (Debye)

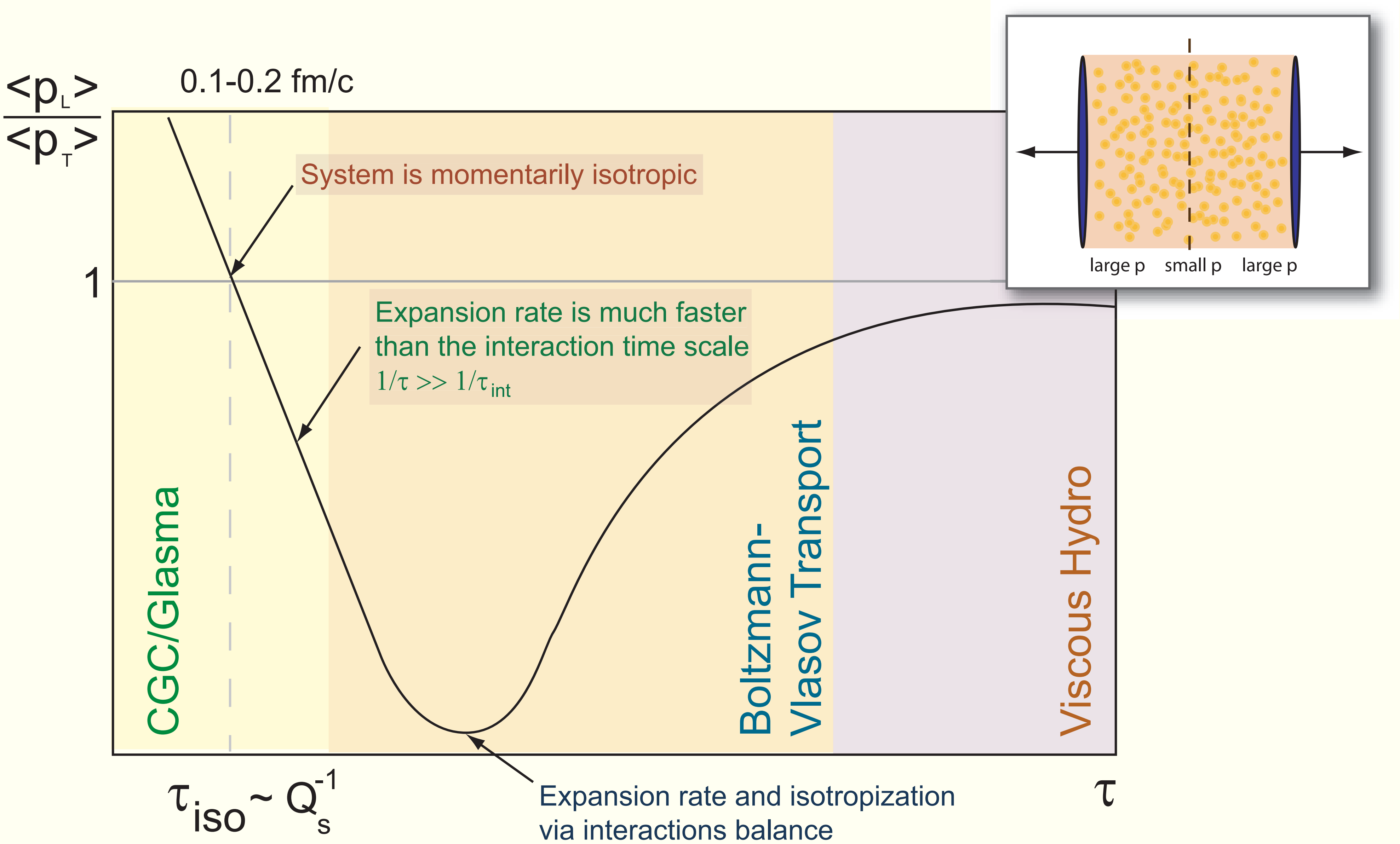
- Non-equilibrium: background fields can have non-trivial dynamics and can have a large effect on the particles’ motion

Improving upon Bottom-Up Thermalization

- Previous leading order perturbative results included 2 $\leftrightarrow$ 2, 2 $\rightarrow$ 3, and 3 $\rightarrow$ 2 processes [R. Baier, A. Mueller, D. Son, and D. Schiff, hep-ph/0009237; Numerical transport implementation: C. Greiner and Z. Xu, hep-ph/0406278]
- "Bottom-up" thermalization : soft modes isotropize and equilibrate first, then the hard modes $\rightarrow$ $\tau_{\rm therm} \sim \alpha_s^{-13/5} Q_s^{-1}$
- At RHIC $Q_s \sim$ 1.5 - 2 GeV and $\alpha_s \sim$ 0.3 $\rightarrow$ $\tau_{\rm therm} \sim$ 2 - 3 fm/c
- Bottom-up calculation ignored effect of local anisotropy in momentum space on soft physics (field dynamics)

In anisotropic systems plasma instabilities are present which will accelerate isotropization and thermalization.

Momentum Space Anisotropy Time Dependence

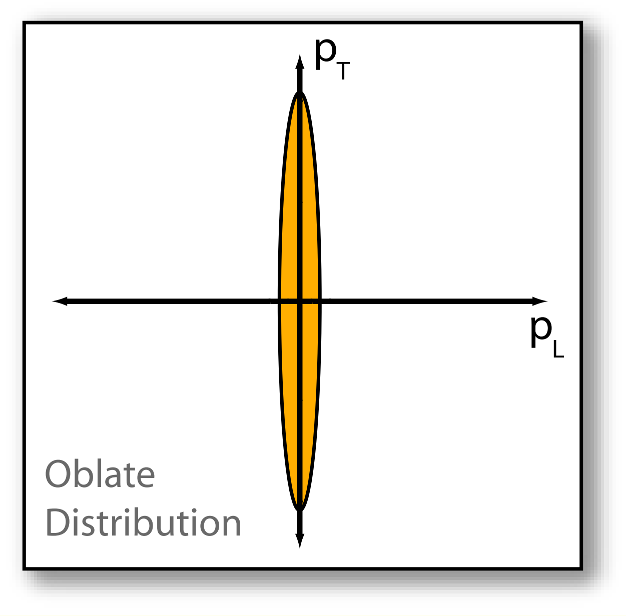


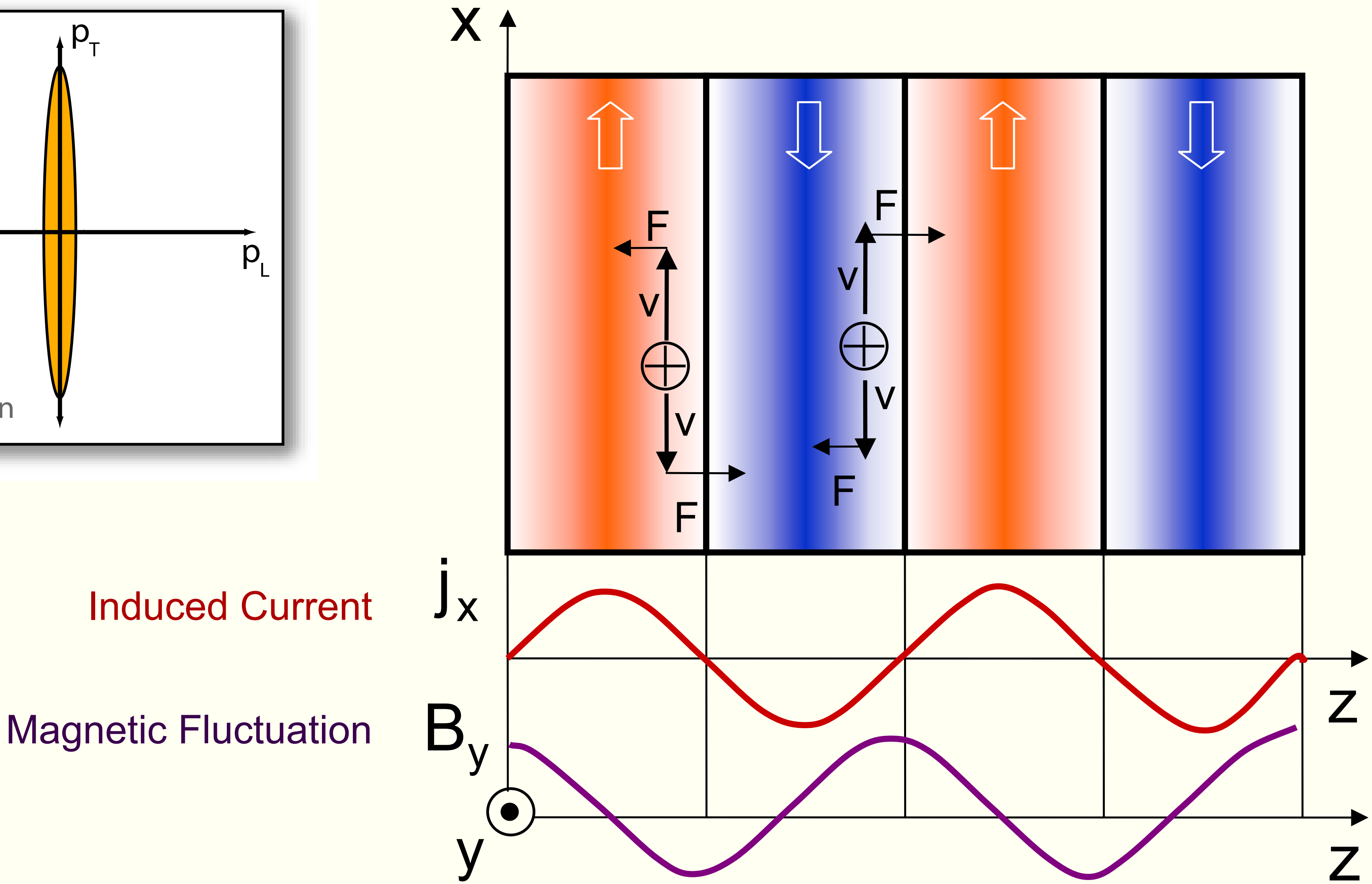
System is momentarily isotropic

Expansion rate is much faster than the interaction time scale $1/\tau >> 1/\tau_{int}$

Expansion rate and isotropization via interactions balance

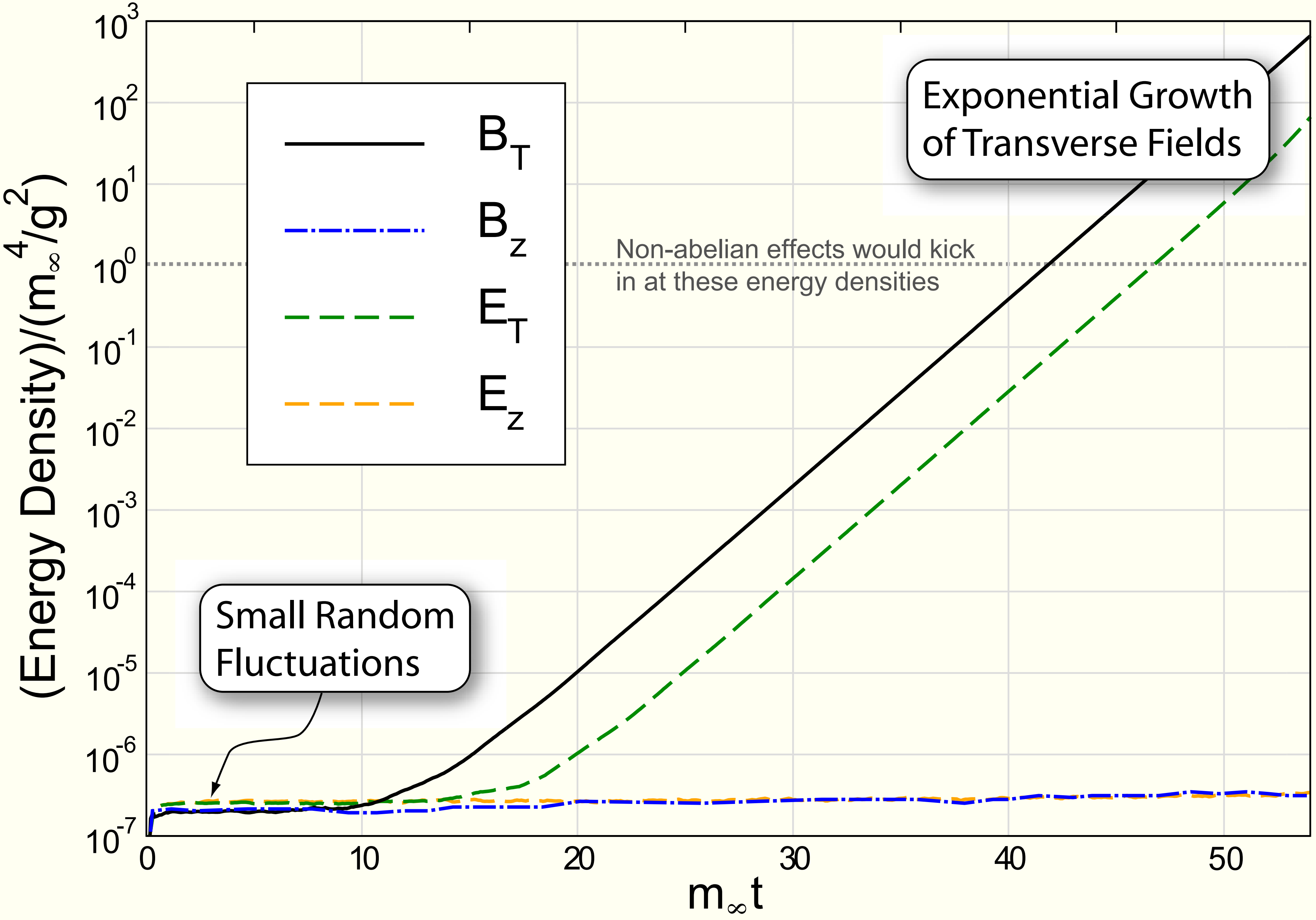
Current Filamentation in Abelian (QED) Plasmas





Right figure adapted from St. Mrówczyński, hep-ph/0511052.

Anisotropic Abelian Plasma – Weibel Instability (1959)



Exponential Growth of Transverse Fields

Non-abelian effects would kick in at these energy densities

Small Random Fluctuations

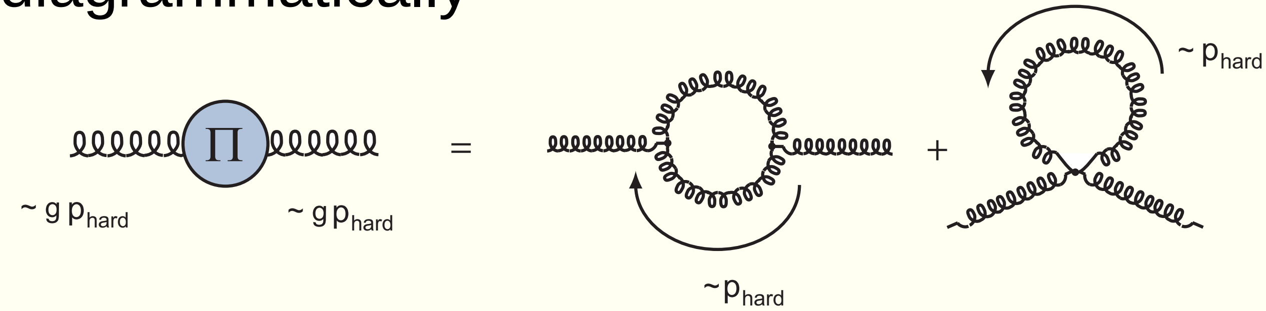
Anisotropic Gluon Polarization Tensor

Hard-loop gluon polarization can be obtained from either linearized Vlasov equation by expanding $f(p,x) \rightarrow f(\mathbf{p}) + \delta f(p,x)$

$$[v \cdot D_x, \delta f(p,x)] + g v_\mu F^{\mu\nu}\, \partial_\nu^{(p)} f(\mathbf{p}) = 0$$

$$D_\mu F^{\mu\nu} = J^\nu = g \int_p v^\nu \delta f(p,x)$$

or diagrammatically



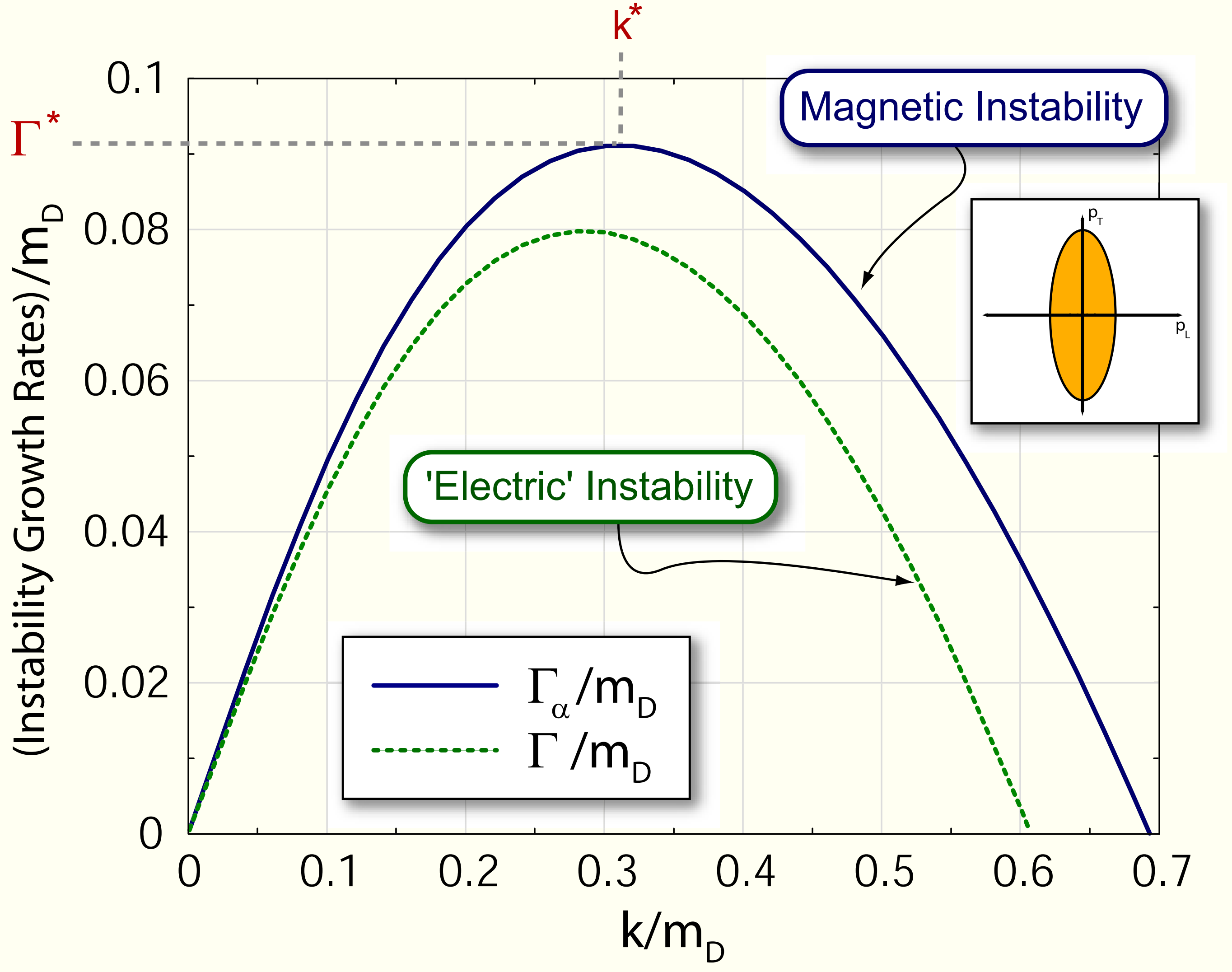
Assuming CGC
$p_{\text{hard}} = Q_s$

In both cases the result for the gluon polarization tensor

$$\Pi^{ij}_{ab}(\omega,\mathbf{k}) = -g^2 \delta_{ab} \int_{\mathbf{p}} v^i \frac{\partial f(\mathbf{p})}{\partial p_l} \left(\delta^{jl} - \frac{v^j k^l}{\omega - \mathbf{v}\cdot\mathbf{k} + i\epsilon} \right)$$

St. Mrówczyński and M. Thoma, hep-ph/0001164; P. Romatschke and MS, hep-ph/0304092

Unstable Mode Spectrum – Oblate Distribution



Using $\alpha_s = 0.3$

$$\begin{aligned} m_D &= g\,Q_s \\ &\rightarrow 3 - 4 \text{ GeV} \end{aligned}$$

$$\Gamma \sim 0.8 - 2.4 \text{ GeV}$$

e-Folding time
0.1 - 0.3 fm/c

Instability growth rates as a function of momentum for $\langle p_T^2 \rangle / \langle p_L^2 \rangle \simeq 10$ and $\theta_{\rm glue} = \pi/8$ with respect to the beamline.

P. Romatschke and MS, hep-ph/0304092

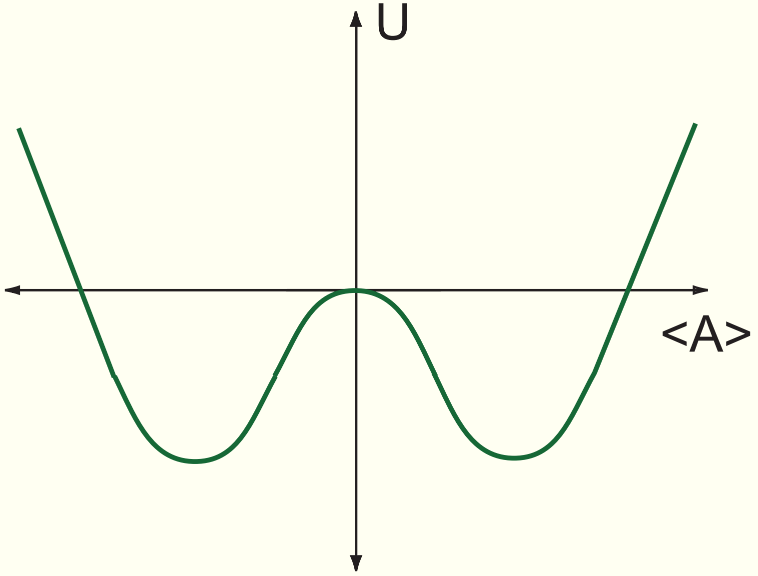
Anisotropic Hard-loop Effective Action

Require gauge invariance

↓

Effective action for soft fields

$$S_{\rm soft} = S_{\rm QCD} + S_{\rm HL}$$



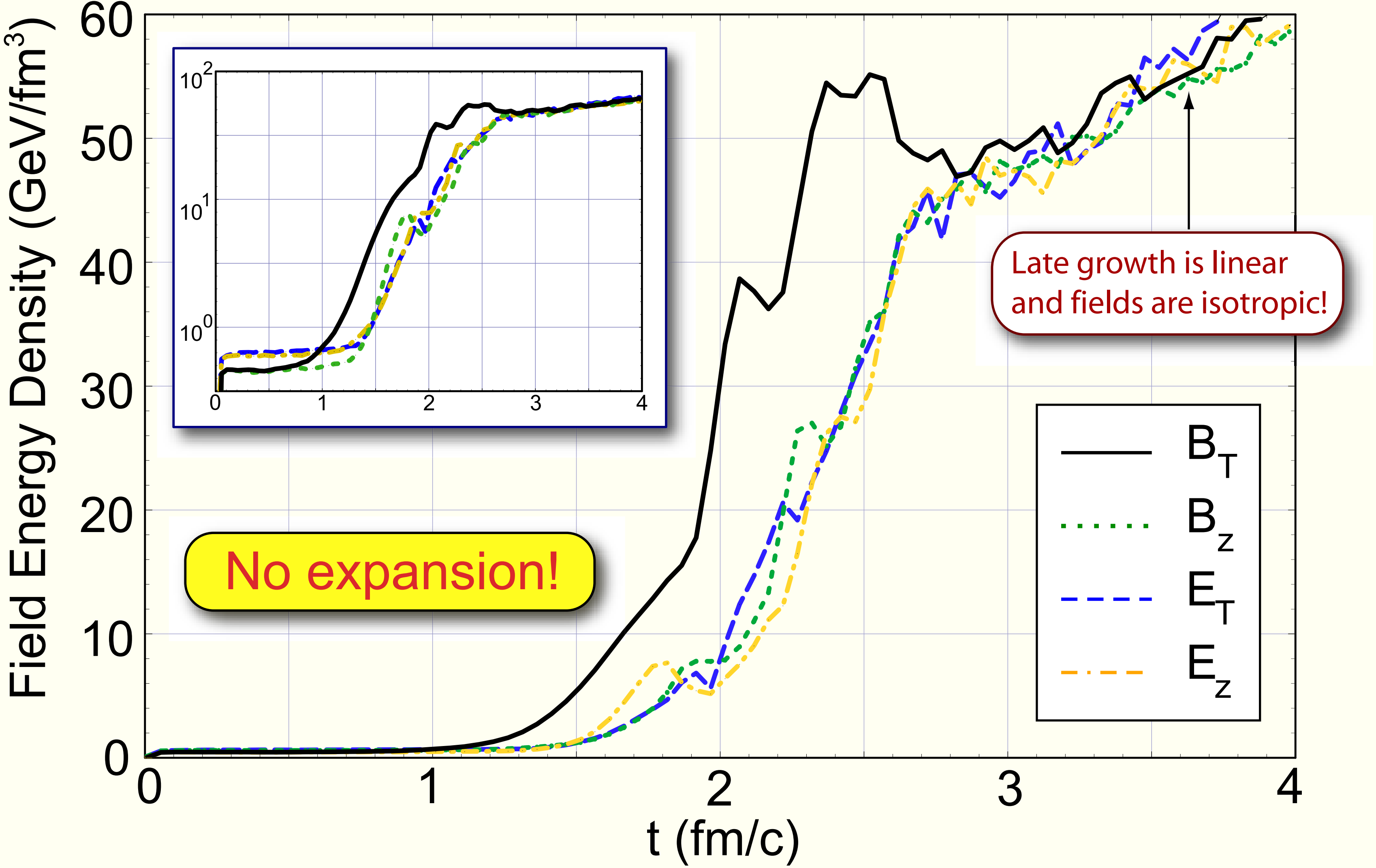
$$S_{\rm HL} \;\; = \;\; \Pi\, A^2 + \Gamma^{(3)}\, A^3 + \Gamma^{(4)}\, A^4 + \ldots$$

Gives gauge covariant non-linear equations of motion for evolution of soft degrees of freedom which can then be solved numerically.

St. Mrówczyński, A. Rebhan, and MS, 2004

3D SU(2) Hard-Loop Results

$\langle p_z^2 \rangle / \langle p_T^2 \rangle = 0.1 \qquad m_D^2 = 10\ \mathrm{GeV}^2$

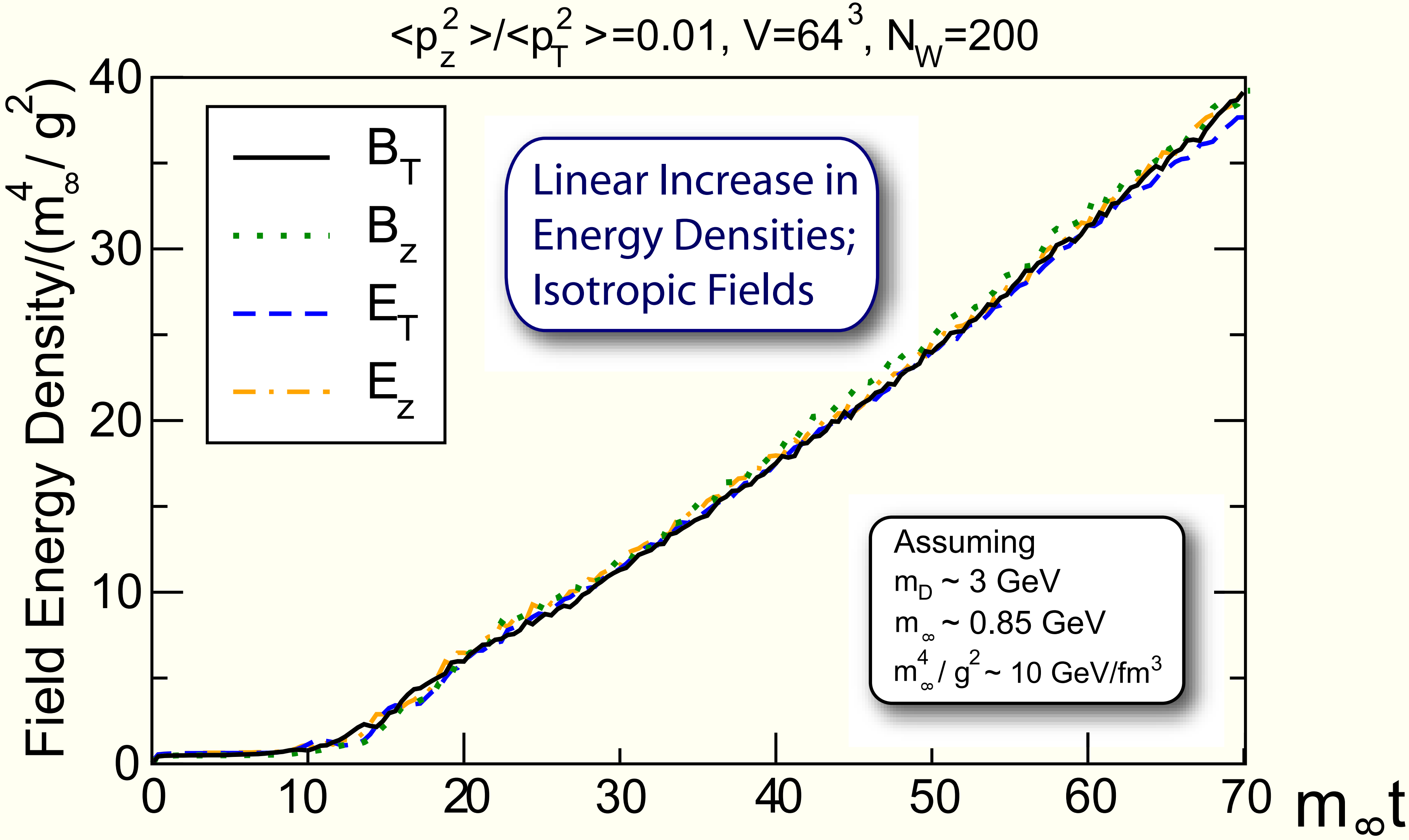


No expansion!

Late growth is linear and fields are isotropic!

A. Rebhan, P. Romatschke, and MS, hep-ph/0505261
P. Arnold, G. Moore, and L. Yaffe, hep-ph/0505212

3D SU(2) Hard-Loop Results – Linear growth



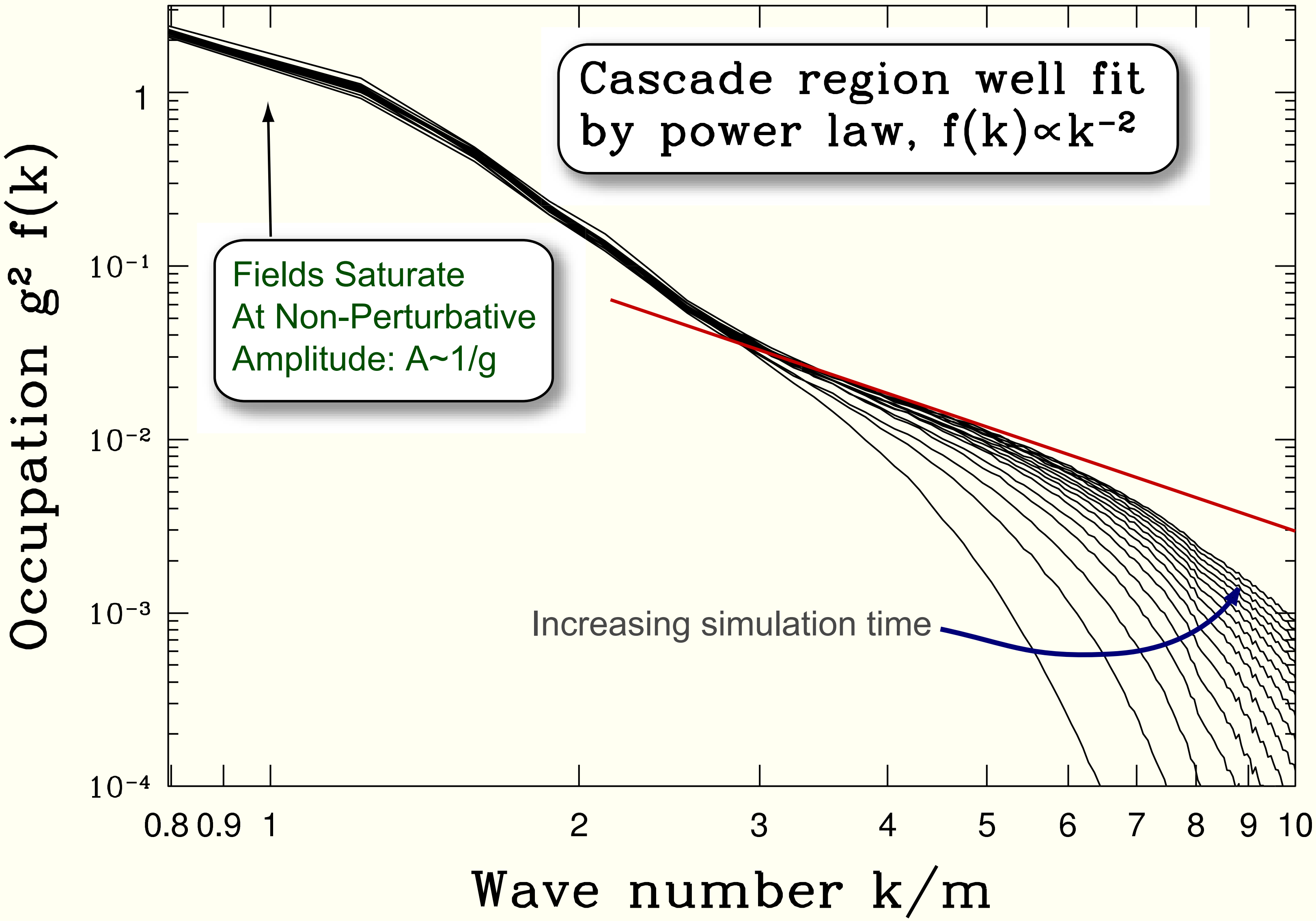
$\langle p_z^2 \rangle / \langle p_T^2 \rangle = 0.01$, $V = 64^3$, $N_W = 200$

Linear Increase in Energy Densities; Isotropic Fields

Assuming
$m_D \sim 3$ GeV
$m_\infty \sim 0.85$ GeV
$m_\infty^4 / g^2 \sim 10$ GeV/fm^3

A. Rebhan, P. Romatschke, and MS, hep-ph/0505261
P. Arnold, G. Moore, and L. Yaffe, hep-ph/0505212

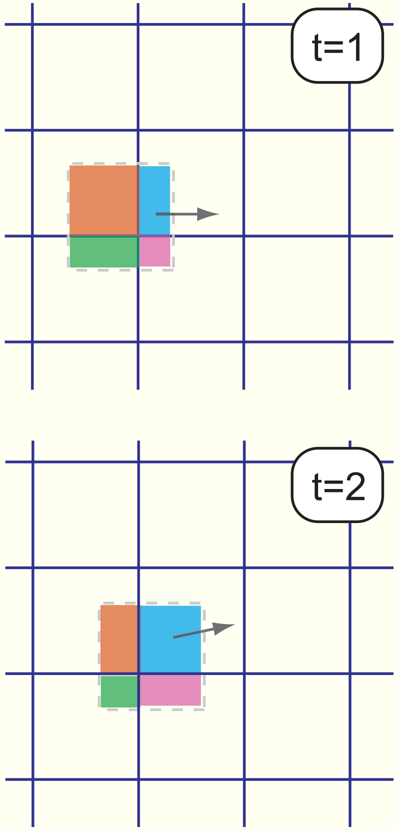
Kolmogorov cascade $\longrightarrow$ Turbulent Fields



P. Arnold and G. Moore, hep-ph/0509206; hep-ph/0509226.

3D Colored-Particle-in-Cell Simulations (CPIC)

Hard-loop approximation strictly only applies when we ignore the back-reaction of the particles on their self-generated fields. How can we go beyond hard-loops?



Include back-reaction by solving collisionless transport equation without linearization

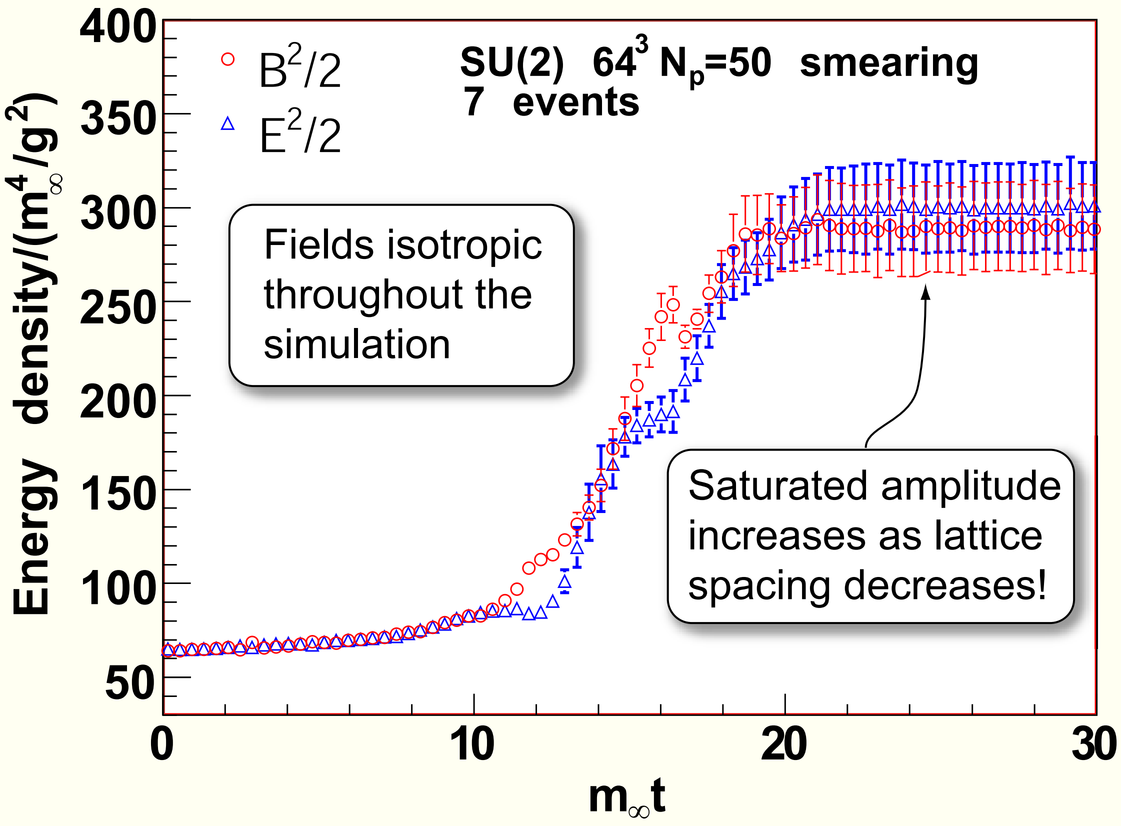
$$p^\mu[\partial_\mu - g q^a F^a_{\mu\nu}\partial^\nu_p - g f_{abc} A^b_\mu q^c \partial_{q^a}] f(x,p,q) = 0$$

Coupled to the Yang-Mills equation for the soft gluon fields,

$$D_\mu F^{\mu\nu} = J^\nu = g \int \frac{d^3p}{(2\pi)^3} dq\, q\, v^\nu f(t,\boldsymbol{x},\boldsymbol{p},q)$$

A. Dumitru, Y. Nara, and MS, hep-ph/0604149

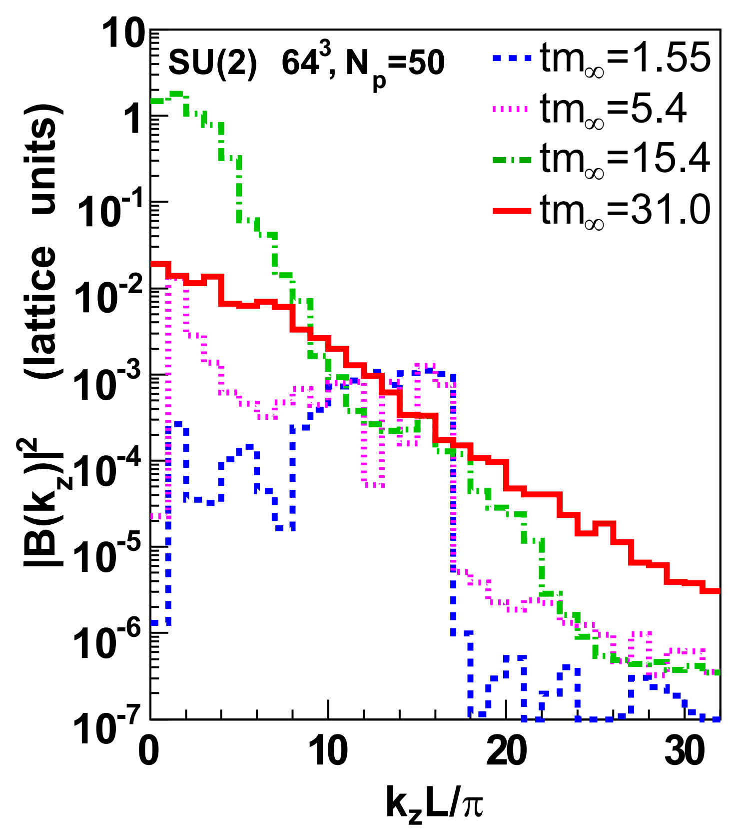
CPIC Results – Ultraviolet Avalanche



Fields isotropic throughout the simulation

Saturated amplitude increases as lattice spacing decreases!

$L = 5$ fm, $p_{\mathrm{hard}} = 16$ GeV, $g^2\, n_g = 10/\mathrm{fm}^3$, $m_\infty = 0.12$ GeV.

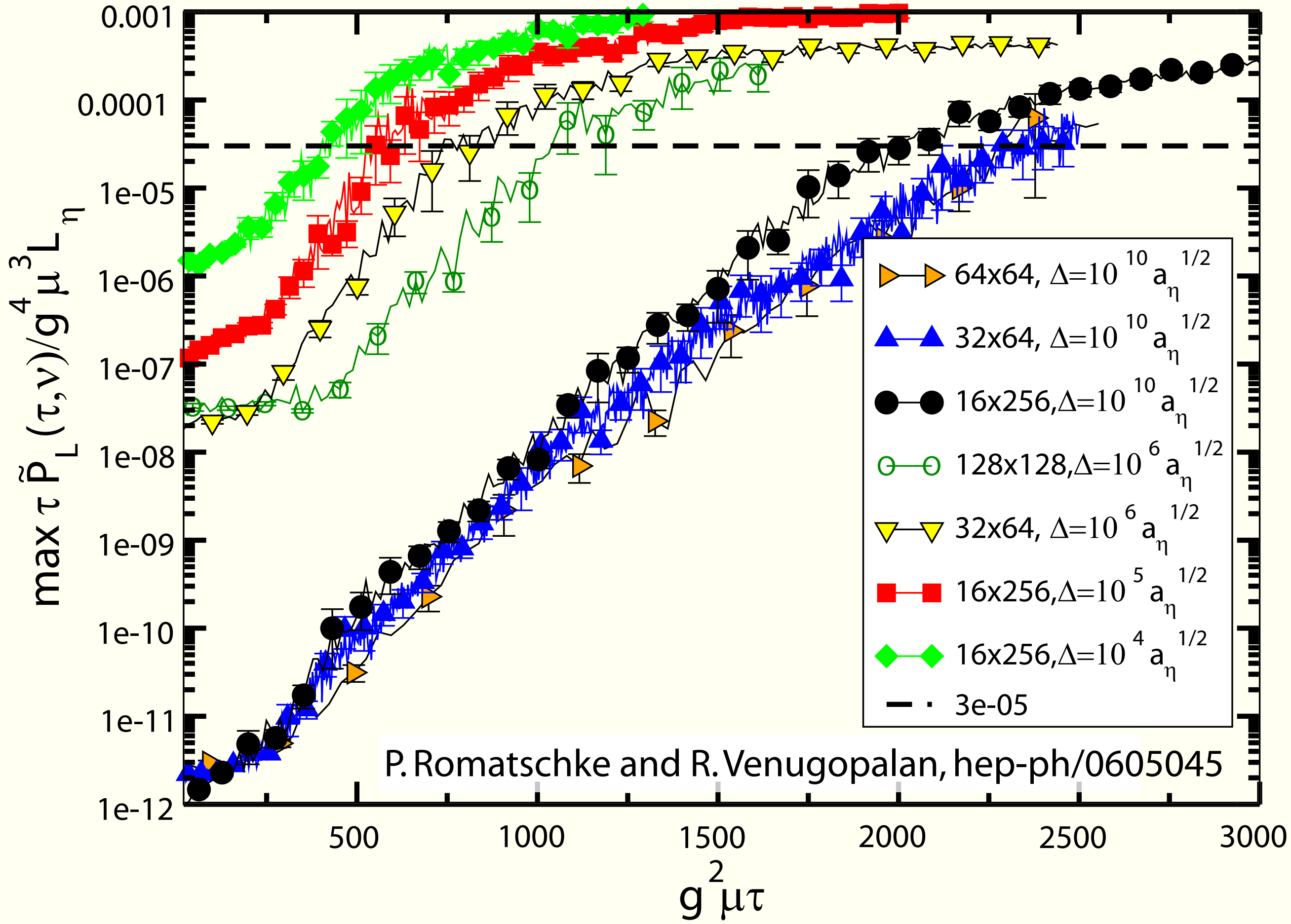


Squares of the Fourier transformed color-magnetic fields at four different times.

A. Dumitru, Y. Nara, and MS, hep-ph/0604149

Instabilities in classical YM – The unstable glasma

Instabilities also seen in expanding classical Yang-Mills solutions which include rapidity fluctuations.



P. Romatschke and R. Venugopalan, hep-ph/0605045

Growth $\sim$ $e^{\sqrt{Q_s\tau}}$ agrees with hard loop estimate!

[P. Arnold, J. Lenaghan, and G. Moore, hep-ph/0307325]

[P. Romatschke and A. Rebhan, hep-ph/0605064]

Initial spectrum of rapidity fluctuations from CGC camp

[K. Fukushima, F. Gelis, and L. McLerran, hep-ph/0610416]

More recent developments

- Effect of hard collisions → even at $\alpha_s \sim 0.3$ instabilities persist

  [B. Schenke, C. Greiner, M. Thoma, and MS, hep-ph/0603029]

- Increased relative longitudinal broadening of jets (“The Ridge”)

  [P. Romatschke and MS, hep-ph/0309093, hep-ph/0408275]

  [P. Romatschke, hep-ph/0309093, hep-ph/0607327]

  [A. Majumder, B. Müller, and S. Bass, hep-ph/0611135]

- Small viscosities generated by turbulent fields

  [M. Asakawa, S. Bass, and B. Müller, hep-ph/0603092, hep-ph/0608270] **PARALLEL SESSION**

- Anisotropic fermionic collective modes → no unstable modes

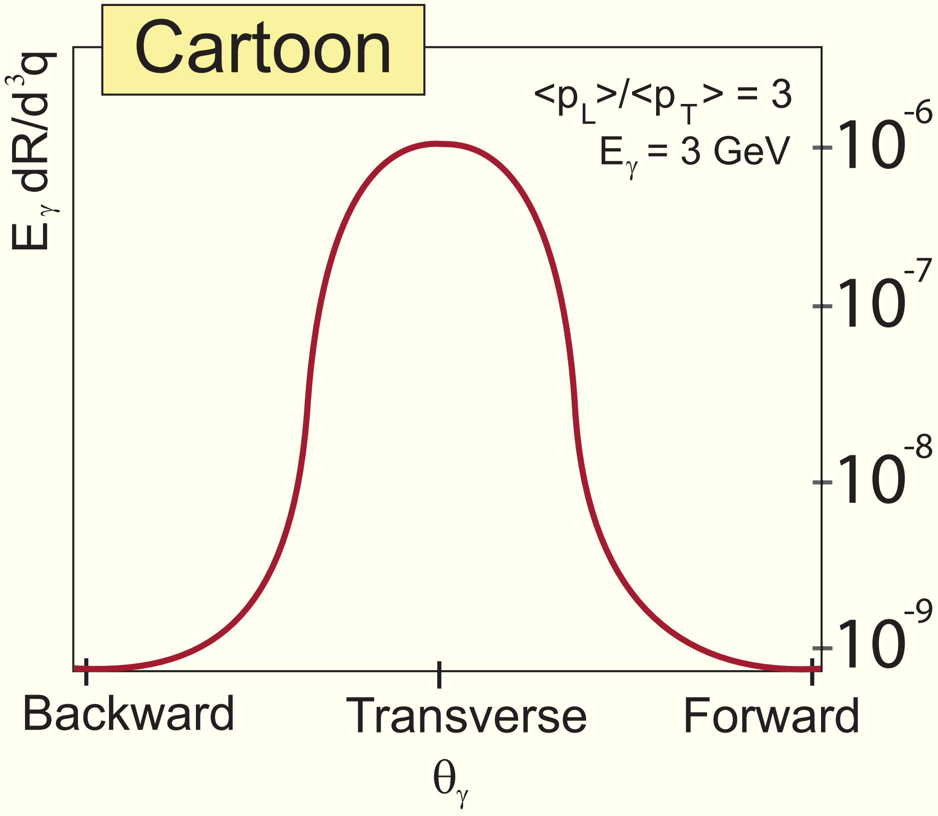
  [B. Schenke, and MS, hep-ph/0606160]

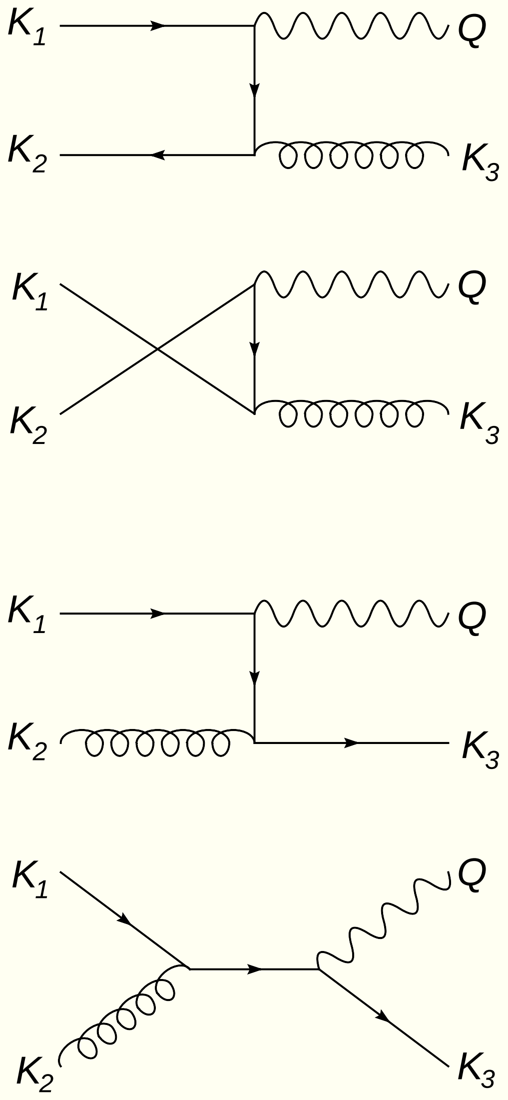
- Chromo-hydrodynamic approach to non-equilibrium QGP

  [C. Manuel and St. Mrówczyński, hep-ph/0606276] **PARALLEL SESSION**

Photons from an Anisotropic Plasma

- Compute hard part from diagrams to the right.
- Compute soft part from HL resummed quark propagator.





B. Schenke and MS, forthcoming.

Conclusions

- Anisotropic plasmas are qualitatively different than isotropic ones.
- Hard-Loop : Fields show isotropic linear growth accompanied by cascade to UV.
- CPIC : Rapid isotropic field growth followed by UV “avalanche”.
- Classical YM : rapidity fluctuations → the “glasma” is unstable to becoming a QGP!
- Systematic calculations of p_T-p_L anisotropy observables such as jet effects and E&M signatures.
- My educated GUESS for the RHIC thermalization time including the Weibel instability is 1.5 - 2 fm/c. Isotropization earlier . . .

Can it be even faster? Hard-loop, classical YM and CPIC simulations including realistic rapidity fluctuations have yet to be performed.

. . . More hard work ahead . . .