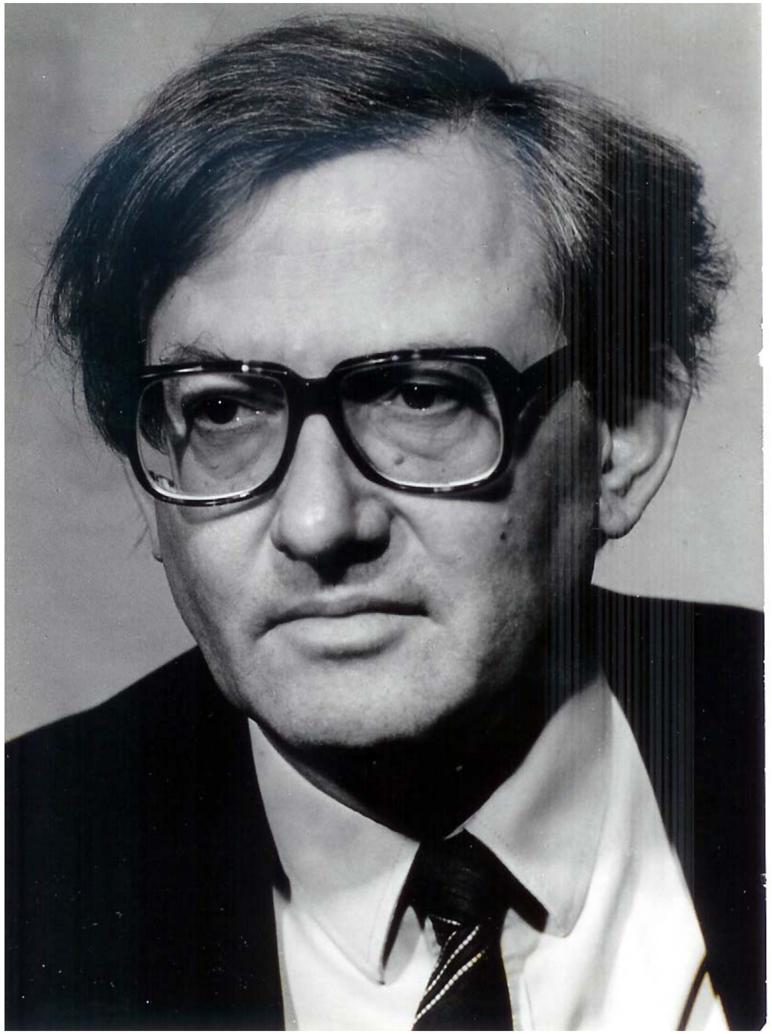
VISCOSITY in the strongly interacting quark matter around the T_c critical temperature

--- Dedicated to the memory of J. Zimanyi ---

P. Lévai (RMKI, Budapest)

with: P. Hartmann, Z. Donkó (SZFKI, Budapest),
G. J. Kalman (Boston Coll., USA)

Quark Matter 2006 Conference
Shanghai, 16 Nov. 2006



József ZIMÁNYI (Jozsó)
(1931-2006)
Honorary chair of QM2005

Founder of the Hungarian relativistic heavy ion physics;

Teacher of the Budapest group;

Mastermind behind the Hungarian membership and activity at CERN;

Reformer of the Hungarian NSF;

Dreamer of the QGP for 30 years !

1954-74: Low energy nuclear physics (KFKI, Budapest)
γ-γ correlation; stripping reaction; optical potential; ...

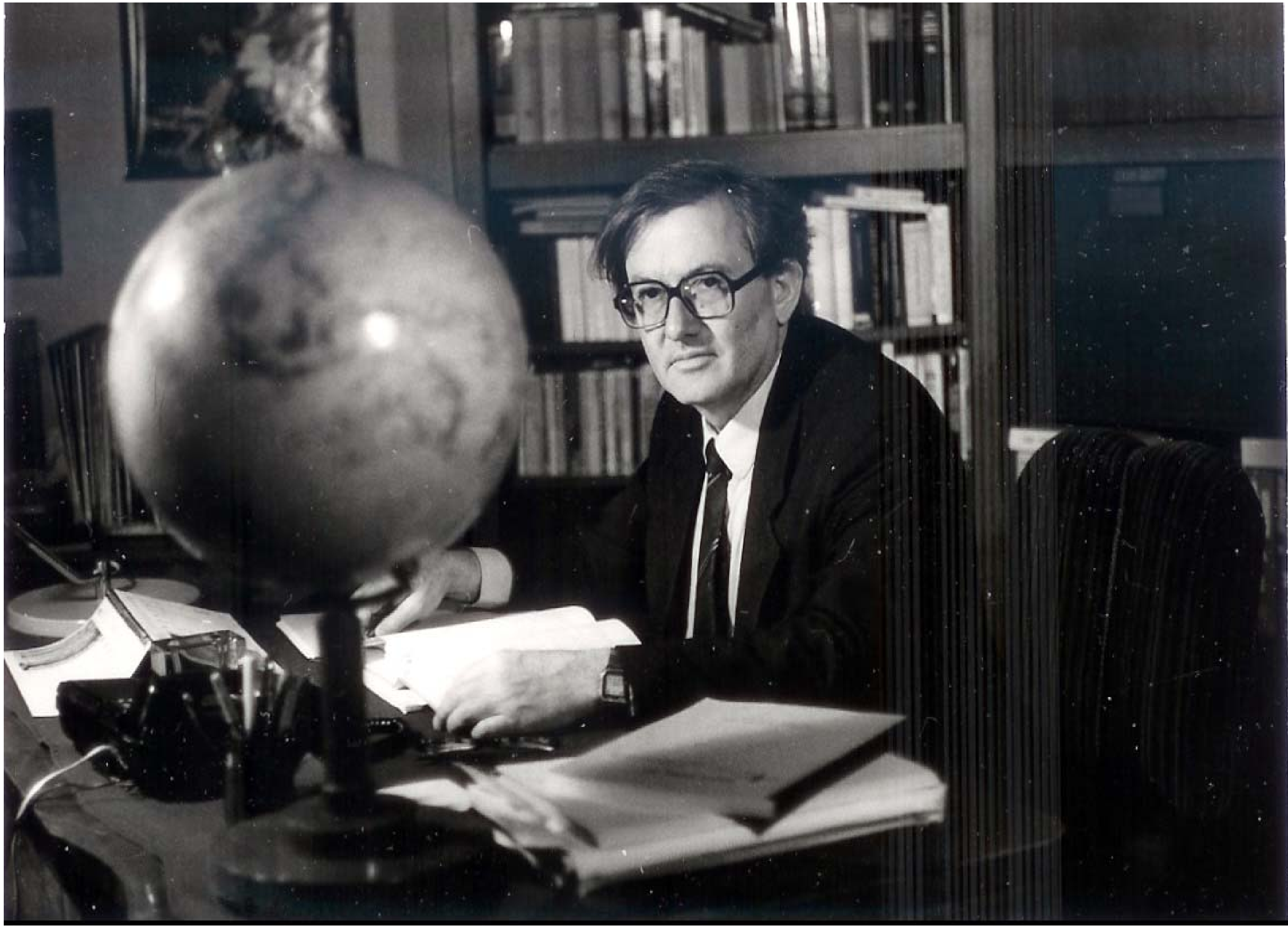
1969-... : Visiting Copenhagen (Ole Hansen, NORDITA)

1974 The birth of QCD and relativistic heavy ion physics (T.D. Lee)

1979-06: ~100 theory articles and papers about HIC

Hydrodynamical model: Bondorf-Garpman-Zimányi, '79
Hadrochemistry: Montvay-Zimányi, '79
Bose condensation: Zimányi-Fai-Jakobsson, '79
Neural network: Csernai-Zimányi, '79
Shock front: Bondorf-Ivanov-Zimányi, '81
Three-fluid model: Lovas-Zimányi-Csernai-Greiner, '81
Quark chemistry: Biró-Zimányi, '82
Entropy and hadrochemistry: Biró-Barz-Lukács-Zimányi,'83
Quark-Gluon Plasma formation: Biró-Zimányi, '85
Relativistic Mean Field Theory: Zimányi-Bondorf-Mishustin, '85
Zimányi-Moskowski, '90
Pion correlation (SPACER) : Csörgő-Zimányi, '90-...
Quark coalescence (ALCOR): Biró-Lévai-Zimányi, '95-...
Quark mass spectra: Zimányi-Biró-Lévai-Ván, '05-...

Member of the CERN SPS NA49, RHIC PHENIX, CERN LHC ALICE !



Zimányi Winter School, Budapest, 11-13 Dec. 2006

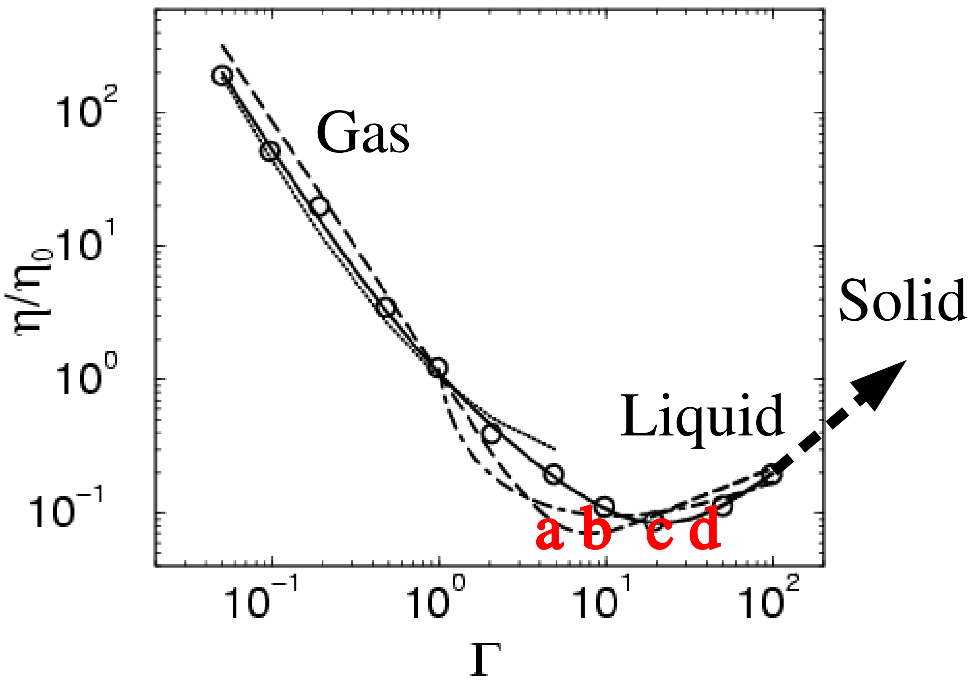
Zimányi Memorial Workshop, Bp, 28-30 June 2007

Motivations: - how large is the viscosity in QGP around T_c ?

- how can we determine it ?

Binary Ionic Mixture (BIM OCP)

S. Bastea, PRE71,2005,056405



$\eta/\eta_0(\Gamma)$ has an U-shape in Γ !!

What about the shape of η/s(T) ???

What about the properties of sQGP ?

What about sQGP ?

SU(3), N_f=2, d= 16 + 12 + 12

$$\Gamma = \frac{4\pi\alpha_s(T)}{a_{WS}T} = C\alpha_s(T)$$

$$a_{WS}^3 = \frac{3}{4\pi n(T)}$$

$$\eta_0 = \sqrt{\frac{\alpha_s(T)M}{a_{WS}^5}\left(\frac{3}{4\pi}\right)^3}$$

T=200 MeV, SU(3), N_f=2

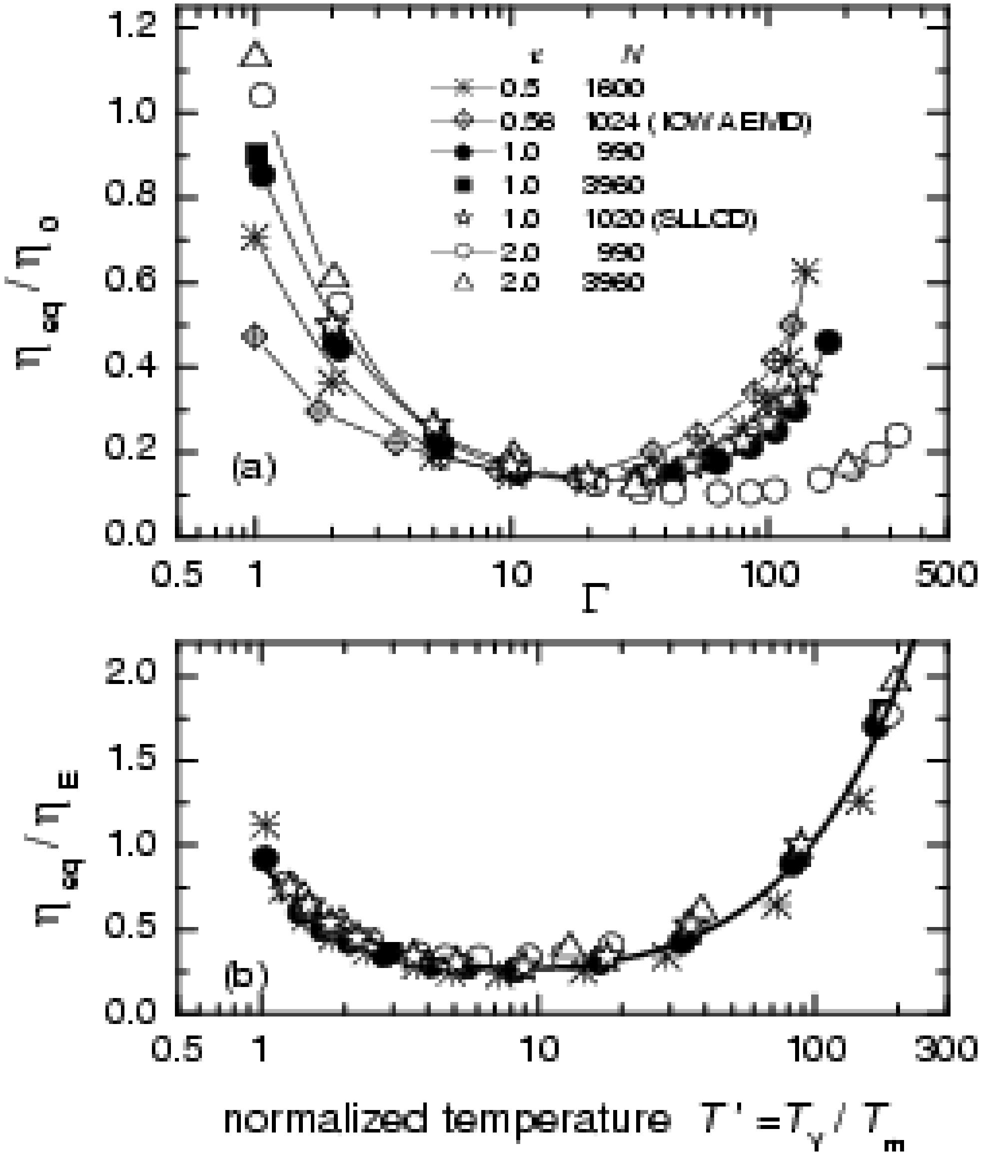
a, $\alpha_s = 0.17$ ↣ $\Gamma = 5.1$

b, $\alpha_s = 0.25$ ↣ $\Gamma = 7.3$

c, $\alpha_s = 1.0$ ↣ $\Gamma = 24$

d, $\alpha_s = 2.0$ ↣ $\Gamma = 40$

<u>Answer in dense plasma physics:</u> η(T) has a U-shape ! η/s ?



Z. Donkó, J. Goree,
P. Hartmann, K. Kutasi:
Shear viscosity in 2D Yukawa-liquid
PRL96(2006)145003

Molecular dynamical simulation with a finite number of particle.

Can we do it in YM case?
How large is the viscosity in other models?

Danielewicz-Gyulassy:
$\eta/s \approx 1$ **PRD31(1985)**

Content:

1. Viscosity in ~free massive quark gas [classical physics]
   in weakly interacting QGP [QCD at high-T]

2. Lattice-QCD results ↣↣↣ sQGP
   --- viscosity in the quasi-particle picture
   --- different models (width, spectral function)

3. Molecular dynamical simulation for massive quasi-particles with Yukawa-interaction
   ↣ ↣ ↣ viscosity at $1 < T/T_c < 2$

4. Viscosity effects on high-p_T jets

5. Summary

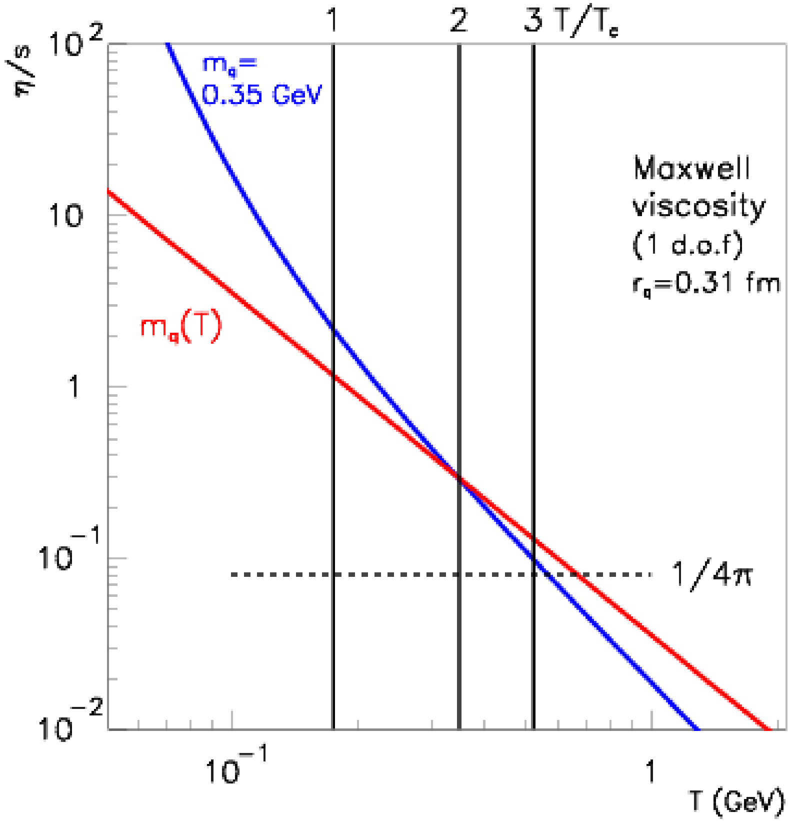
Viscosity of ~free massive fermi (q) gas [1 degree of freedom]

A, constant mass, $m_q = 0.35$ GeV

B, temperature dep. mass, $m_q(T) = g\, T/\sqrt{3}$ [$g=1.77$, $\alpha_s=0.25$]

$\eta(T)$: Maxwell viscosity ➔ $\eta = \frac{m_q}{3\sqrt{2}\,\pi\, d^2}\sqrt{\frac{8T}{\pi\, m_q}}$ $[d = 2 r_q]$ $[r_q = 0.31\ fm, \sigma = 3\ mb]$

$s(T) = (\varepsilon(T) + P(T))/T$ for free massive fermi gas, $s(T) \sim T^3$



NO Minimum (?)

$\eta/s = 1/4\pi$ at $T \approx 3\ T_c$

$\eta/s = 0.2 - 2$ at $2 > T/T_c > 1$

Did we find the wanted "perfect fluid" in the classic description of a free massive gas ???

[Where is the U-shape ??!]

Viscosity of ~free massive fermi (q) gas [1 degree of freedom]

A, constant mass, $m_q = 0.35$ GeV

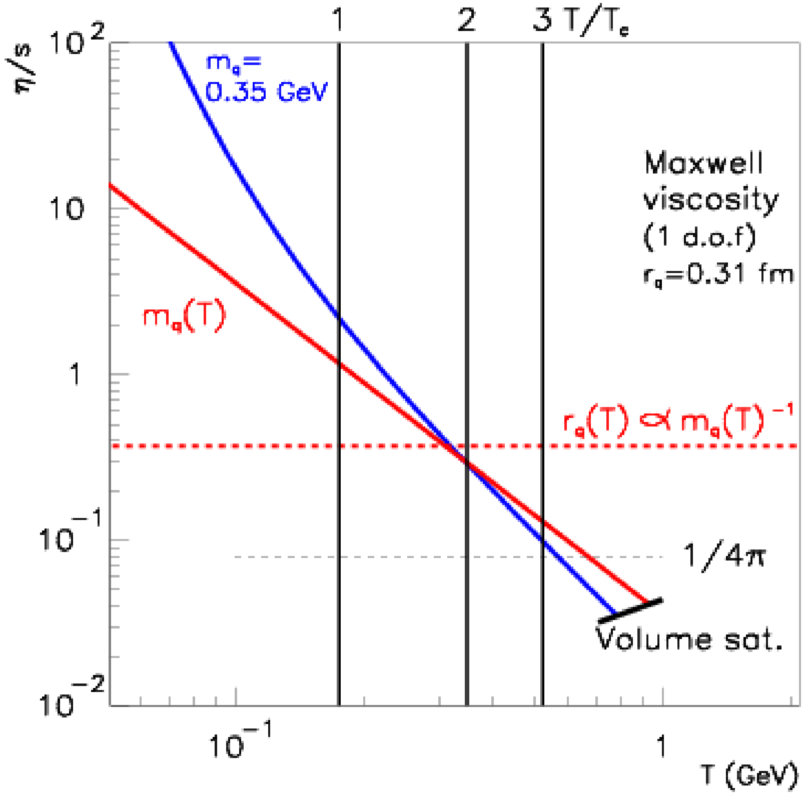
B, temperature dep. mass, $m_q(T) = g\,T/\sqrt{3}$ [g=1.77, α_s=0.25]

$\eta(T)$: Maxwell viscosity ➔ $$\eta = \frac{m_q}{3\sqrt{2}\,\pi\, d^2}\sqrt{\frac{8T}{\pi\, m_q}}$$

$[d = 2 r_q]$

$[\sigma = \pi r_q^2 \sim (g T)^{-2}]$

C, ---------- $$r_q(T) = \frac{1}{2}\frac{\hbar c}{m_q(T)} \sim \frac{1}{g T}$$



A & B : Constant volume

↣ Volume saturation

$\eta/s \approx (1/4\pi)/2$ at $T \approx 5\, T_c$

C, -------- $\eta \sim T^3,\ s \sim T^3$

$$\frac{\eta}{s} = const.$$

More T-dependence is needed in η for the U-shape

Viscosity in weakly coupled QGP [SU(3), gluon + $N_f = 2$, g=const.]

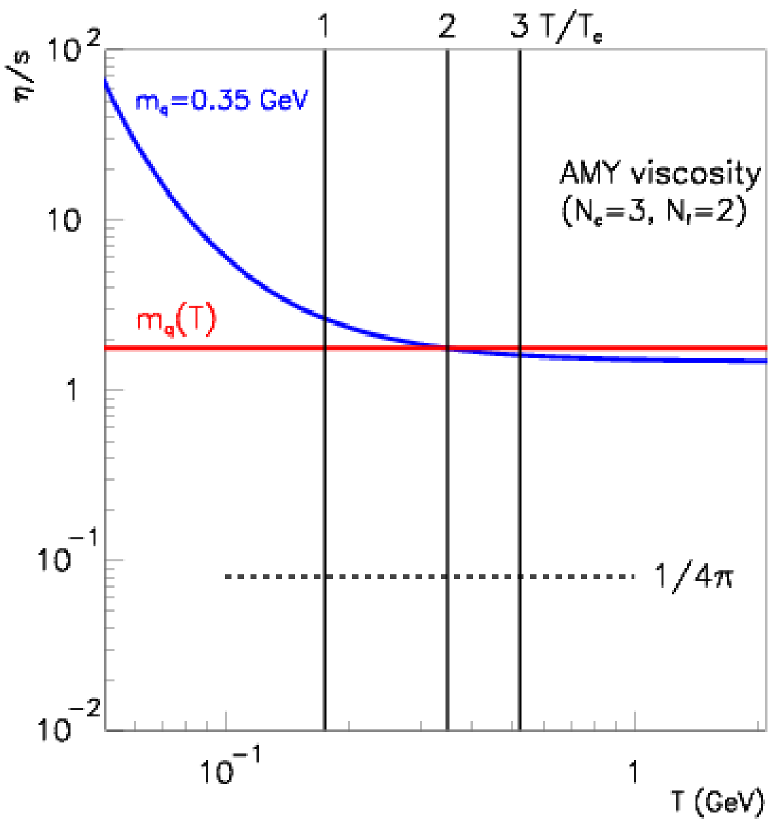
A, constant mass, $m_q = 0.35$ GeV, $m_g = \sqrt{2}\, m_q$

B, temperature dep. mass, $m_q(T) = g\, T/\sqrt{3}$ [g=1.77, α_s=0.25]

$\eta(T)$: AMY viscosity ➔ $\eta = \kappa \dfrac{T^3}{g^4 \ln g^{-1}}$ $[\kappa = 86.47]$

(Arnold, Moore, Yaffe, JHEP 2003)

$s(T) = (\varepsilon(T) + P(T))/T$ for massive fermi and bose gas, $s(T) \sim T^3$



NO Minimum

If T ↣ ∞ then

η/s ↣ $20(1/4\pi)$

At $1 < T/T_c < 2$

$\eta/s = 2 - 3$

This matter is not a "perfect fluid" !

Viscosity in weakly coupled QGP [SU(3), gluon + $N_f = 2$, g(T)]

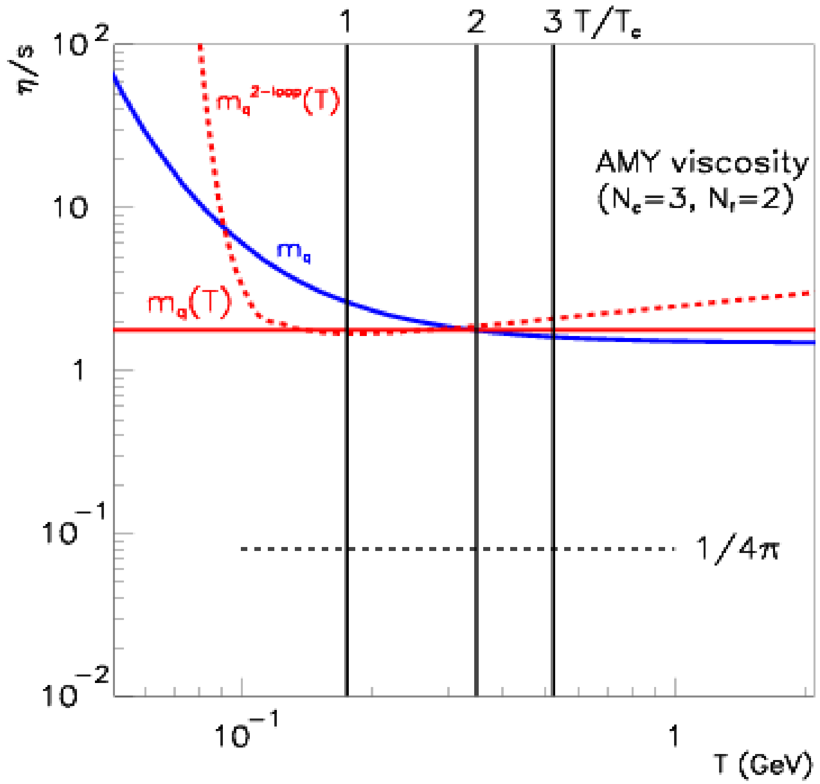
A, constant mass, $m_q = 0.35$ GeV, $m_g = \sqrt{2}\, m_q$

B, temperature dep. mass, $m_q(T) = g\, T/\sqrt{3}$ [$g=1.77$, $\alpha_s=0.25$]

$\eta(T)$: AMY viscosity ➔ $$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$ $[\kappa = 86.47]$

(Arnold, Moore, Yaffe, JHEP 2003)

C, ---- 2-loop renorm. $$g(T)^{-2} = \frac{9}{8\pi^2} \ln \frac{T}{\mu_0} + \frac{4}{9\pi^2} \ln\left(2 \ln \frac{T}{\mu_0}\right)$$ $[\mu_0 = 0.03\, GeV]$



C: Minimum at

$T \approx T_c$, $\eta/s = 20\ (1/4\pi)$

$1 < T/T_c < 2$, $\eta/s \approx 2$

$T \rightarrow \infty$, $\eta/s \sim \ln^2 T$

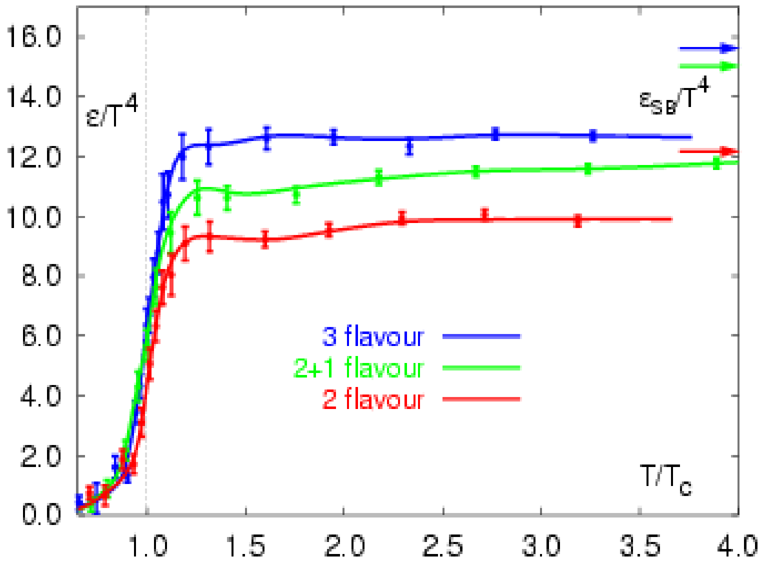
So far this is the best candidate but it is not "perfect fluid".

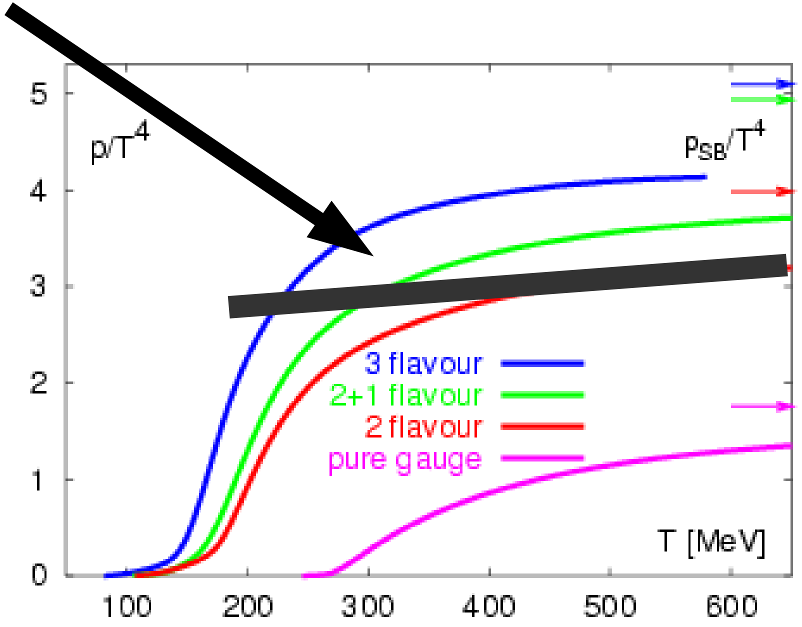
"Low-viscosity matter"
Csernai, Kapusta, McLerran, PRL97, 152303, 2006

Lattice-QCD results: EOS for strongly interacting QGP

Testing $g^{2\text{-loop}}(T)$ ➔ $\varepsilon(T)$, $P(T)$ in SU(3), $N_f = 2$

$m_q(T) = g(T)T/\sqrt{3}$; $m_g(T) = \sqrt{2}\, m_q(T)$





(Karsch et al. 1992)

$g^{2\text{-loop}}(T)$ is maybe good asymptotically, but not for $T < 2\, T_c$

Non-ideal EOS ↣ quasi-particle picture of strongly interacting QGP

Can we extract viscosity in the quasi-particle description ???

Introducing quasi-particle picture:

SU(3) Gluon EOS with free quasi-gluons + B(T) bag

Fix degrees of freedom (d=16)

$$P(T)=\frac{d}{(2\pi)^3}\int d^3p\ \frac{p^2}{3\sqrt{p^2+M(T)^2}}\ \left[\exp\frac{\sqrt{p^2+M(T)^2}}{T}-1\right]^{-1}-B(T)$$

$$\varepsilon(T)=\frac{d}{(2\pi)^3}\int d^3p\ \sqrt{p^2+M(T)^2}\ \left[\exp\frac{\sqrt{p^2+M(T)^2}}{T}-1\right]^{-1}+B(T)$$

ϵ(T), P(T) ↣ M(T), B(T)

Mass + Interaction

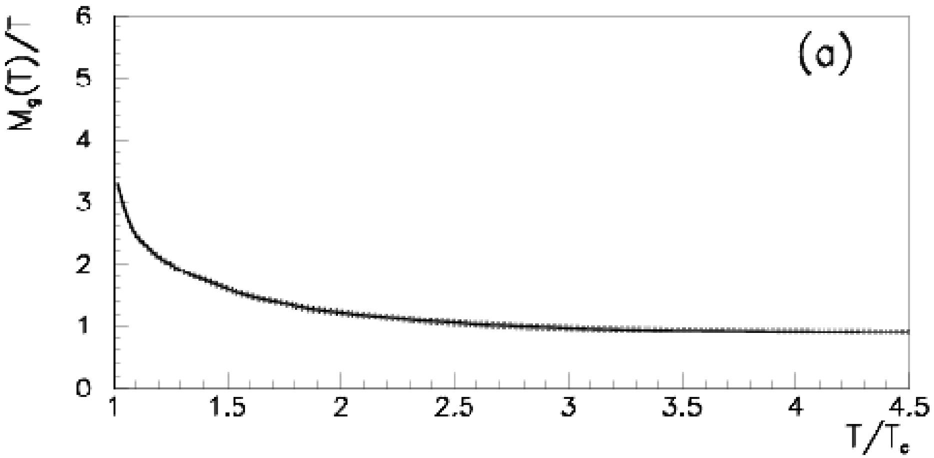
P. L., U. Heinz, 1996, PRC51, 3326.

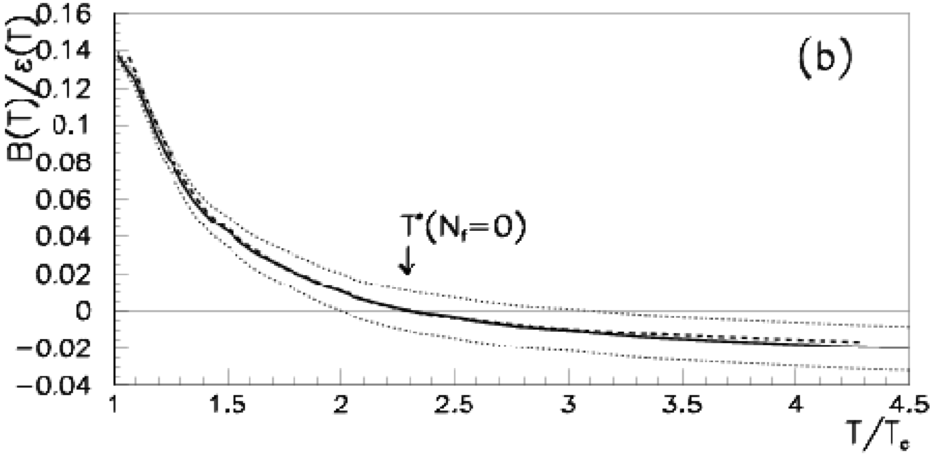
A. Peshier, B. Kampfer, ..., 1996,PRD54, 2399.

Extracting g(T) and B(T) from lattice-QCD results:

SU(3) Gluon EOS with free quasi-gluons + B(T) bag

Fix degrees of freedom (d=16)





$\epsilon(T)$, P(T) ↣ M(T), B(T)

$T > 2\,T_c$

$M(T)/T = c \cdot g(T)$ [$c=1/\sqrt{2}$]

$T \to 5T_c$, $\alpha_s = g^2/4\pi \to 0.15$ ✓

B(T) is small (≈0)

Free quasi-gluons !

$T < 2\,T_c$

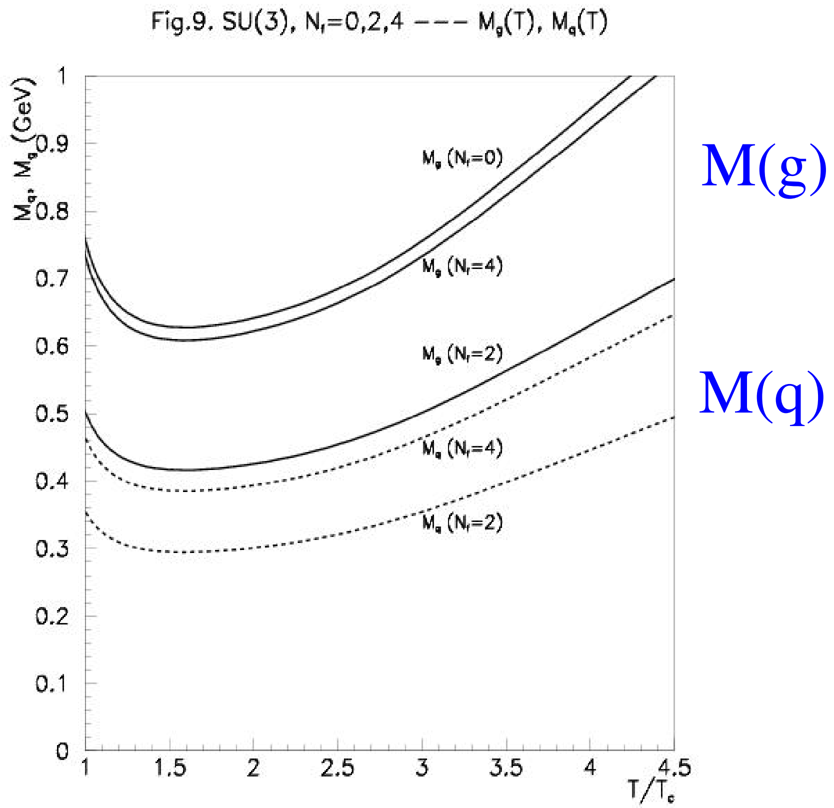
Interacting quasi-gluons

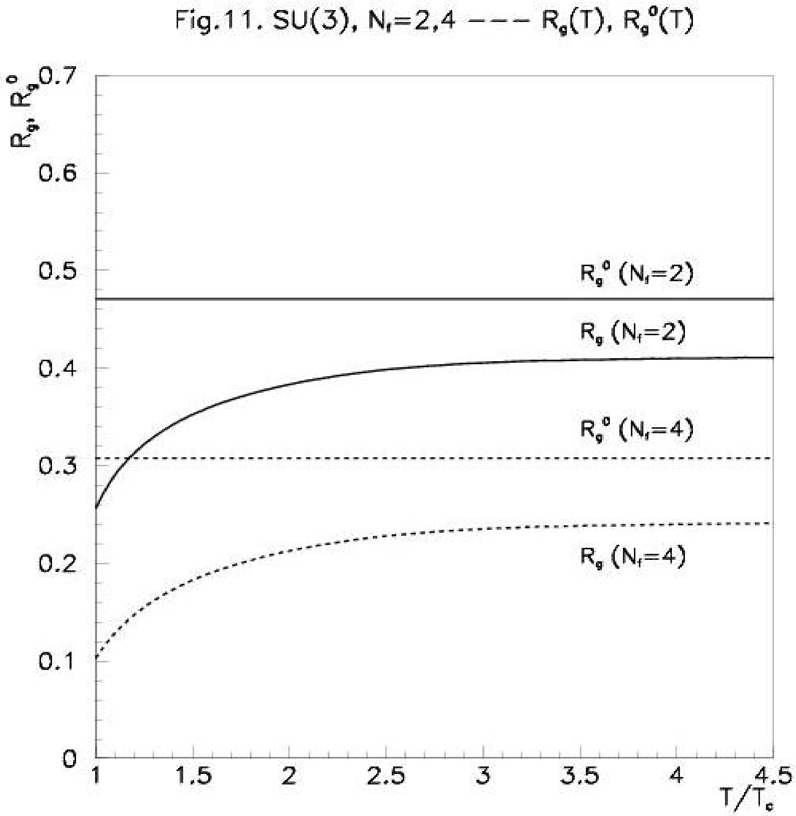
or new excitations ?

Or both ?

Quark matter formation in heavy ion collisions

Lattice-QCD results around T_c, SU(3), N_f=0,2,4 μ=0 (1990 - ...)





Massive quasi-particles: $M_q \simeq$300 MeV, $M_g \simeq$ 500-800 MeV

Dilute system with the suppression of the heavy gluons: $(R_g = N_g / (N_g + N_q + N_{anti\text{-}q})$

[L.P, Heinz U., 1996, PRC51,3326]

➔ Quark and antiquark dominated matter (QAP)

HADRONIZATION ⇔ QUARK COALESCENCE (ALCOR '95)

('Cross-over' phase transition) **[T.S. Biro, P.L., J. Zimányi]**

Model for interacting massive gluonic quasi-particles [SU(3), N_f=0]

B(T) ↔ interaction between gluons

$P_{tot}(T) = P_{kin}(M(T),T) - B(T)$

$\varepsilon_{tot}(T) = \varepsilon_{kin}(M(T),T) + B(T)$

B(T) ➣ attractive (effective) scalar field

$$B(T) = \frac{1}{2} m_\sigma^2 \sigma^2 = \frac{1}{2} \frac{g^2}{m_\sigma^2} n^2$$

M(T) ➣ effective mass

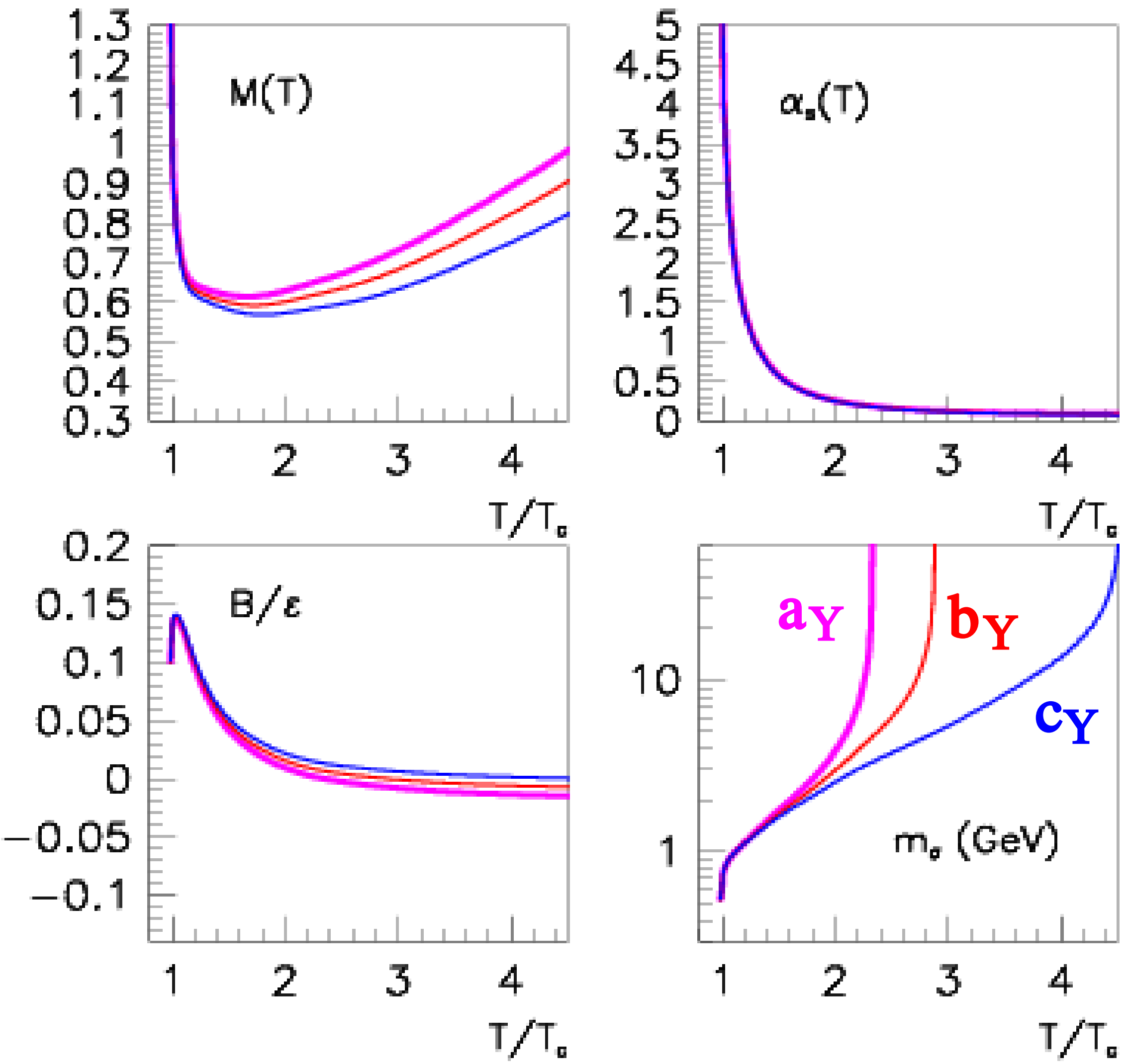
$$M(T) = M_0(T) - g\sigma = M_0(T) - \frac{g^2}{m_\sigma^2} n$$

U(r) ➣ effective potential between octet gluons

$$U(r) = \langle \lambda_i \lambda_j \rangle \alpha_s \frac{e^{-m_\sigma r}}{r} = -\frac{3}{2} \alpha_s \frac{e^{-m_\sigma r}}{r}$$

Numerical results for interacting massive gluonic quasi-particles

$\varepsilon(T)$, $P(T)$ ➤➤➤➤ $M(T)$, $B(T)$ ➤➤➤➤ $\alpha_s(T)$, $m_\sigma(T)$



Uncertanties (0-2-4 %) in the lattice results:

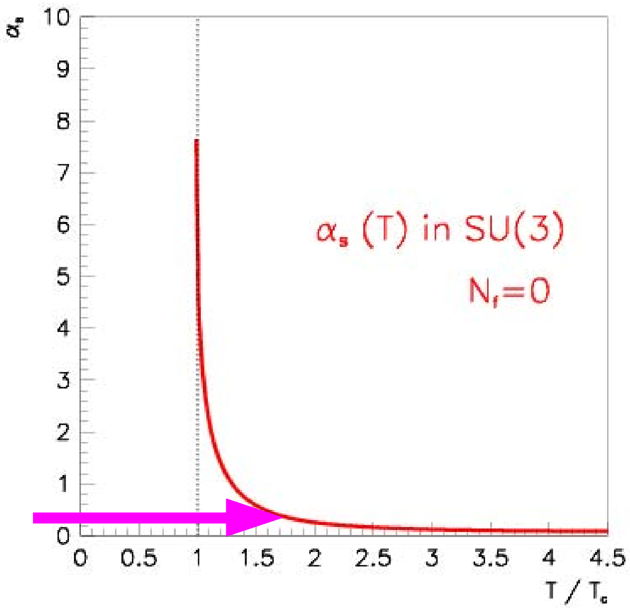
$\alpha_s(T)$ is robust

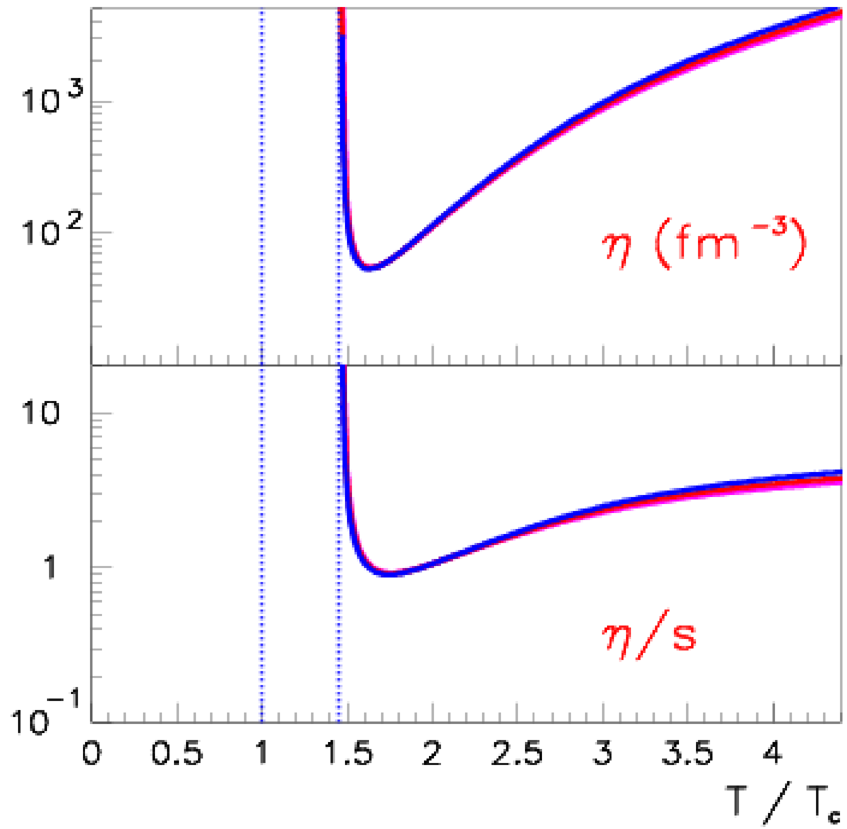
$m_\sigma(T)$ is sensitive
[Yukawa-int.]

$m_\sigma(T)$ is divergent, where B becomes negative

We can compress infinity into a finite T/T_c region.

AMY formula for the quasi-gluon gas [SU(3) N_f=0]





$$\eta_{NLL} = \frac{27.12\,T^3}{g^4 \ln(2.765/g)}$$

$\eta/s = 0.9$ at $T = 1.75\, T_c$

$T \rightarrowtail T_c$ ➢➢➢➢➢ $\alpha_s(T)$ becomes extremely large

If $\alpha_s(T) > 1$, then the simple viscosity formula does not work.

AMY: formula is good for $\alpha_s(T) < 0.3$!!! $T/T_c \geq 1.8$, $\eta/s \geq 0.9$

How to improve quasi particle description, especially in the region $1 < T/T_c < 1.5$

1. Multi-gluon and multi-quark states around T_c

E.V. Shuryak, I. Zahed, PRC70 (2004) 021901.

P.L., A. Nemeth, in preparation [10-15 %]

2. Width of the quasi particles .

A. Peshier, W. Cassing, PRL94(2005)172301. [Diverg.]

3. Mass distribution of the quasi particles (spectral function)

T.S. Bíró, P.L., P. Ván, J. Zimányi

JPG31,2005,711 [Interaction in

hep-ph/0606076 the $F(m_q)$.]

OR: Molecular dynamical simulations for SU(3) quark matter!

P. Hartmann, Z. Donkó, G. Kalman, P.L.

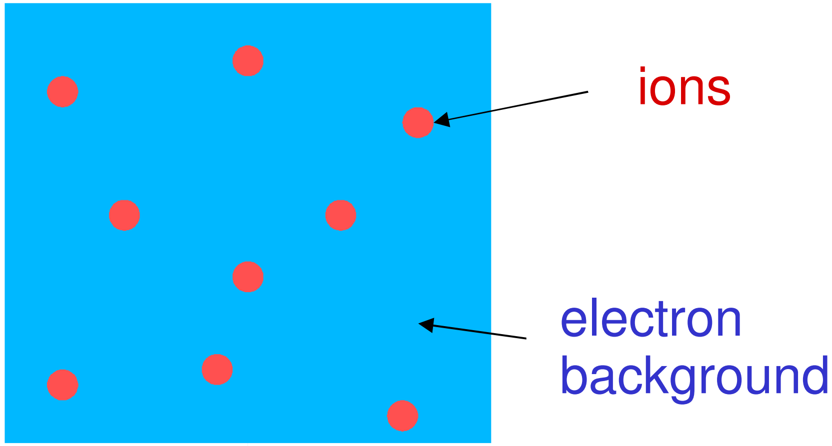
Nucl. Phys. A774, 2006, 881

Classical strongly coupled plasmas

The simplest system: classical one-component plasma (OCP).

OCP: charged heavy particles immersed into a homogeneous neutralizing background.

interaction (Coulomb) potential: $V(r) = \frac{Q^2}{r}$



particle density n
particle mass m
electric charge Q
Temperature T

→

plasma coupling parameter $\Gamma = Q^2 / (a_{WS} k_B T)$

ion sphere radius $a_{WS} = \sqrt[3]{3/(4\pi n)}$

plasma frequency $\omega_p = \sqrt{4\pi Q^2 n/m}$

investigated properties:

- structure (pair correlation function, static structure function)
- thermodynamics (internal energy, compressibility, equation of state)
- transport phenomena (thermal conductivity, shear viscosity, diffusion)
- collective dynamics (density and current fluctuations, dispersion relations)

Our sQGP model is rooted on the classical OCP model. The links are:

classical OCP		QGP model
ions	→	*quarks (massive)*
electron background (neutralizing)	→	*gluon background* interacting !!!
$V(r_{ij}) = \frac{Q^2}{r_{ij}}$	→	$V(r_{ij}) = \frac{\langle \lambda_i \lambda_j \rangle \alpha_S}{r_{ij}}$
$\Gamma = \frac{Q^2}{a_{WS} k_B T}$	→	$\Gamma = \frac{C \quad _S}{a_{WS} k_B T}$

The numerical simulation is based on the classical molecular dynamics scheme:

- calculating the forces acting on each particle due to all other particles
- integrating the equation of motion for all particles in each time-step
- using periodic boundary conditions to handle long range forces
- implementing color rotation due to random gluonic interaction

Potential model for QCD interaction

color dependent interaction potential between quark i and j: $$V = \langle \lambda_i \lambda_j \rangle \frac{\alpha}{r_{ij}}$$

possible two-quark states (***R***, ***G*** and ***B*** are the single-quark color states):

$\|RR\rangle$	$\Psi = \Psi_1(R)\Psi_2(R)$
$\|GG\rangle$	$\Psi = \Psi_1(G)\Psi_2(G)$
$\|BB\rangle$	$\Psi = \Psi_1(B)\Psi_2(B)$
$\|RG\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(G) + \Psi_1(G)\Psi_2(R)]$
$\|RB\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(B) + \Psi_1(B)\Psi_2(R)]$
$\|GB\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(G)\Psi_2(B) + \Psi_1(B)\Psi_2(G)]$
$\|RG\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(G) - \Psi_1(G)\Psi_2(R)]$
$\|RB\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(B) - \Psi_1(B)\Psi_2(R)]$
$\|GB\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(G)\Psi_2(B) - \Psi_1(B)\Psi_2(G)]$

color factor:

$$\langle \lambda_i \lambda_j \rangle = \frac{1}{2}\left[\langle(\lambda_i + \lambda_j)^2\rangle - \langle \lambda_i^2 \rangle - \langle \lambda_j^2 \rangle\right]$$

- $+1/3$ symmetric (6)
- $-2/3$ antisymmetric ($\bar{3}$)

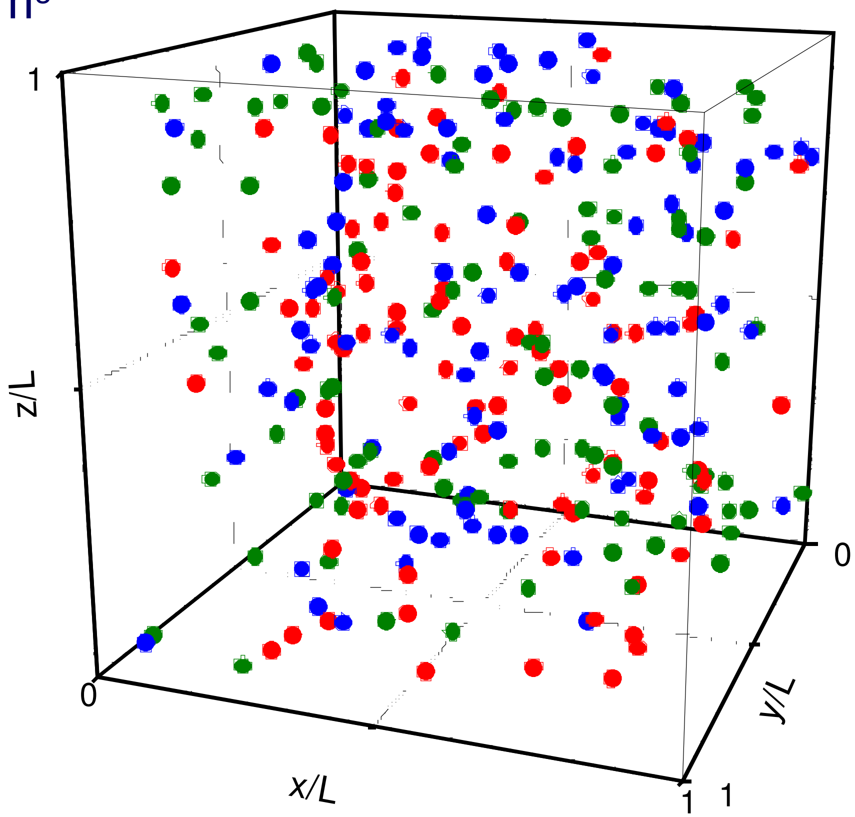
MD results for pair correlation

In the following we present molecular dynamics results for quark plasma with physical parameters:

- kinetic temperature, T_0 = 180 - 350 MeV
- particle density, n = 5-100 quarks / fm^3
- interaction strength, α_S = 1-2
- quark mass, m = 300-400 MeV

and technical parameters:

- number of particles, N = 300
- starting positions = random
- initialization time, $t_i = 10^{6+1}\, dt$
- measure time, $t_m = 2\text{x}10^5\, dt$
- time-step, $dt = 5\text{x}10^{-5}$ fm



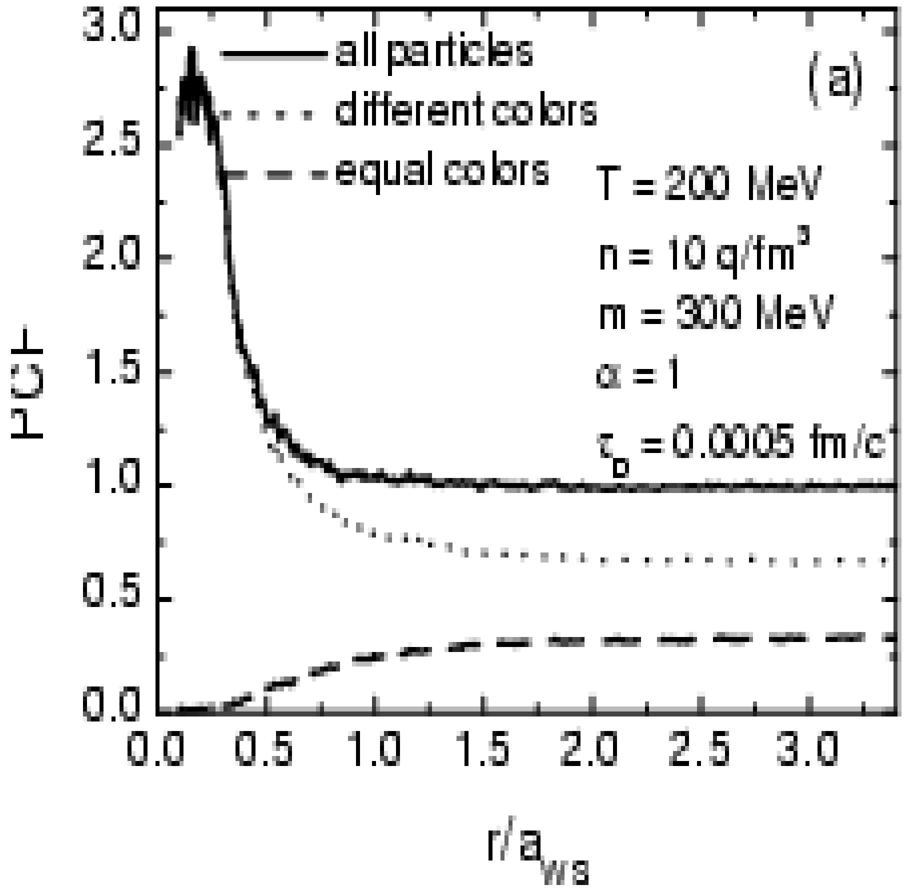
measured parameters are:

- kinetic temperature, $T(t)$
- pair correlation function, $g(r)$
- viscosity, η

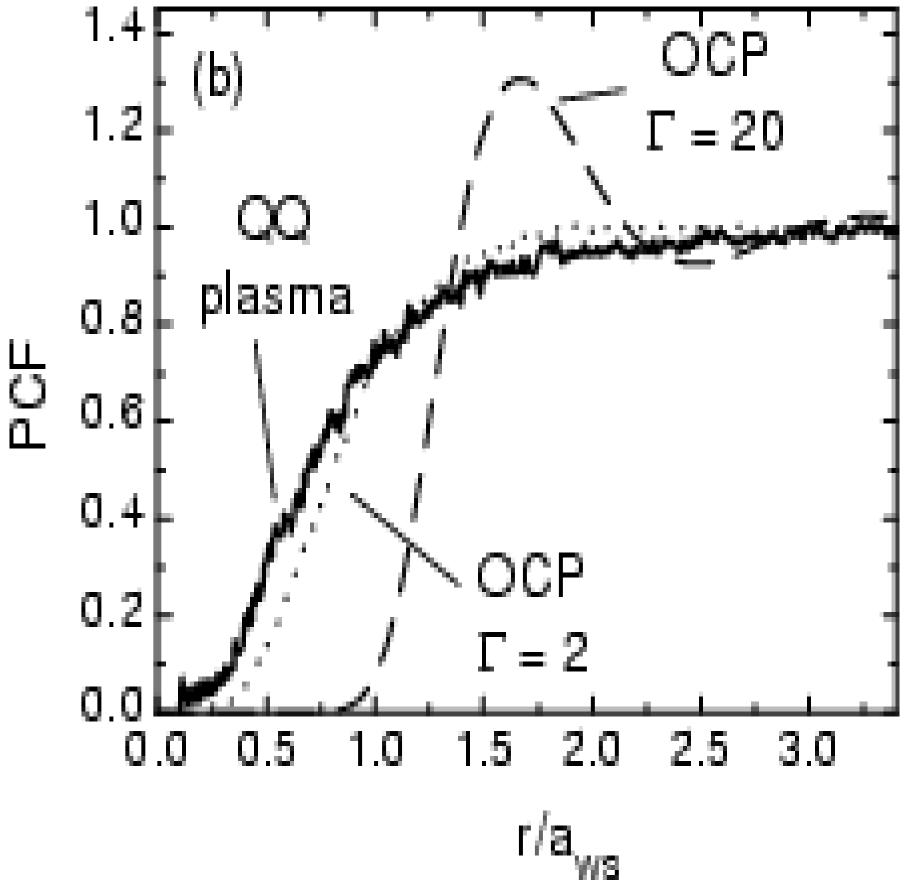
g(r) pair-correlation function: liquid or gas ? ⇒⇒⇒ "gas" !

repulsive species: gas-like behaviour
attractive species: strong correlation (clusterization)

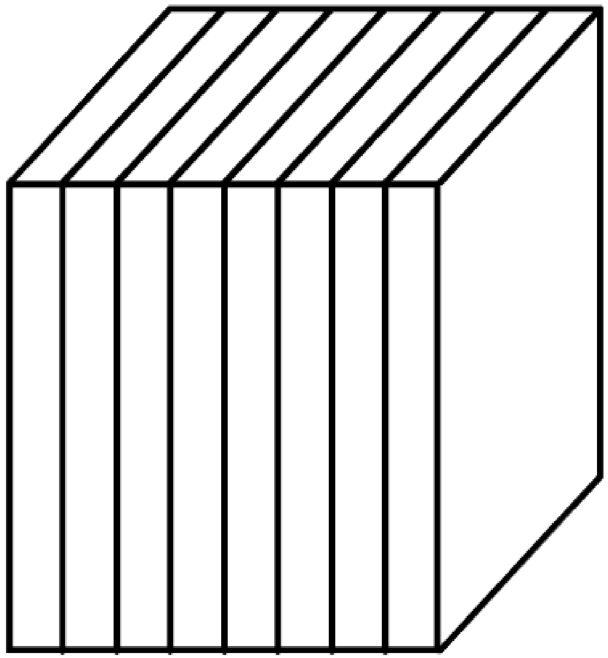
QQ plasma



QQ and ee plasma

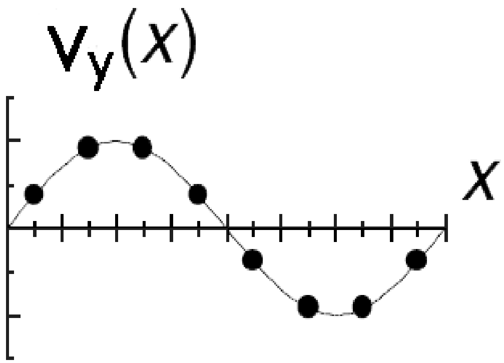


MD measurement for viscosity – 1



Z. Donkó, B. Nyíri, L. Szalai, S. Holló, Phys. Rev. Lett. 81 (1998) 1622.

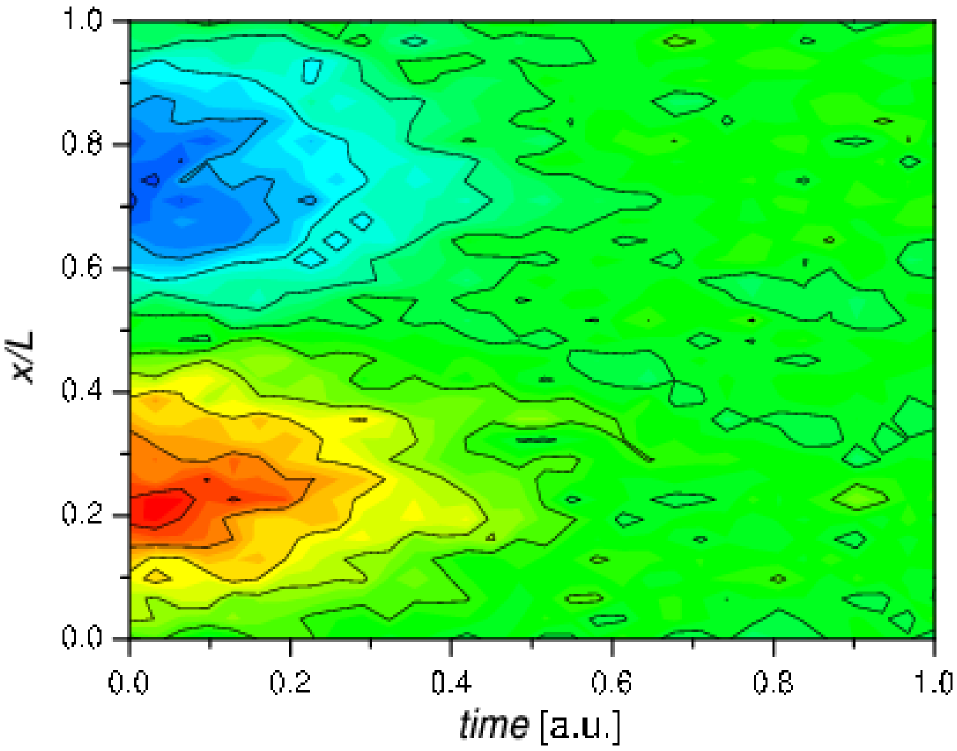
Periodic boundary condition.
(Separation into 8 bin.)

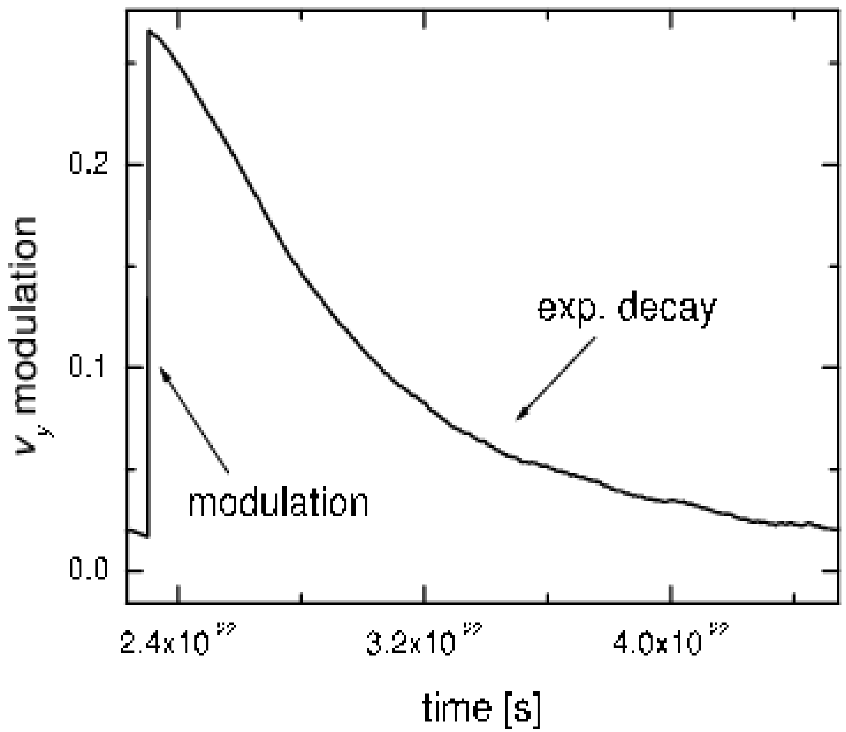


Sinusoid velocity perturbation (artifical shear in the system):

$$\Delta v_y(x, t=t_0) = v_{m0} \sin(2\pi x/L)$$

MD measurement for viscosity – 2





The relaxation of this shear is exponential (see Navier-Stokes):

$$v_m(t) = v_{m0} \exp(-(t - t_0)/\tau)$$

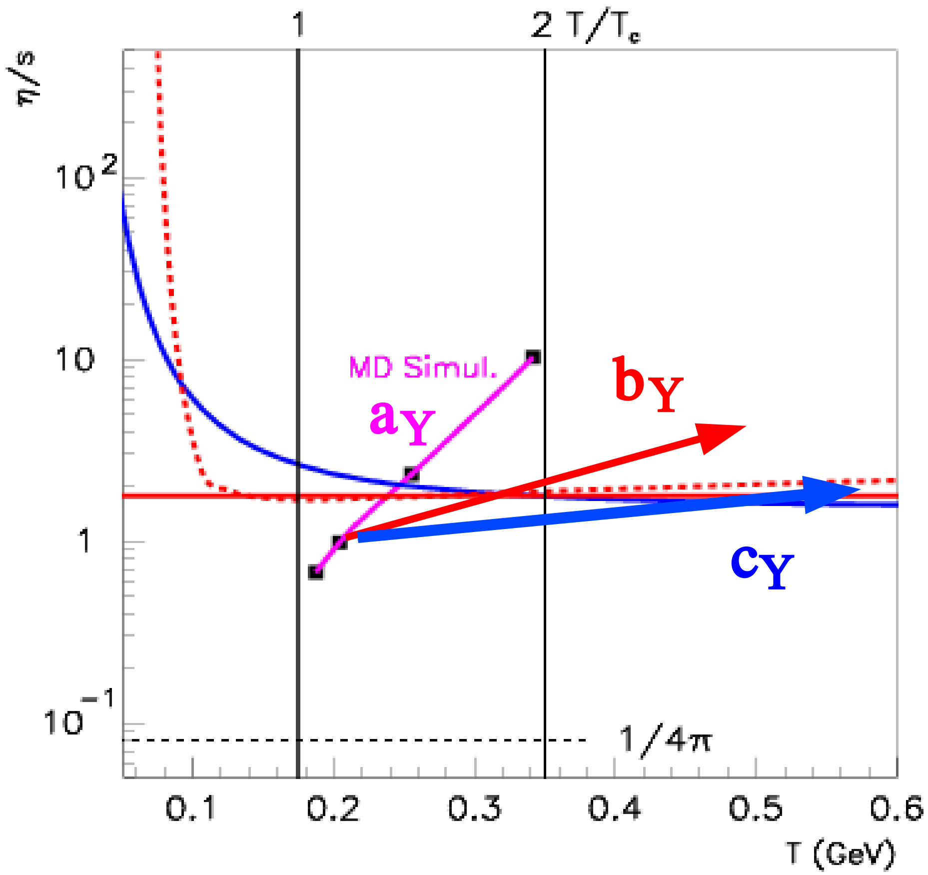
Here τ is related to the dynamical viscosity η_D :

$$\eta_D = \frac{\rho\, m_q}{\tau} (L/2\pi)^2$$

ρ : density [1/fm^3]

L : simulation box size [fm]

RESULTS from MD simulation for viscosity (90 000 part.):



'a_Y' MD simulation: Minimum

$T = T_c \Rightarrow\Rightarrow \eta/s \approx 10\ (1/4\pi)$

Result is smaller than others.

[No MD for $T < T_c$]

$1 < T/T_c < 2 \Rightarrow \eta/s = 0.6 - 10$!

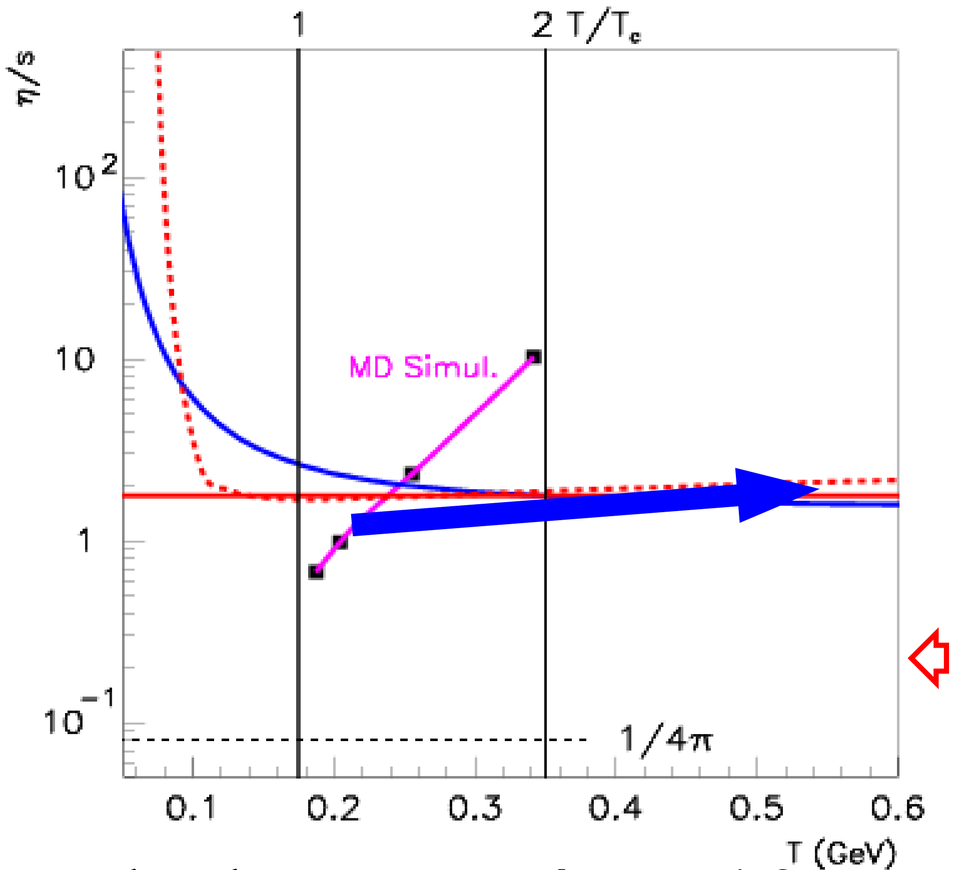
'b_Y' and 'c_Y' simulation:

$T/T_c < 1.3$: result is the same

T/T_c large: free gas limit

$\eta \approx$ 1-2 : large or small ???

RESULTS from MD simulation for viscosity:



If Yukawa ↣ 0 at T/T_c ↣∞

then η/s ↣ 0.8 ≈ 20 (1/4π)

Smooth change at T/T_c ≈ 3-4

from dilute quasi-particle gas

to dense gluon field

[η/s =0.8≈20 (1/4π)]

Dense gluon field:

Anomalous viscosity (η_A)

in turbulent gluon fields,

$\eta_A \ll \eta$

What happen at T/Tc ↣ 1 ?

Cross-over phase transition in MD?

$\eta_A/s \approx C\ (1/4\pi)$ C=?? (2-3)

[Asakawa, Bass,Müller,PRL96,2006]

Summary:

1. Viscosity can be calculated in many ways:

   Classical description can give very small value for η/s
   seems to generate "perfect fluid" (?)

   QM and QCD give $\eta/s \approx 2$ at $T/T_c \geq 1$
   Quasi-particle picture gives $\eta/s \geq 0.9$ at $T/T_c \geq 1.8$
   MD for quark matter gives $\eta/s \geq 0.6$ at $T/T_c \geq 1.1$

   Further studies are needed, especially close to T_c.

2. How large is the anomalous viscosity, η_A/s?

3. Viscosity and jets, direct experimental measurement ?
   Especially, if the viscosity is large: $\eta/s \geq 0.6$ at $T \approx 200$ MeV
   $\eta/s \geq 1.5$ at $T > 250$ MeV!