

# pQCD Calculations of Heavy Quark and J/ψ Production

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## Outline



J/ψ production



Heavy quark production

Each would require a stand-alone talk.

Hard to cram two into a single one.

Will try to choose some highlights

# (Tough) choices

For both  $J/\psi$  and open heavy quark production:

- neglect all matter effects
- restrict to standard collinear factorization

## $J/\psi$

- Introduce Non-Relativistic QCD
- Present (very) selected results
- Point out outstanding issues

Reviews:

M. Kraemer, [hep-ph/0106120](#)  
Quarkonium Working Group, [hep-ph/0412158](#)  
J.P. Lansberg, [hep-ph/0602091](#)

## Heavy quarks

- pQCD calculations and their uncertainties
- Non-perturbative issues
- Comparisons with collider data

(see Lourenço, Woerhi, [hep-ph/0609101](#),  
for fixed target data)

In both cases, get a feeling for pQCD predictivity,  
and set a benchmark for other calculations

# Heavy Quarkonium

Heavy quark  
mass

Heavy quark  
momentum

binding  
energy

Various energy scales ( $m$ ,  $mv$ ,  $mv^2$ ) leading to small expansion parameters:

Coupling:

$$\alpha_S(m_c) \simeq 0.35$$

$$\alpha_S(m_b) \simeq 0.2$$

From virial theorem and Coulomb-like potential:  $v \sim \alpha_S(mv)$

$\Rightarrow$

$$v_{charm} \sim 0.6$$

$$v_{bottom} \sim 0.3$$

This suggests a **double expansion** in  $\alpha_S$  and  $v$

**Tool:** an effective theory of QCD,  
**Non Relativistic QCD (NRQCD)**

# Heavy Quarkonium production in NRQCD

Bodwin, Braaten and Lepage, 1994

Factorization 'theorem' for heavy quarkonium production:

$$\sigma[H] = \sum_n \sigma_n(\mu) \langle O_n^H(\mu) \rangle$$

Short-distance cross-section for production of QQ state with color-spin-angular momentum n

Long-distance NRQCD matrix element. Describes the 'hadronization' of the QQ pair

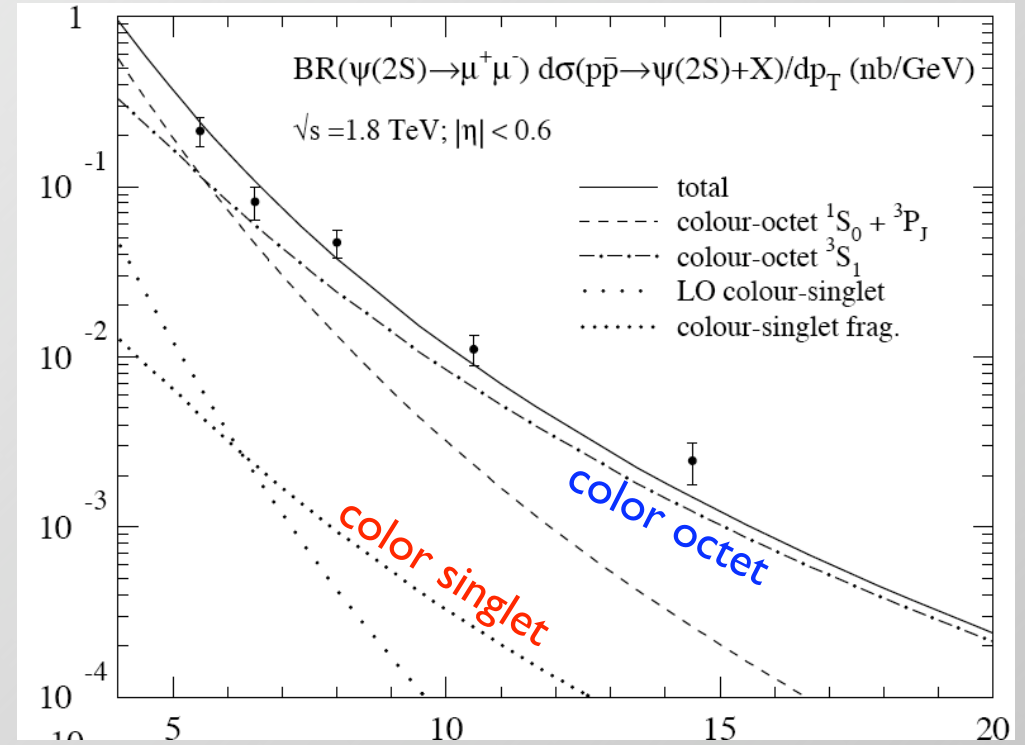
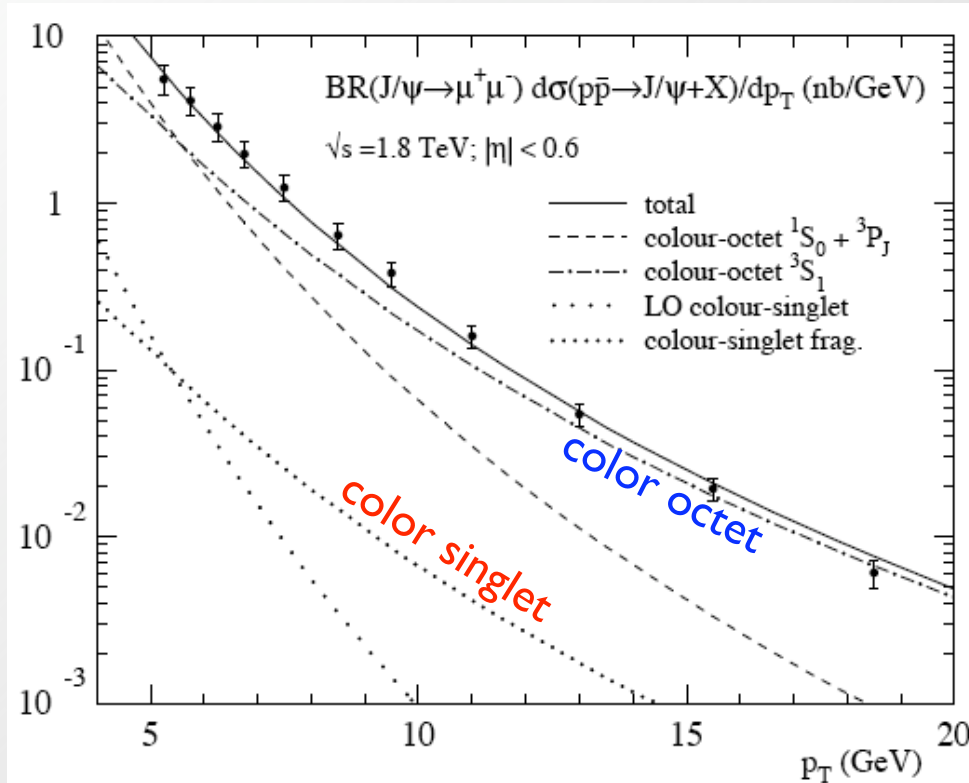
NB: The sum runs over production of both singlet and octet color states

The **NRQCD approach** (often mistakenly called the color octet model) extends the Color Singlet Model in a systematic way:

- no divergences should be left in higher-order calculations
- a controlled truncation in terms of powers of  $\alpha_S$  and  $v$  becomes possible

# NRQCD: early successes - I. Phenomenological

## Prompt $J/\psi$ and $\psi'$ production at Tevatron



Matrix elements:

**Singlet**

**Octet**

$J/\psi$

$1.16 \text{ GeV}^3$

$1.2 \times 10^{-2} \text{ GeV}^3$

$\psi'$

$0.76 \text{ GeV}^3$

$0.5 \times 10^{-2} \text{ GeV}^3$

Fitted to data

Compatible with  $\frac{v^4}{2N_c}$  scaling

Fixed by decays or potential models

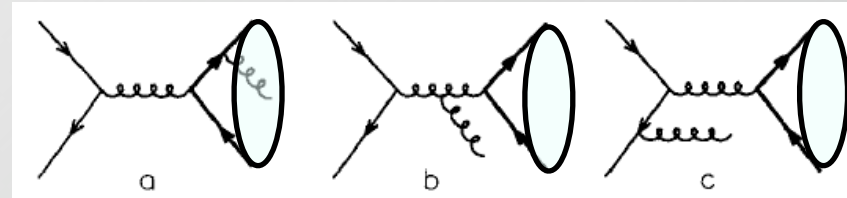
$$(v^4/2N_c \simeq 0.6^4/2/3 \simeq 0.02)$$

# NRQCD: early successes - 2. Theoretical

Full NLO calculation of all  $ij \rightarrow \text{Quarkonium}$  processes: **no divergences**

(Petrelli et al.)

Example:  $q\bar{q} \rightarrow {}^3P_J^{[1]} g$



$$\sigma^H[q\bar{q} \rightarrow {}^3P_J^{[1]} g] = -\frac{C_F}{D_F^2} \frac{256\pi^2 \alpha_s^3 \mu^{4\epsilon}}{9(2m)^7} \beta^{-4\epsilon} f_\epsilon(s) \left( \frac{1}{\epsilon} + \frac{4}{3} \right) \delta(1-x) \langle \mathcal{O}_1^H({}^3P_J) \rangle$$

$$+ \frac{\pi^2 \alpha_s^3 \mu^{4\epsilon}}{(2m)^7} \left( \frac{1}{1-x} \right)_\rho f_{q\bar{q}}[{}^3P_J^{[1]}](x) \langle \mathcal{O}_1^H({}^3P_J) \rangle \quad [J=0, 1, 2], \quad (186)$$

**Long-distance divergence:  
breakdown of Color Singlet Model**

(Known since late Seventies: Barbieri, Caffo, Gatto, Remiddi, ...)

# NRQCD: early successes - 2. Theoretical

Solution: sum all color singlet **and color octet** processes contributing at a given order in  $v$

singlet octet

$\sigma^H [q\bar{q} \rightarrow {}^3P_J^{[1]} g] + \sigma^H [q\bar{q} \rightarrow {}^3S_1^{[8]}]$  is **finite**

$\Downarrow$

$\sigma [q\bar{q} \rightarrow {}^3S_1^{[8]}] \langle O_8^H ({}^3S_1) \rangle$   $\rightarrow$  Corrections to this matrix element cancel the divergence

To **NNLO**: need to update NRQCD matrix elements definitions.  
Checked for spin-independent operators

(Nayak, Qiu and Sterman)

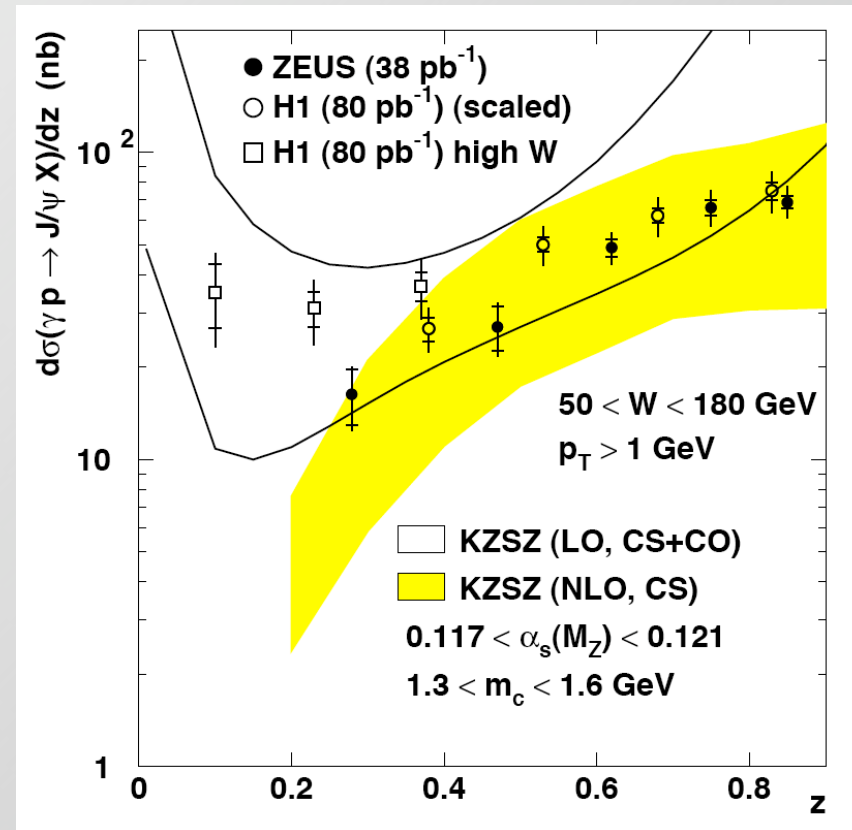
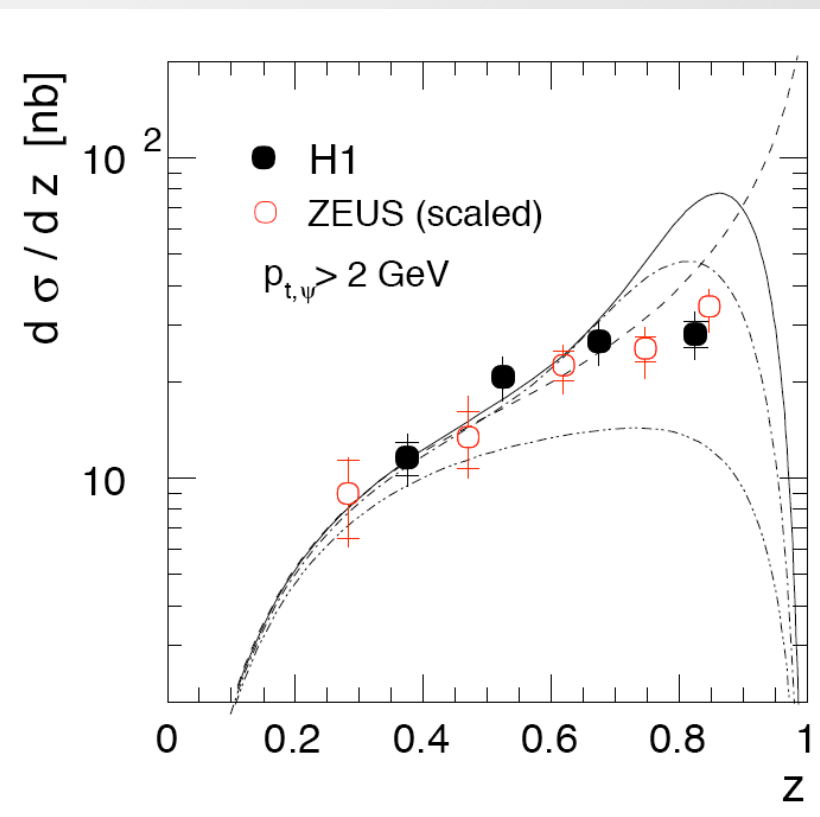
# NRQCD: problems (and fixes)

## $J/\psi$ photoproduction:

addition of color octet terms with matrix elements fitted at Tevatron seems to overestimate the cross section at large  $z$

**However:** in the large- $z$  region both pQCD and the velocity expansion of NRQCD break down

⇒ need to resum to all orders



resummation improves  
large- $z$  behaviour  
(Beneke, Rothstein, Wise;  
Beneke, Schuler, Wolf,  
Fleming, Leibovich, Mehen)

**Message/reminder:** many NRQCD calculations are still leading order, hence subject to large uncertainties

# NRQCD: problems (and fixes)

$$e^+e^- \rightarrow J/\psi + \eta_c \quad \text{rate}$$

**Belle**  $25.6 \pm 2.8 \pm 3.4 \text{ fb}$

**BaBar**  $17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb}$

**LO NRQCD**  $3.8 \pm 1.3 \text{ fb}$   
 $5.5 \text{ fb}$

(Braaten, Lee)  
(Liu, He, Chao)

Hopeless? A number of corrections can (and should) be considered:

 higher order corrections in  $\alpha_S$ : **K-factor = 1.96** (Zhang, Gao, Chao)

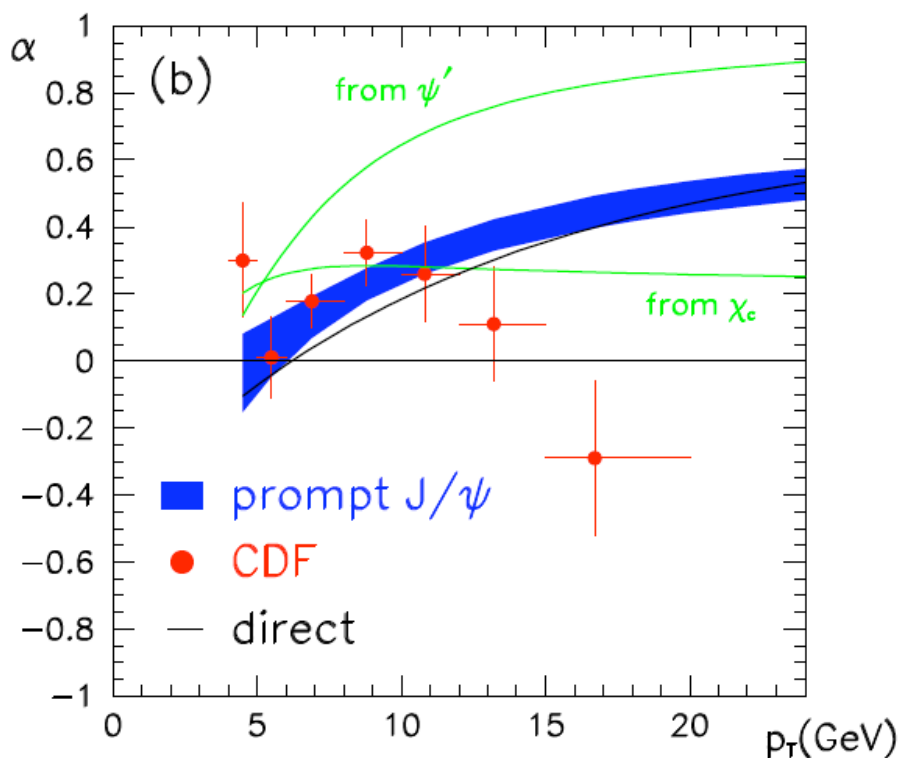
 resummation of relativistic corrections in  $v^2$ : **K-factor = 1.8**  
(Bodwin et al.)

Combining everything, the upgraded prediction becomes  **$17.5 \pm 5.7 \text{ fb}$**

Once more, the message is that LO NRQCD can still be far from being reliable

# NRQCD: problems

## J/ $\psi$ polarization in proton-antiproton collisions

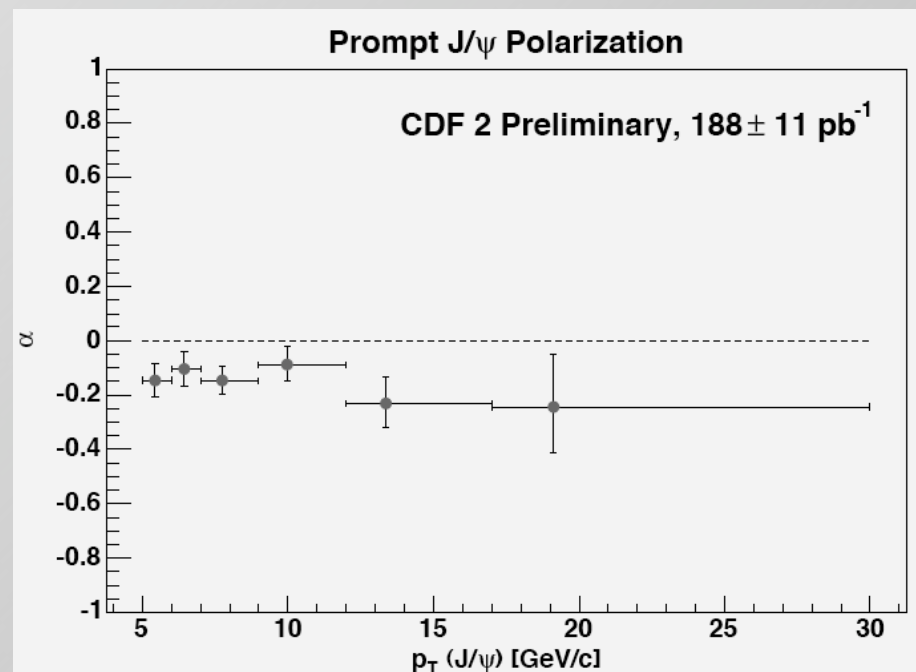


NRQCD expects **transversely polarized J/ $\psi$ 's** at large  $p_T$ , as they should mainly come from fragmenting gluons. Feeddowns are predicted to change this view only slightly

**CDF Run I data seem to beg to differ....**

Preliminary Run II data don't change the picture

Velocity scaling rules might hold the key



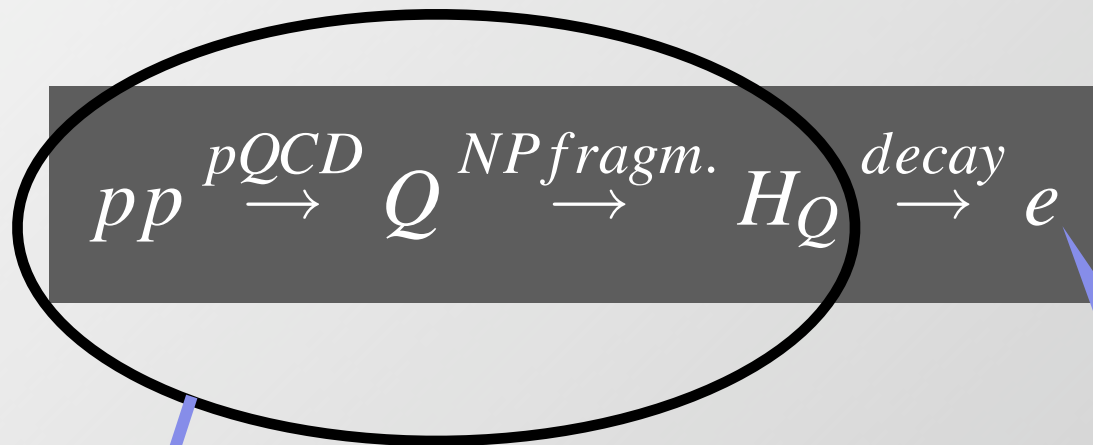
# Quarkonium summary

- NRQCD probably established as a correct effective theory of QCD in the  $v \ll 1$  limit
- Factorization theorem not proved, but it certainly holds at least up to NLO
- Phenomenology looks qualitatively correct, especially when potentially large terms are resummed
- What happens with polarization at the Tevatron? Are velocity scaling rules to be reconsidered?

As far as absolute cross sections are concerned, heavy quarkonium predictions can only be as good as the underlying heavy quark calculations

⇒ next part of the talk tries to see how we fare there

# A generic heavy quark production process



A generic final state observable

This part is QCD.  
How accurately can we predict it?  
What ingredients do we need?

# Factorization 'theorem' for heavy quark hadroproduction

Collins, Soper, Sterman, Nucl. Phys. B263 (1986) 37

$$\sigma_Q(S, m^2) = \sum_{i,j \in L} \int dx_1 dx_2 \hat{\sigma}_{ij \rightarrow QX}(x_1 x_2 S, m^2; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) F_{i/A}(x_1, \mu_F) F_{j/B}(x_2, \mu_F) + O\left(\frac{\Lambda}{m}\right)^P$$

**Light** flavours only

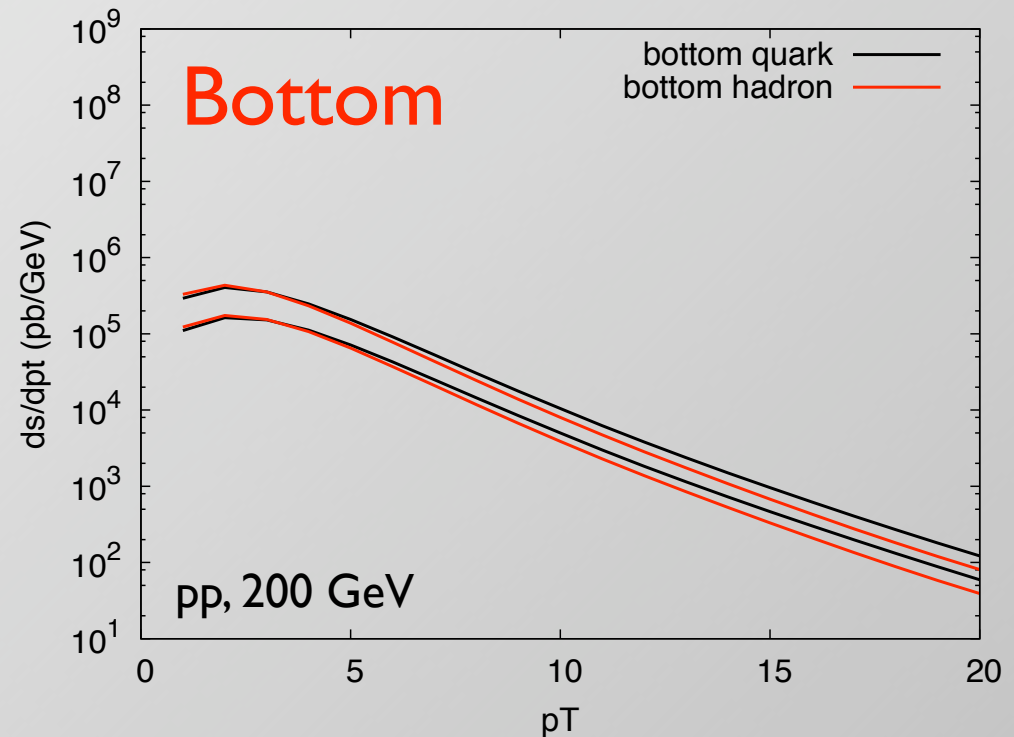
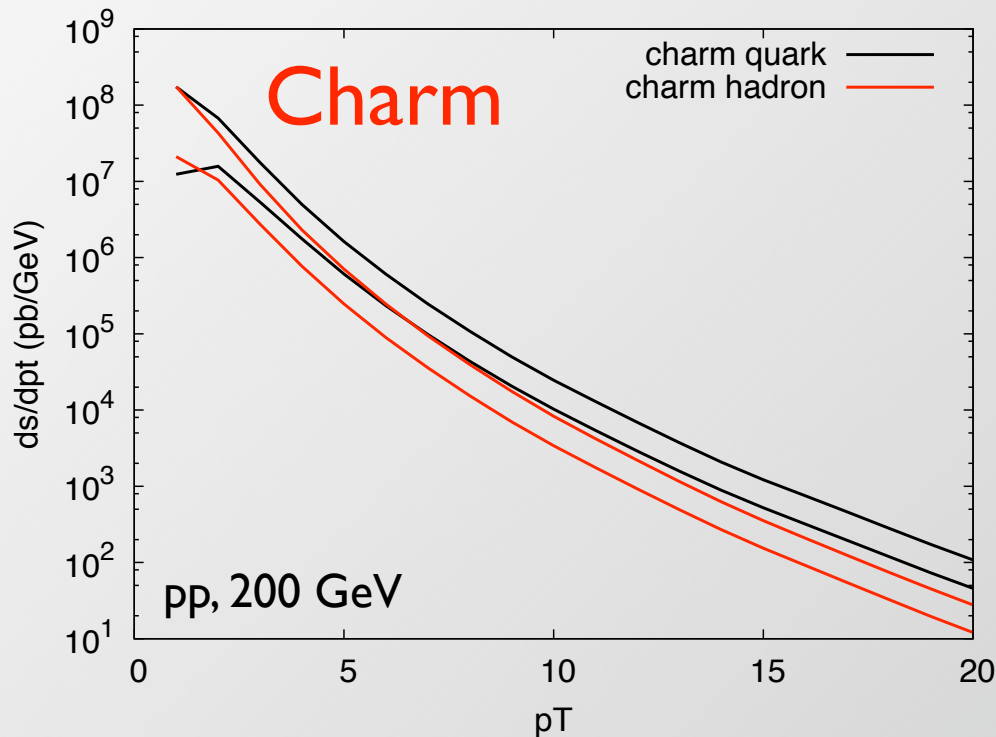
contribute most of the total cross section. The **hard scattering function** is perturbatively calculable in an expansion in powers of  $\alpha_s(M)$ : potential singularities in  $H$  have been factorized into the **parton distribution functions**. Corrections to this formula are suppressed by **powers of (hadron mass scale/ $M$ )**.

We have by no means proved this result in this paper, but we believe that the analysis given here should make the result plausible. We are arguing that heavy

# Heavy quark cross sections

Heavy quarks are special:  
their **total number** (and that of **heavy hadrons**)  
is a **genuine prediction of pQCD**

Not so for **differential** distributions: hadrons and quarks differ



However, the non-perturbative correction is expected  
(and observed) to be **parametrically small**,  $\mathcal{O}(\Lambda/m)$

(Still, at large  $p_T$  the effect can be large)

# Non-perturbative fragmentation

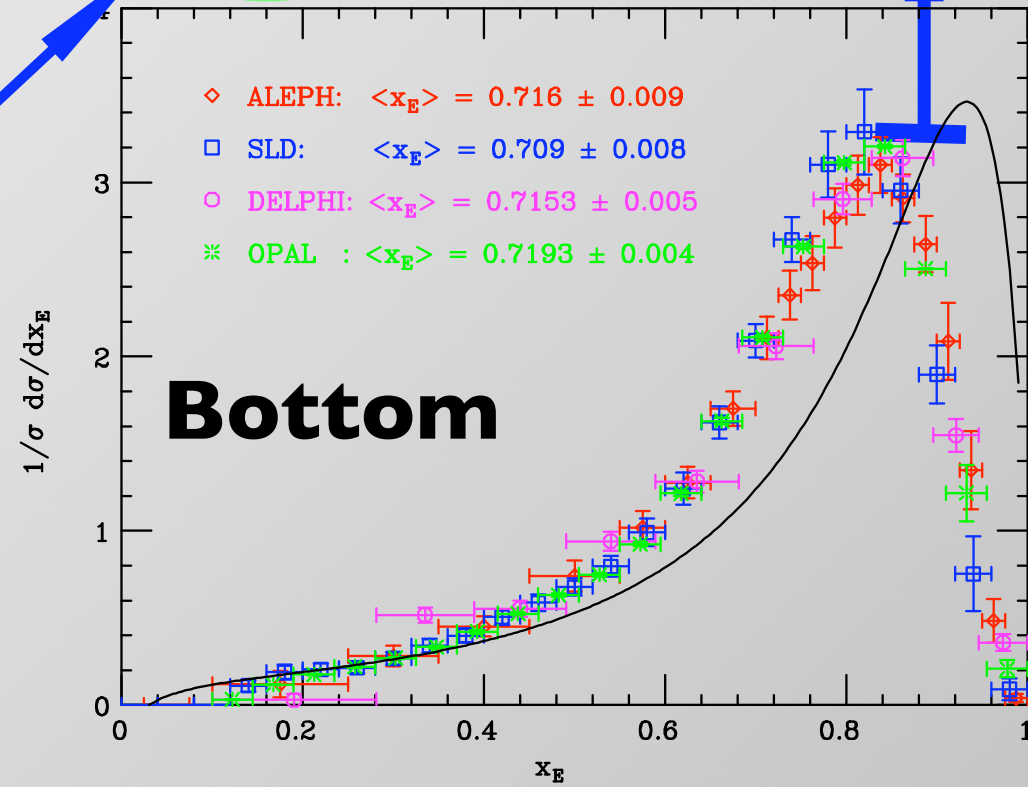
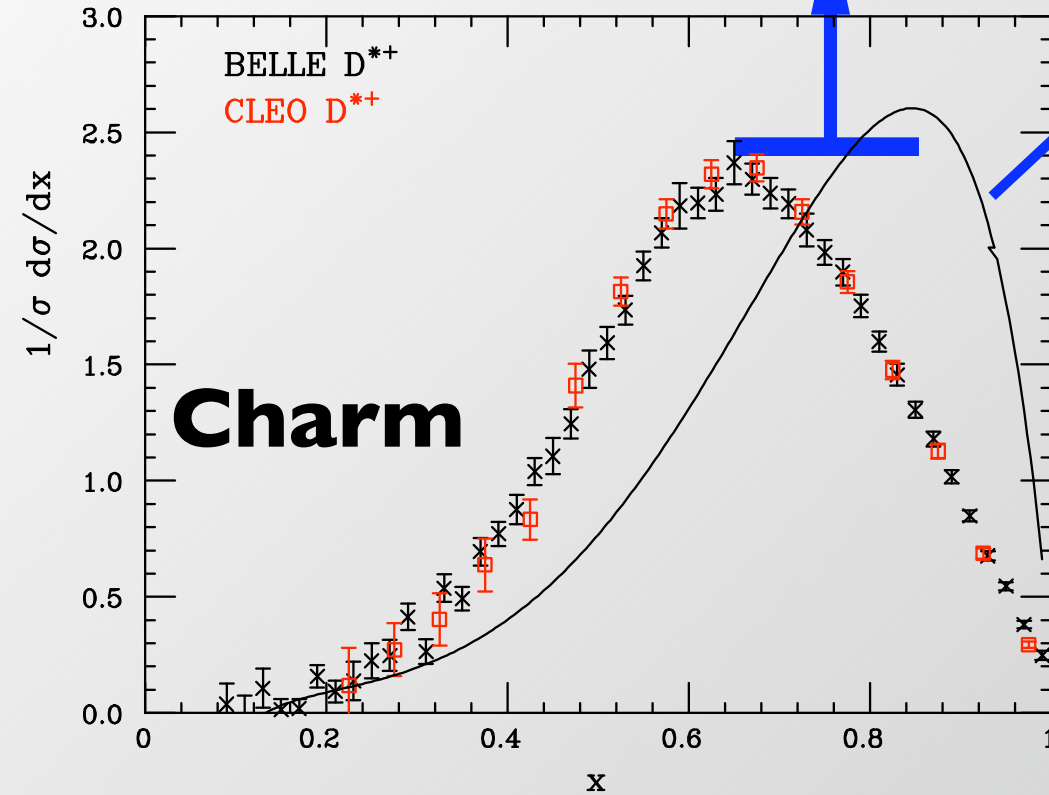
$$e^+e^- \rightarrow QX \rightarrow H_Q X$$

non-perturbative contribution

$\mathcal{O}(\Lambda/m_{\text{charm}})$

pQCD

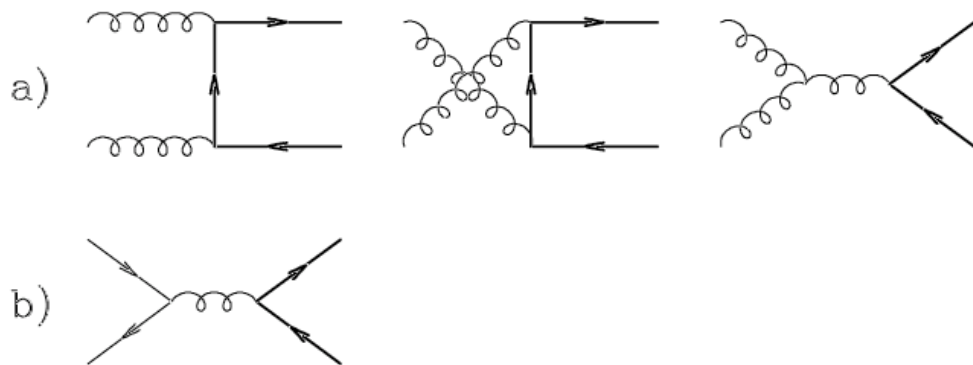
$\mathcal{O}(\Lambda/m_{\text{bottom}})$



 non-perturbative contribution limited in size and compatible with expectations

 high-accuracy expt. data allow it to be precisely determined

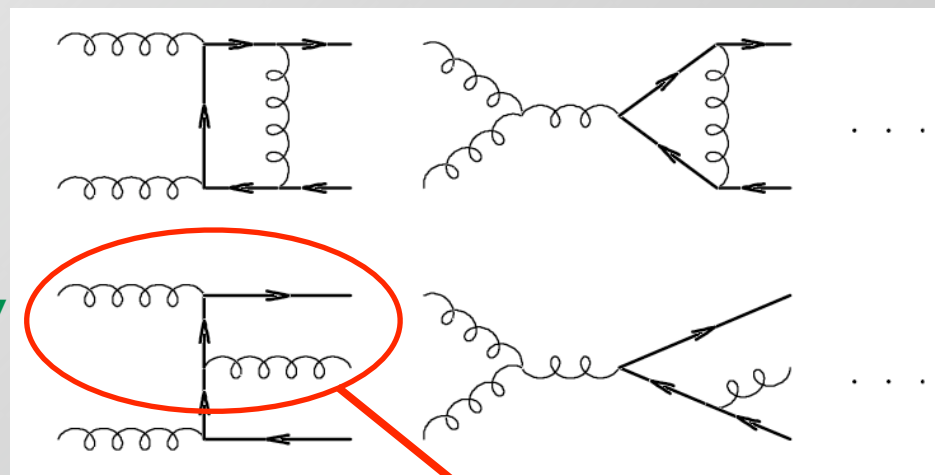
# NLO implementation of factorization theorem



Leading order diagrams

(Some of the) Next-to-Leading order diagrams

Nason, Dawson, Ellis, NP B327 (1989) 49, NP B303 (1988) 607  
Beenakker, van Neerven, Meng, Schuler, Smith, NP B351 (1991) 507



'flavour excitation'



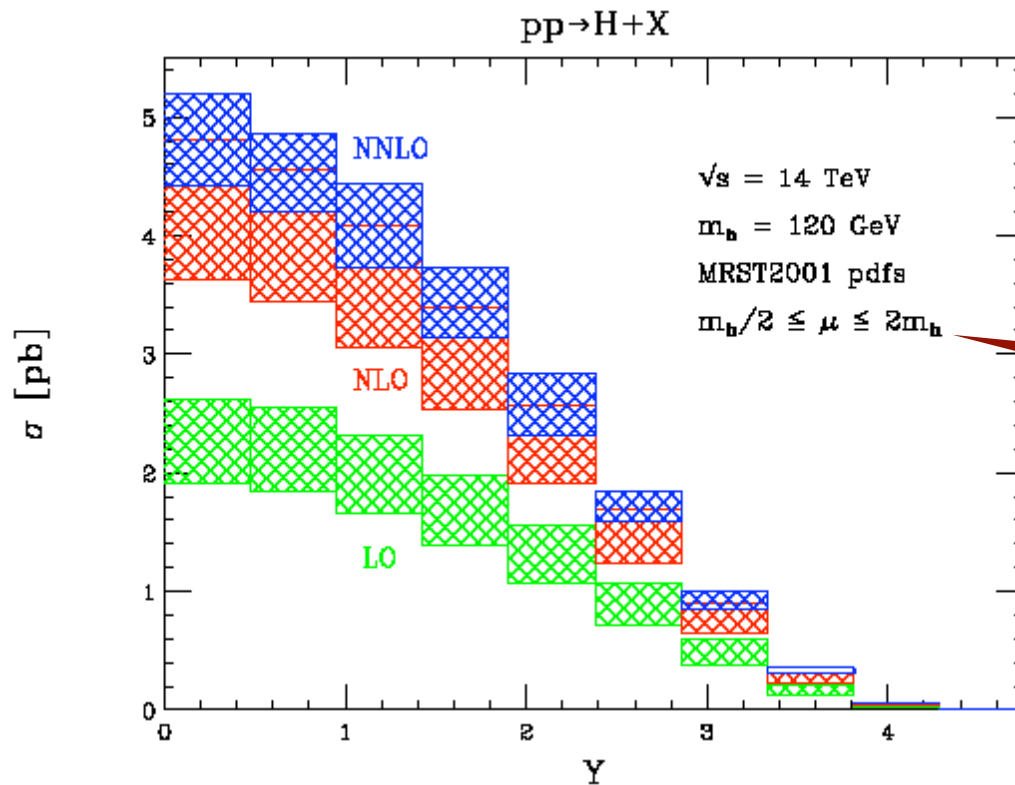
No need to have,  
e.g., charm in the  
proton

This is still the state of the art for fixed order perturbative calculations, and should be the building block of all phenomenological predictions:

- it incorporates in a rigorous manner production “channels” like flavour excitation and gluon splitting which Monte Carlo or ‘improved’ leading order calculations have to include by hand (beware MC tunes and recipes!!)
- it allows a **rough estimate of the theoretical uncertainty**

# The rule of thumb on uncertainties

- A **LO** calculation gives you a **rough estimate** of the cross section
- A **NLO** calculation gives you a **good estimate** of the cross section and a **rough estimate** of the uncertainty
- A **NNLO** calculation gives you a **good estimate** of the uncertainty



Example of a **NNLO** calculation:  
Higgs boson production at the LHC

Anastasiou, Melnikov, Petriello,  
hep-ph/05011030

**Scale variations**

**NB.** This example shows that the center of the NLO band has nothing to do with the most accurate theoretical prediction.

**Theoretical uncertainty bands are not gaussian errors!**

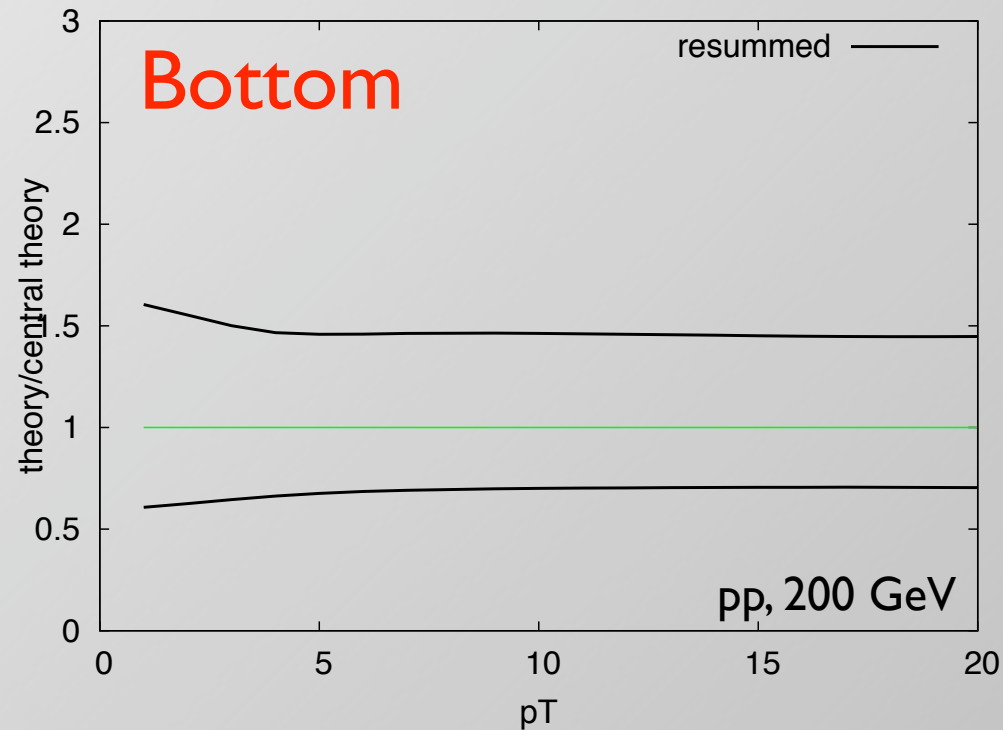
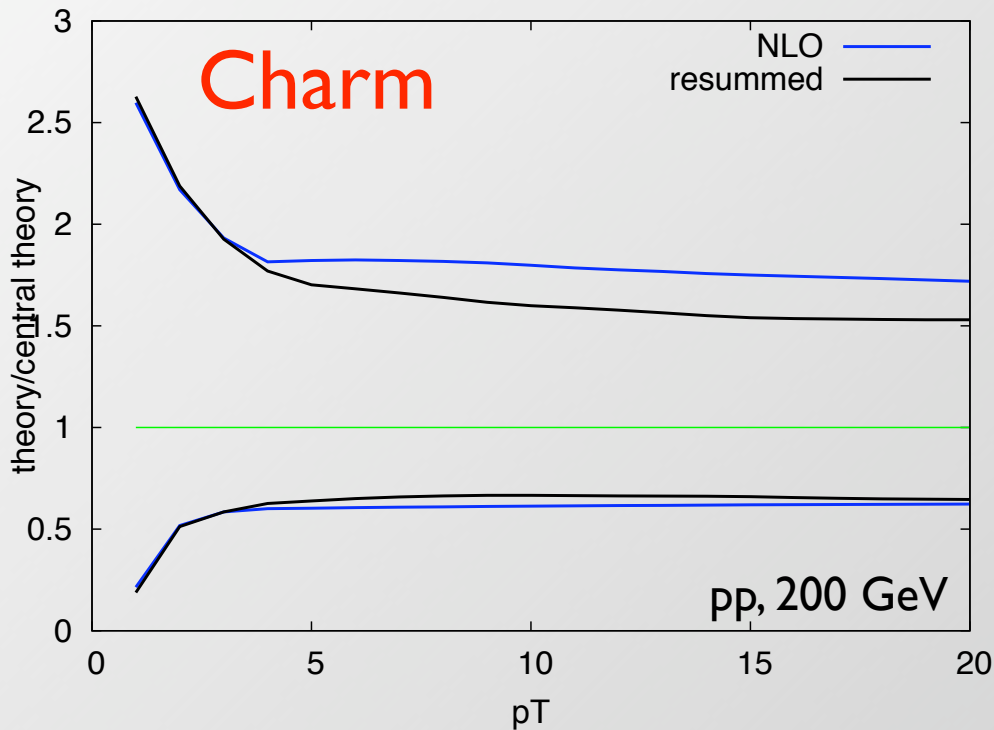
# Perturbative uncertainties

**max and min of theory**



Vary scales independently, vary mass, sum in quadrature, take max and min

**central theory**



No big effect of resummation in this region. But its big contribution lies in the accurate determination of the non-perturbative component from  $e^+e^-$  data

# Non-perturbative uncertainties

The non-perturbative FF is usually employed in hadronic collisions by writing

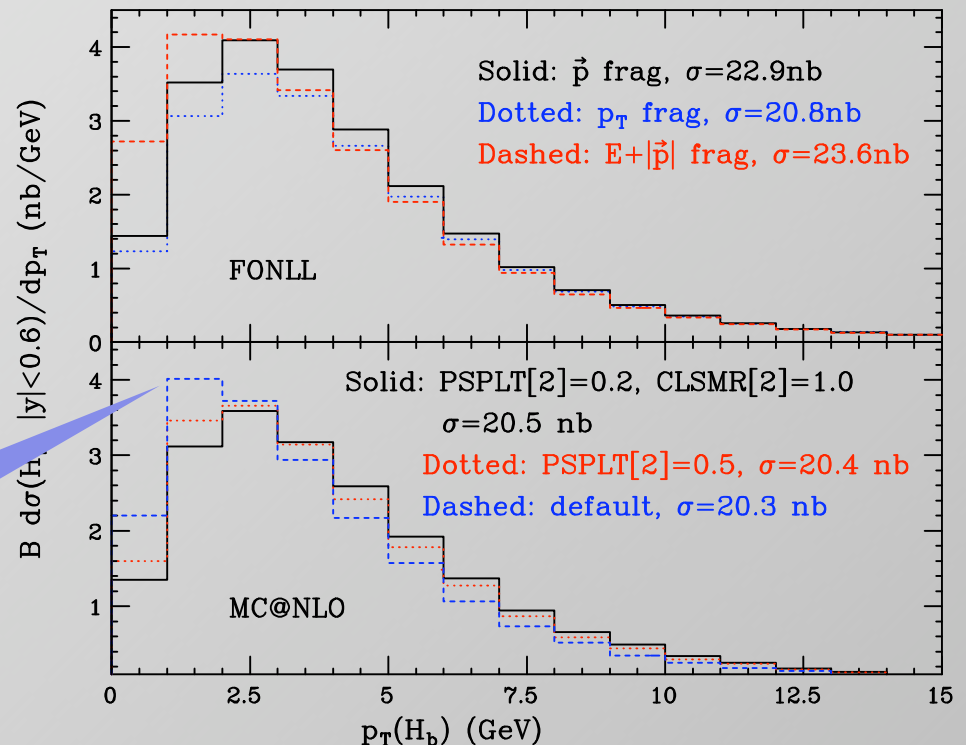
$$E_H \frac{d^3 \sigma_H(p_H)}{dp_H^3} = E_Q \frac{d^3 \sigma_Q(p_Q)}{dp_Q^3} \otimes D_{Q \rightarrow H}^{np}$$

Besides the uncertainties in its extraction from data (usually small with modern data), bear in mind that when the transverse momentum is small two things happen:

1. The “independent fragmentation” picture fails, as factorization-breaking higher twists grow large. So, whatever the result of the convolution above, there will be further uncertainties looming over it

2. Scaling a massive particle’s 4-momentum is an ambiguous operation. One can scale the transverse momentum at constant rapidity, the 3-momentum at constant angle in a given frame, etc.

Different fragmentation choices



# Comparisons of phenomenological QCD predictions and experimental data

LEP, HERA, Tevatron, RHIC

Charm, Bottom and top

But first.....



# Real observables

We cannot always measure a total cross section, and Monte Carlo based extrapolations can be dangerous (theoretical bias seeping into a 'measurement')

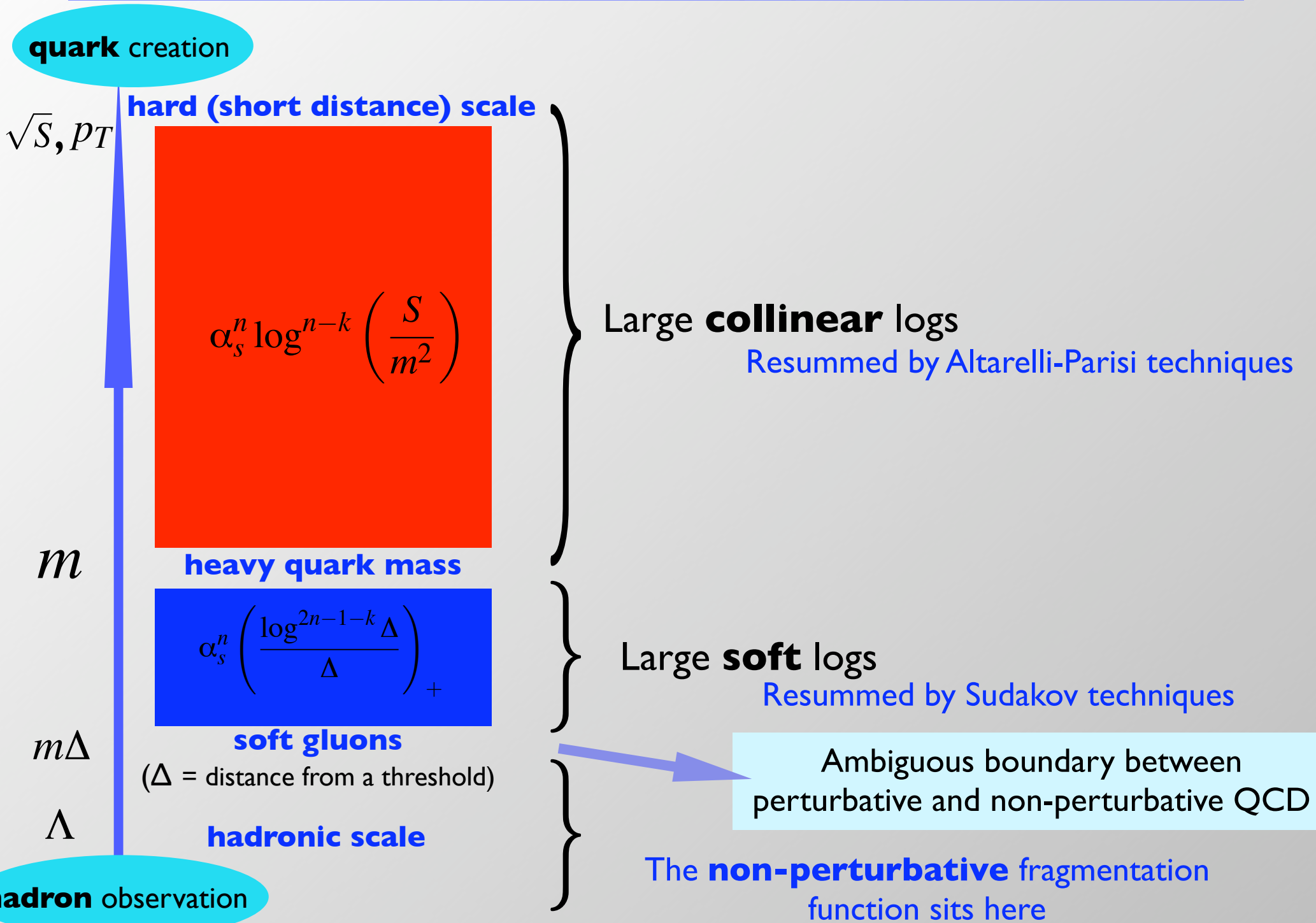
Ideally, we'd like to use differential observable quantities for comparing to theoretical predictions

Unfortunately, as usual, all good things come at a price:

Theoretical predictions for differential observables are **harder**

-  Any multi-scale quantity in QCD will display possibly **large logarithms** in the perturbative expansion. These logs will tend to spoil the convergence of the series. Hence, **resummations** will be needed
-  Eventually, resummations will not be enough, and genuinely **non-perturbative** contributions will need to be added

# The many scales in heavy quark production



# Putting things together

A modern tool for producing phenomenological predictions for heavy quarks at the differential level will

- 1- properly **resum** (say to next-to-leading log accuracy) the large logarithms
- 2- **match** the resummation to a full NLO fixed order calculation
- 3- properly **extract** from data (and **use** for predictions) **non-perturbative** fragmentation functions describing the hadronization of the heavy quarks

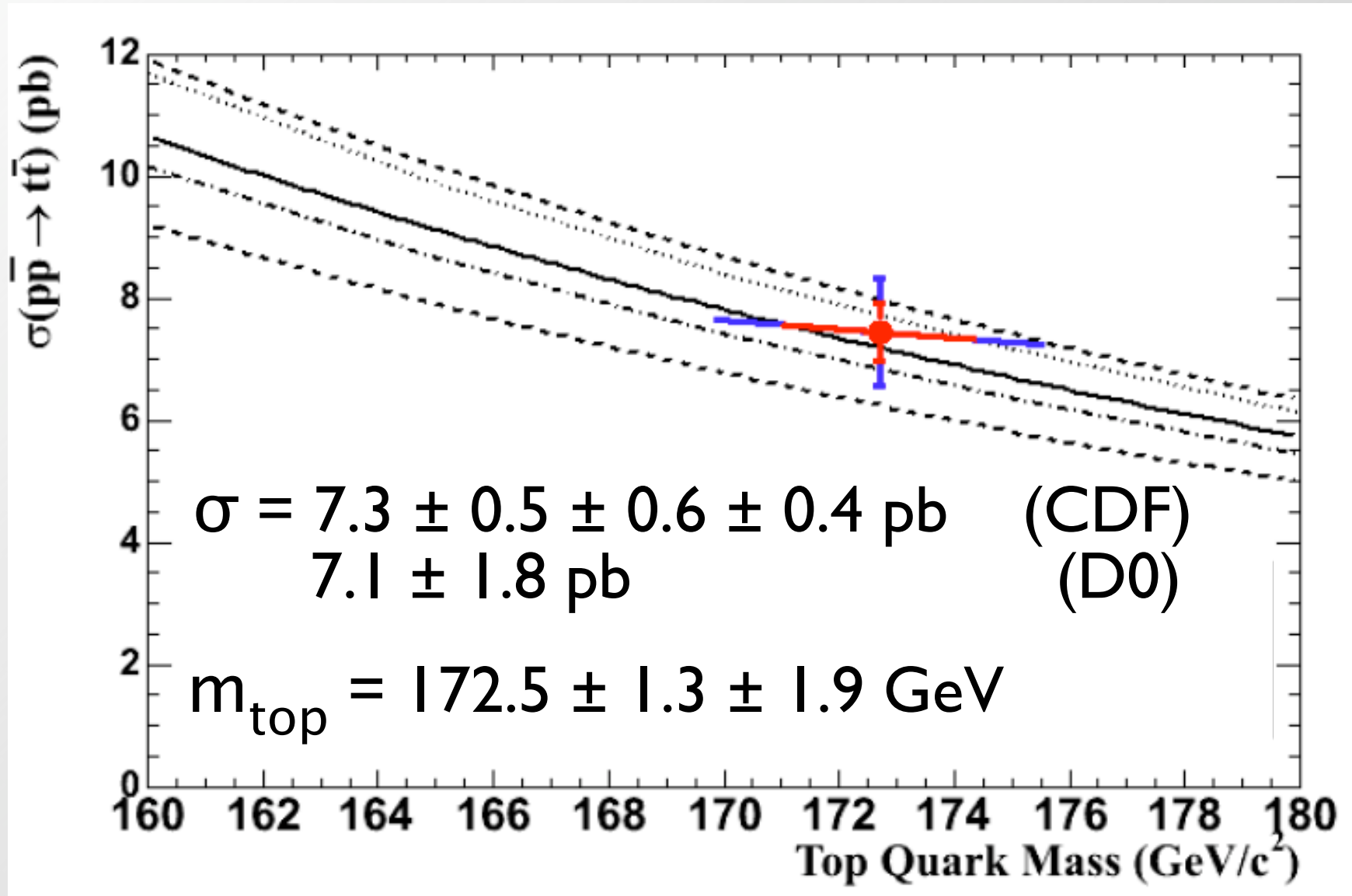
NB. Whether you need all this or not depends, of course, on your accuracy goal. If you are happy with a factor of two uncertainties or more none of this is probably necessary: take the 15 years old NLO calculation, slap on it any non-perturbative fragmentation function, and go ahead (but then don't come to me complaining of discrepancies with QCD)

On the other hand, if you aim at a few 10% accuracy then you need this stuff.

Top

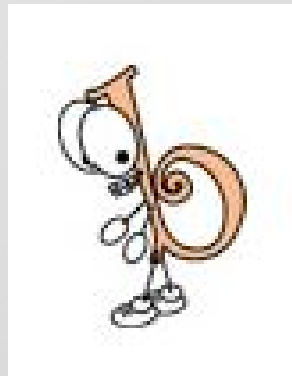


# top @ Tevatron Run II

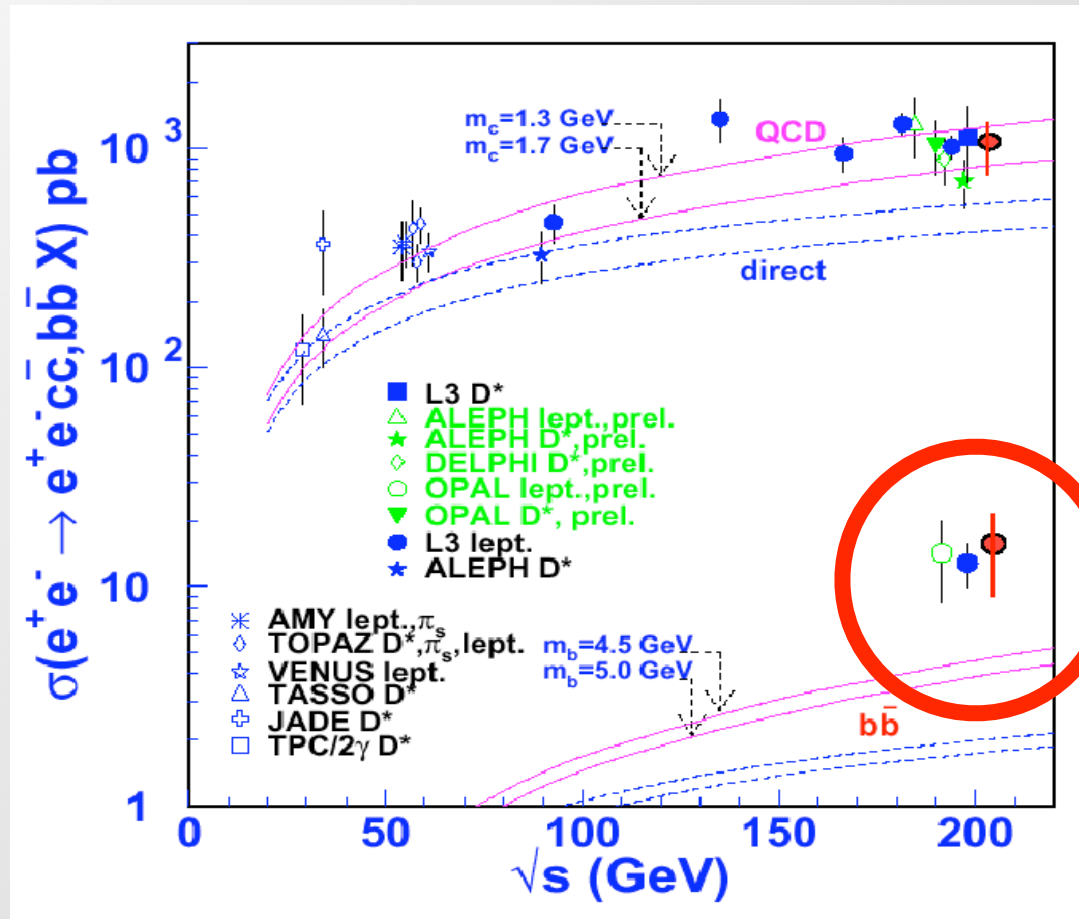


Experimental uncertainties of the same order as the theoretical ones.  
pQCD works perfectly here.

Bottom



# Bottom production @ LEP 2

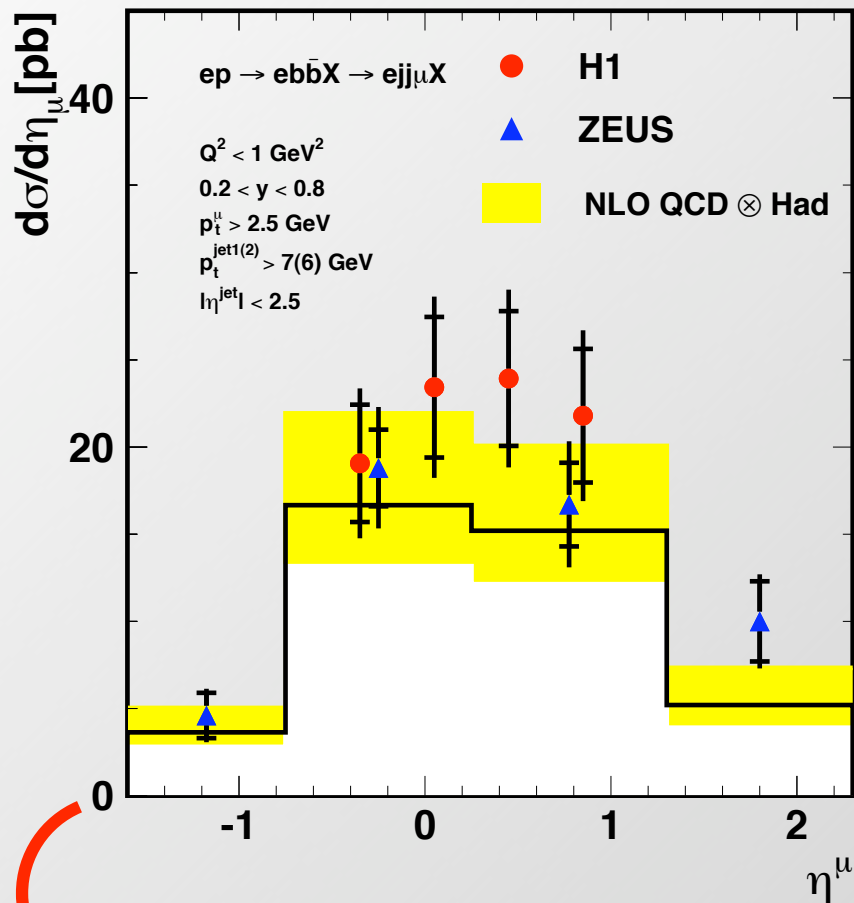


Worst alleged disagreement presently known (I think)

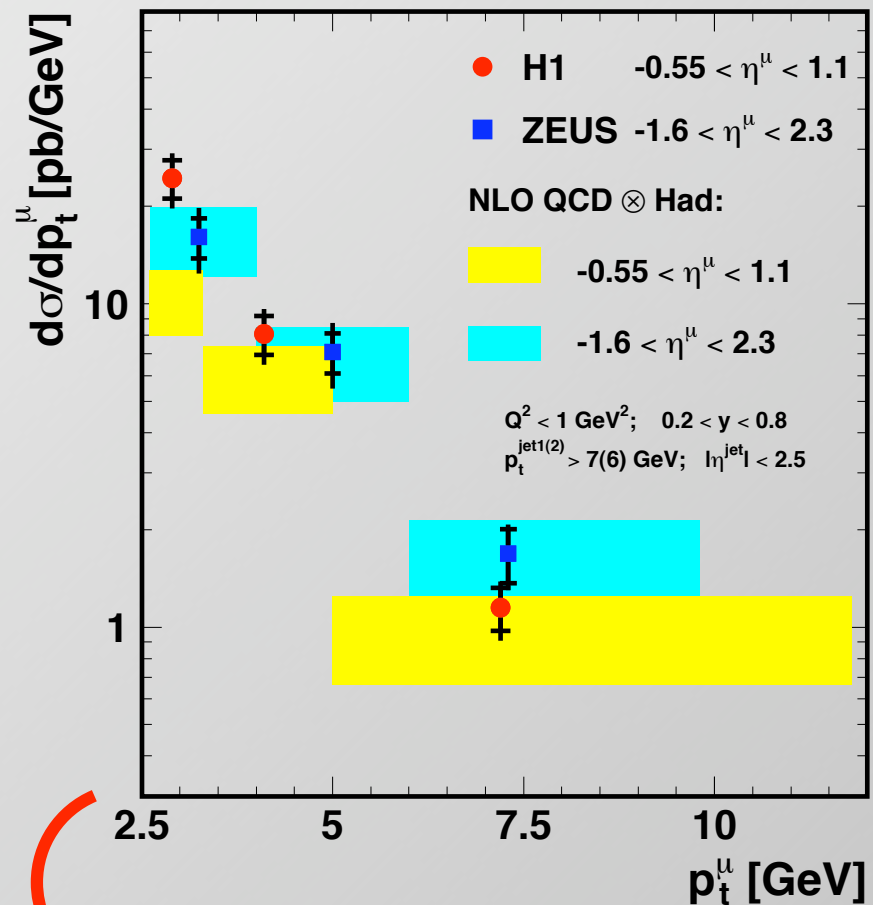
But.... data extrapolated to total cross section. Not a real differential observable

(more differential distributions are available. Awaiting comparison with a theoretical calculation)

# Bottom photoproduction @ HERA

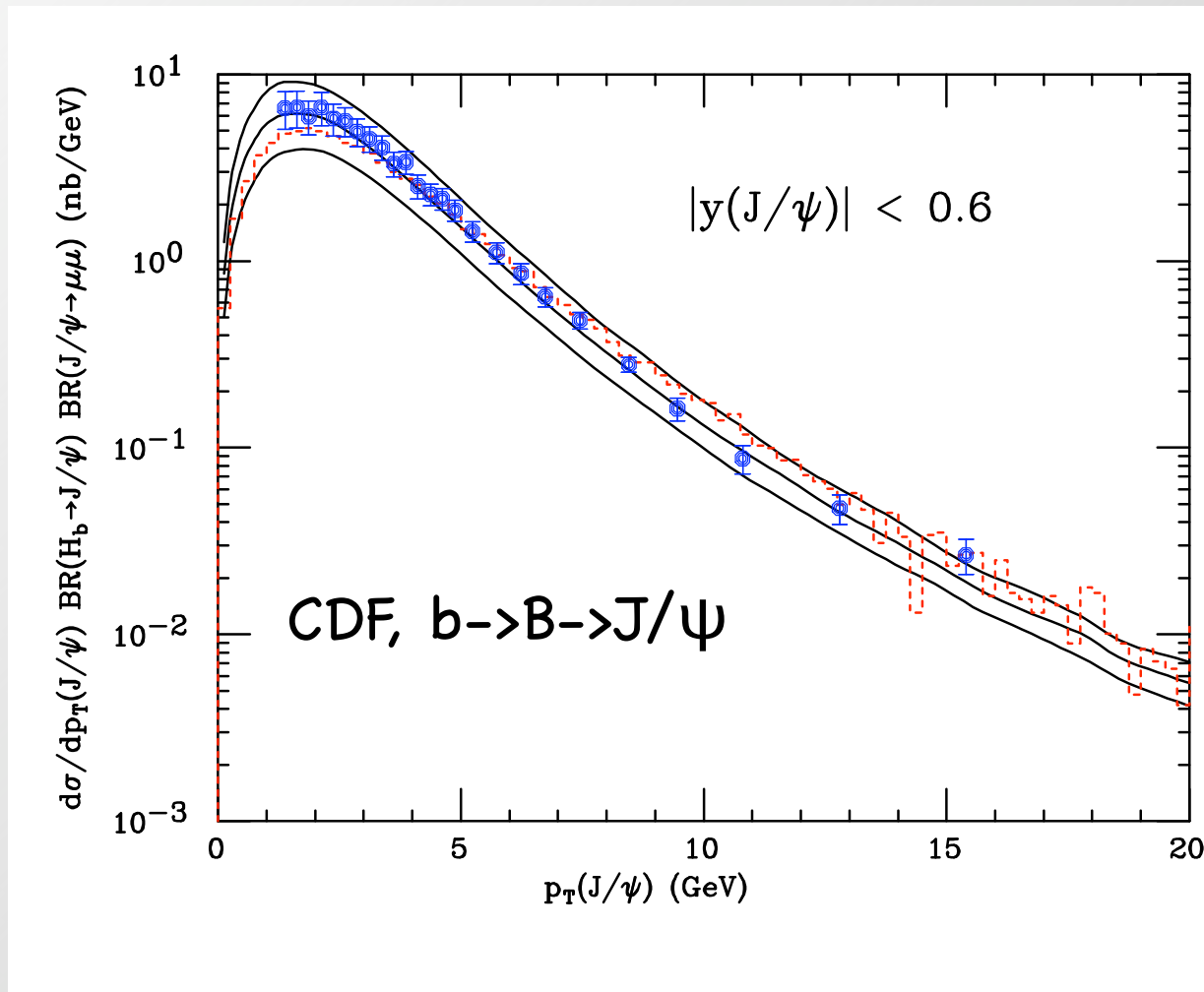


Good agreement  
H1 vs ZEUS



ZEUS: No excess  
at low  $p_t^\mu$

# Bottom production @ Tevatron Run II

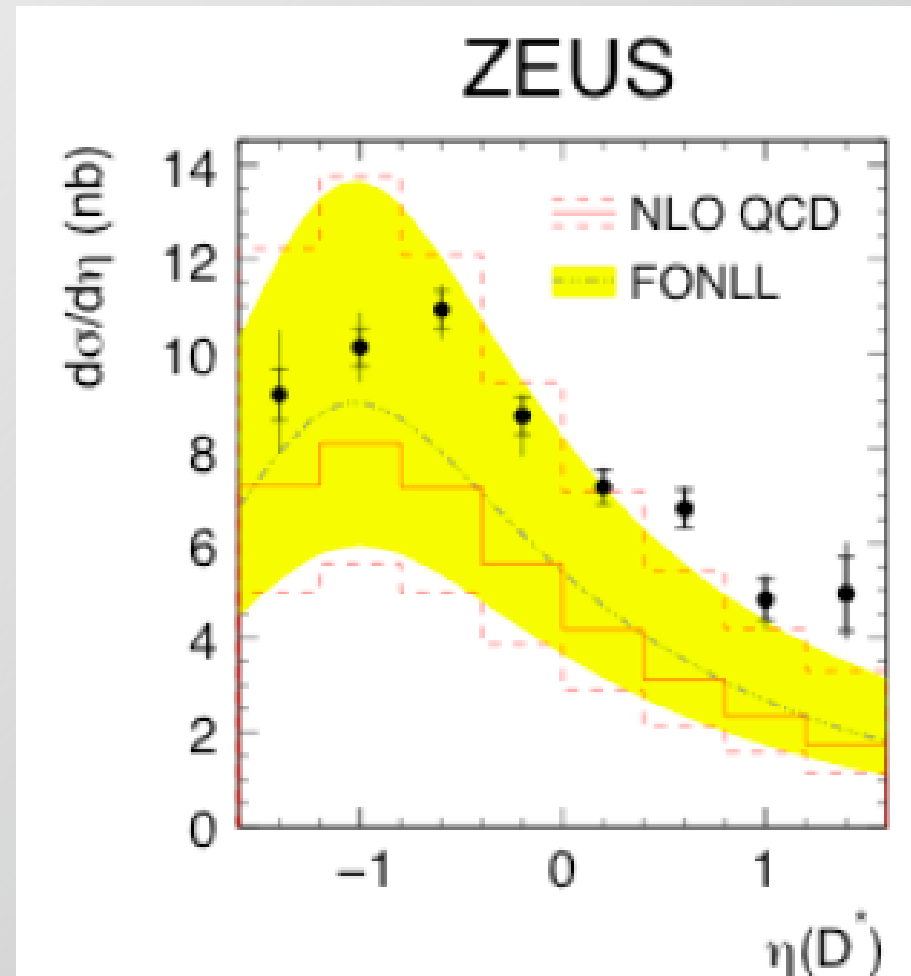
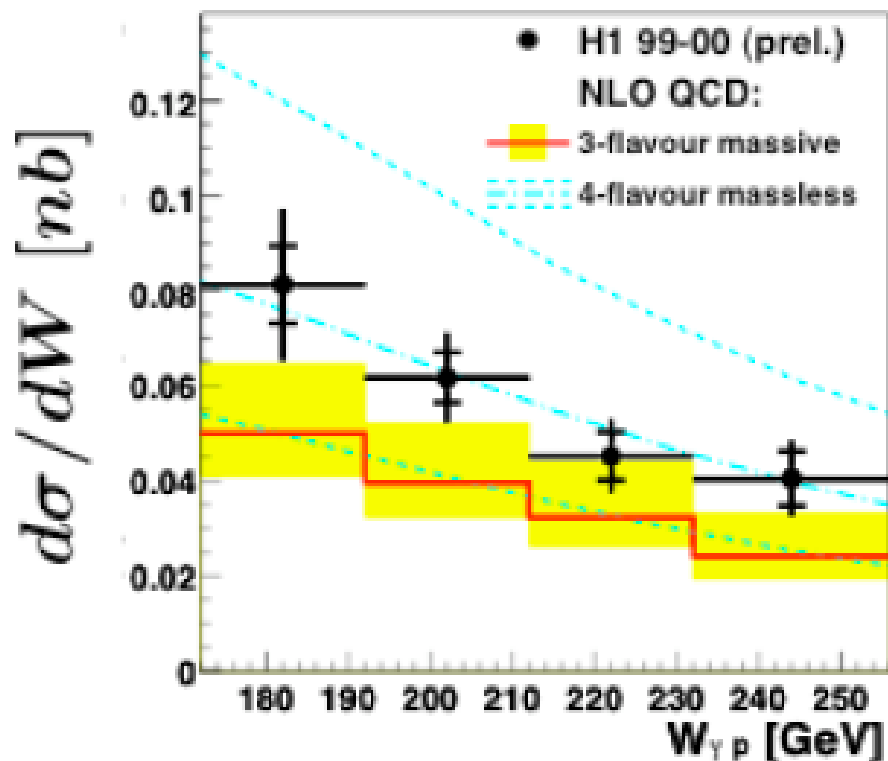


pQCD (+ small non-perturbative contribution) in good agreement with Tevatron bottom data

# Charm

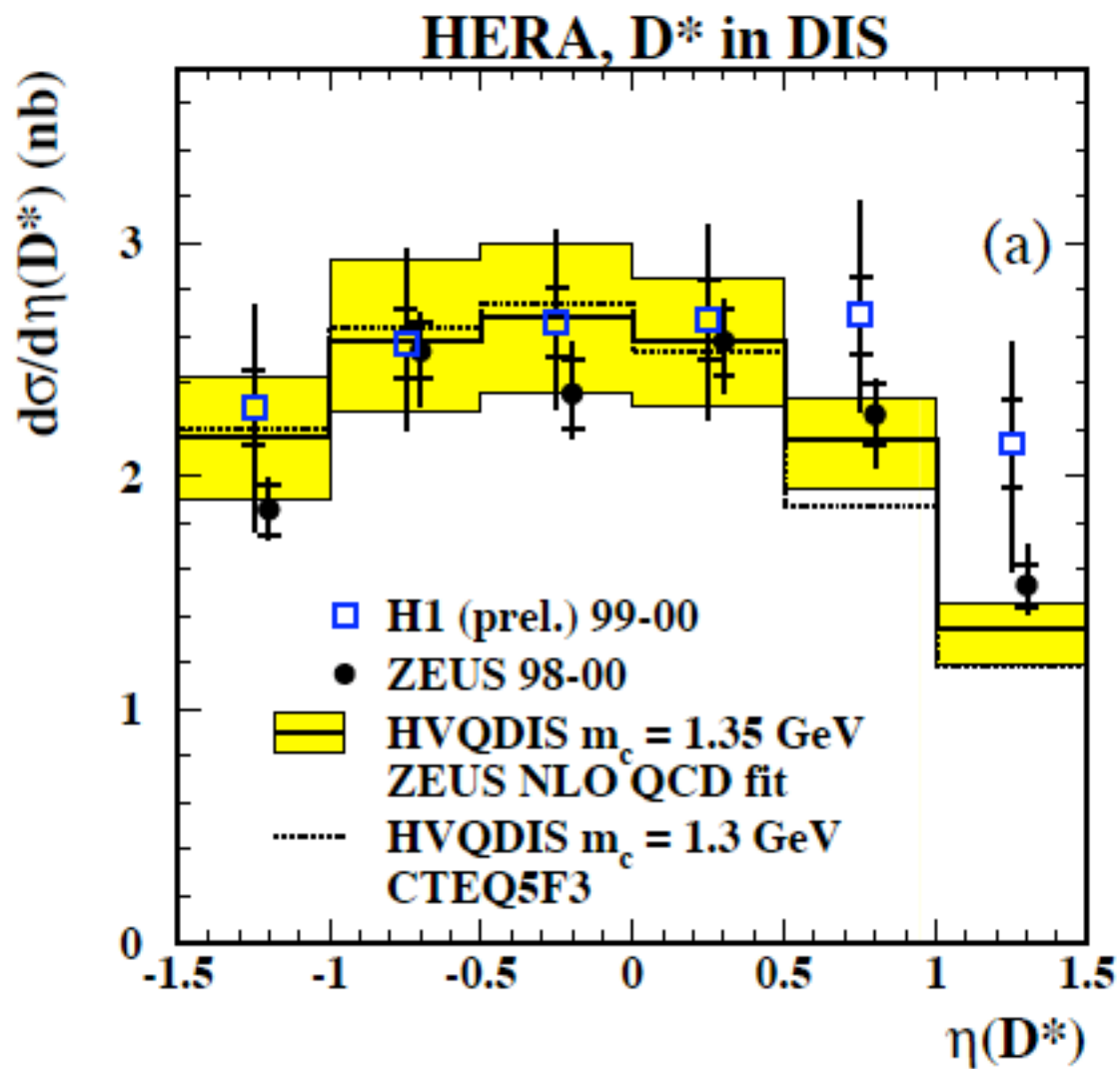


# Charm photoproduction @ HERA



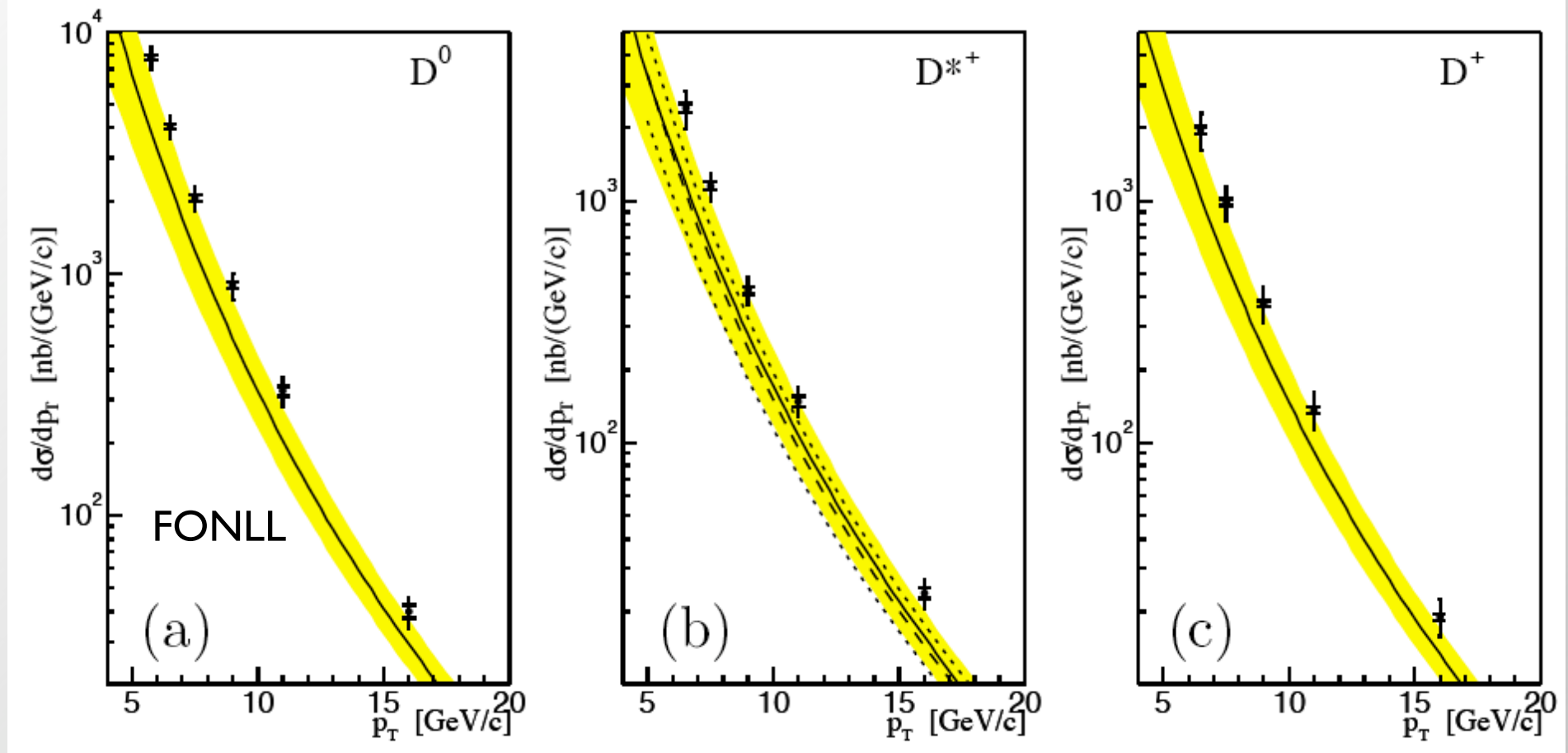
At least 50% theoretical uncertainty. Expt. data more precise

# Charm production in DIS @ HERA



# Charm production @ Tevatron Run II

CDF Run II  $c \rightarrow D$  data [PRL 91:241804,2003]



The non-perturbative charm fragmentation needed to describe the  $c \rightarrow D$  hadronization has been extracted from moments of ALEPH data at LEP.

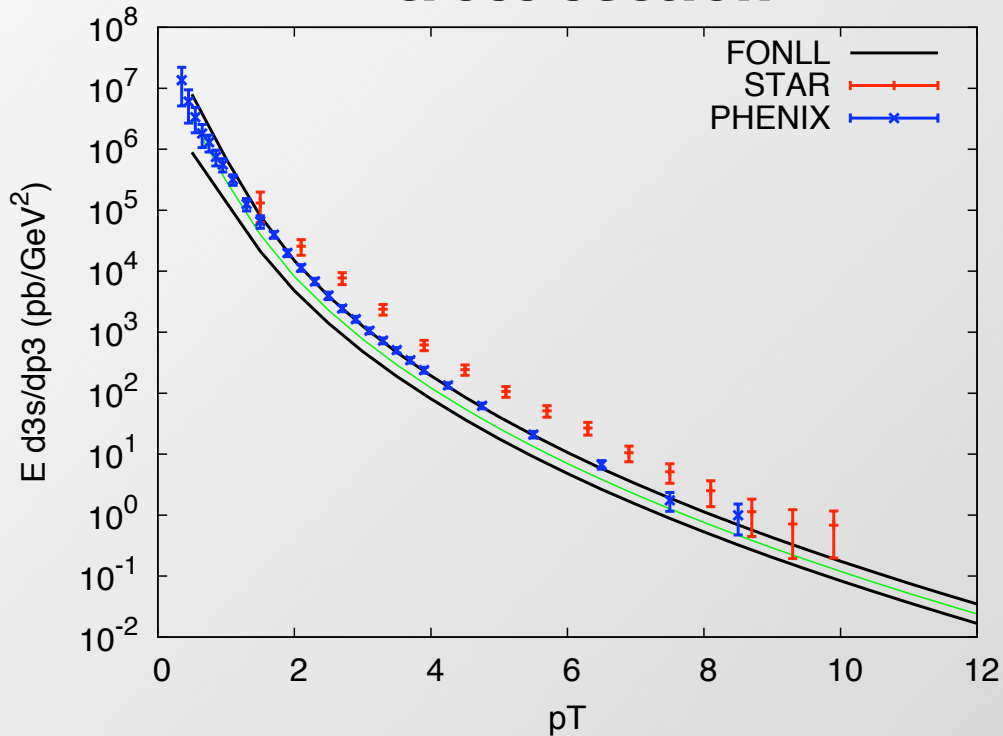
# Charm and bottom at RHIC



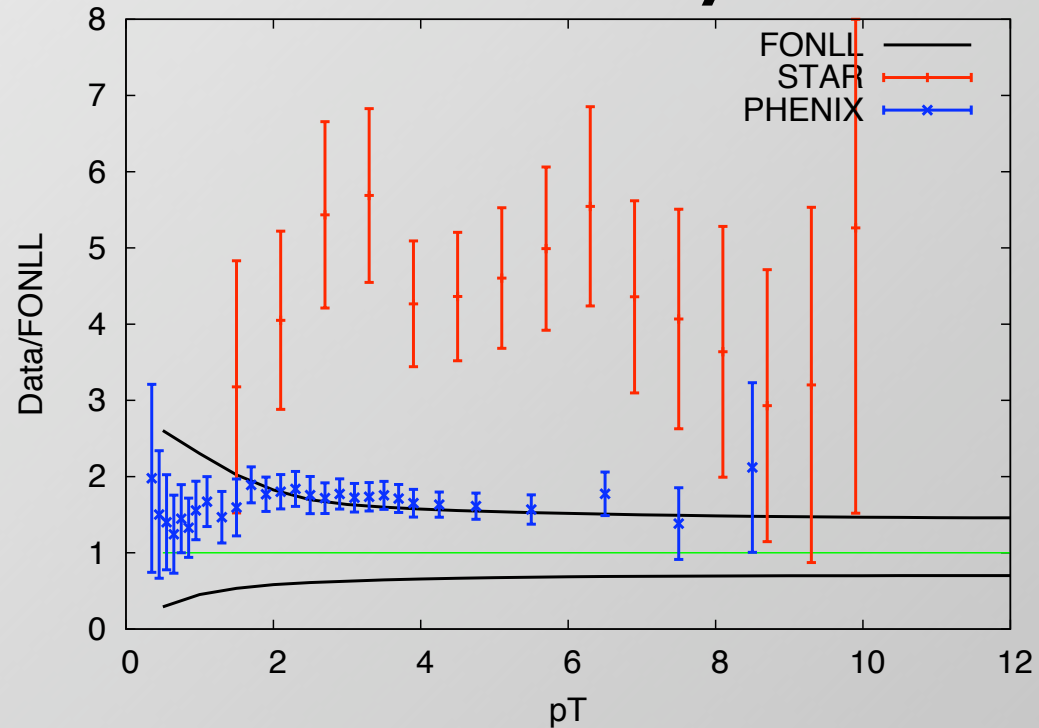
# Charm and bottom production @ RHIC

Non-photonic electrons from charm and bottom

**cross section**



**data/theory**



Phenix data in good agreement.

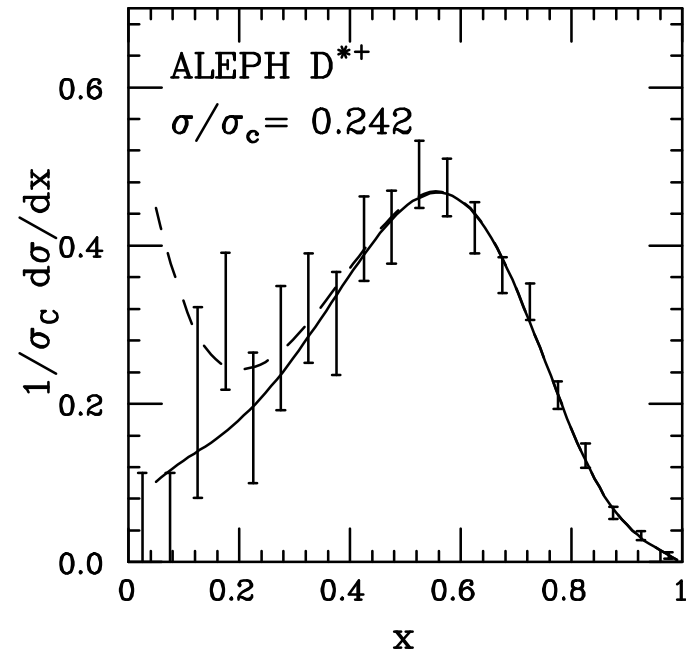
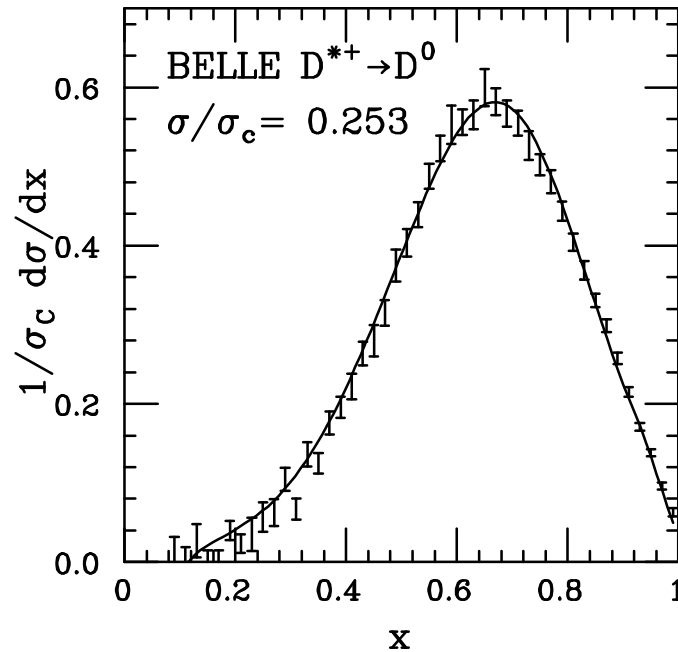
STAR data a little larger than most theory v. charm data comparisons

# Conclusions

- Heavy quark phenomenology is mature and has the tools to produce predictions in many **realistic** situations. These predictions can include all the available knowledge for calculating heavy quark production in QCD. Since they are implemented in a rigorous framework, it is usually possible to also provide a (more or less reliable) estimate of the **theoretical uncertainty**
- Most predictions seem to agree well with Tevatron and HERA data for charm and bottom production. They should provide a solid benchmark for pp collisions at RHIC. Even if you don't believe this, the apparent STAR excess looks a little puzzling, given the better agreement of many other measurements
- Final note: given the size of intrinsic pQCD uncertainty, it is very unlikely that effects of the order of a few (tens of) percent will ever be visible just by comparing to the absolute value of the cross sections. This might (might!) only be doable with a NNLO calculation

**Backup slides**

# D\* fragmentation from BELLE and ALEPH

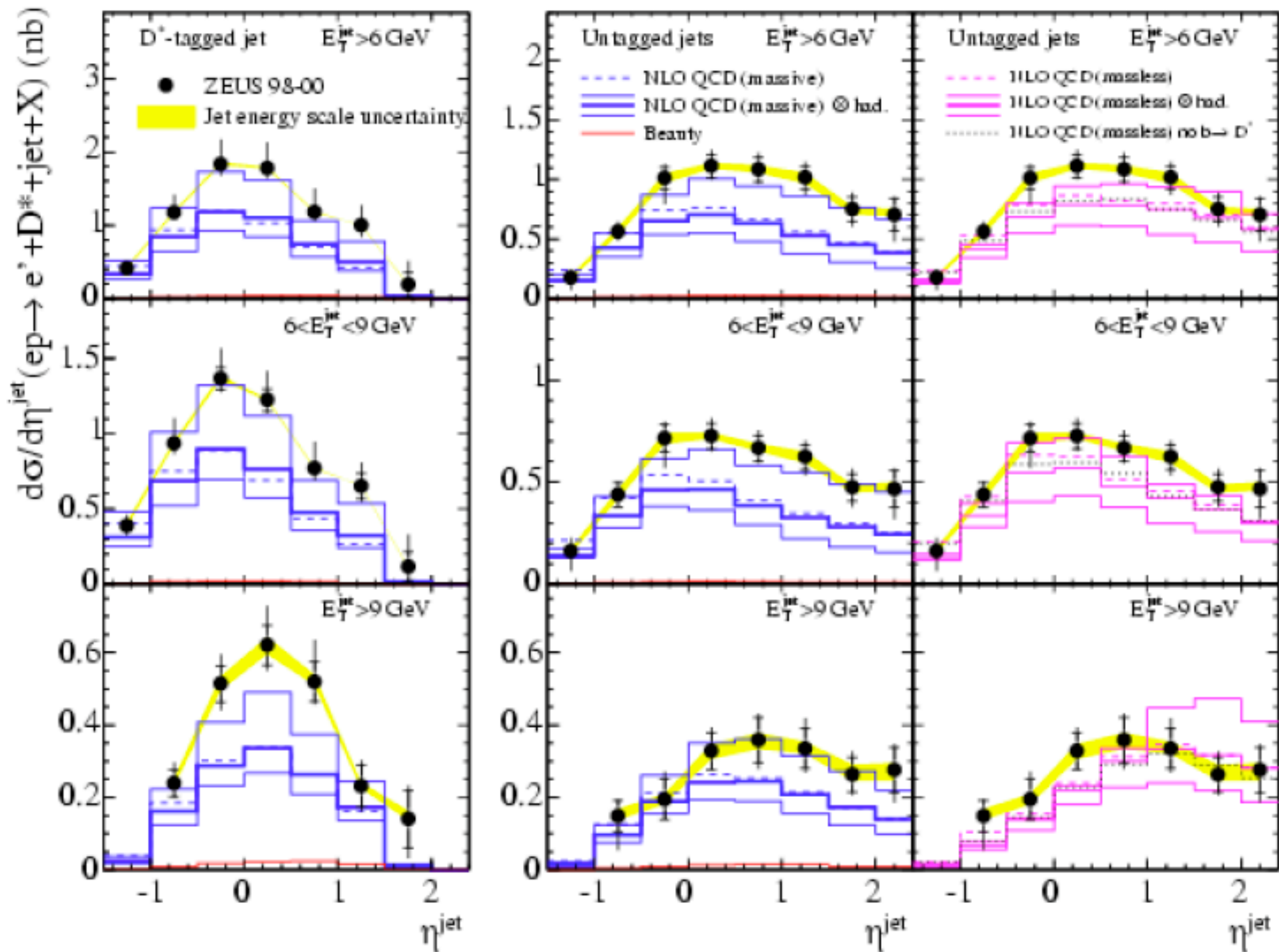


These curves contain two contributions:  
a perturbative one convoluted with a non-perturbative term

The experimentally observed fragmentation is different at the two machines (of course!), but thanks to collinear resummation **the extracted non-perturbative contributions coincide to within about 10%**

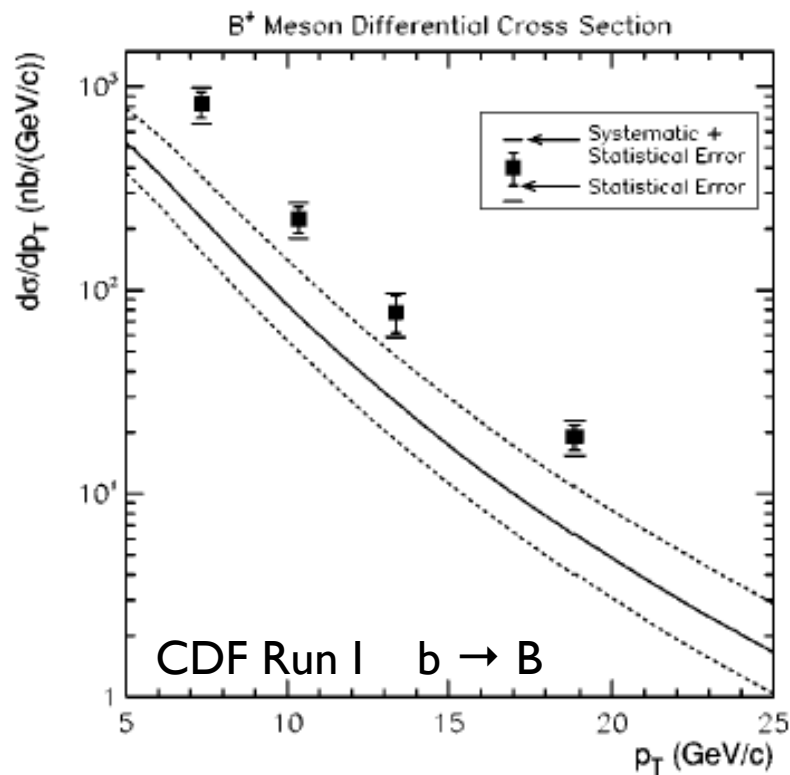
$\Rightarrow$  not much uncertainty on the hadronic production rates, once fragmentation is properly implemented

# D\* + jet @ HERA



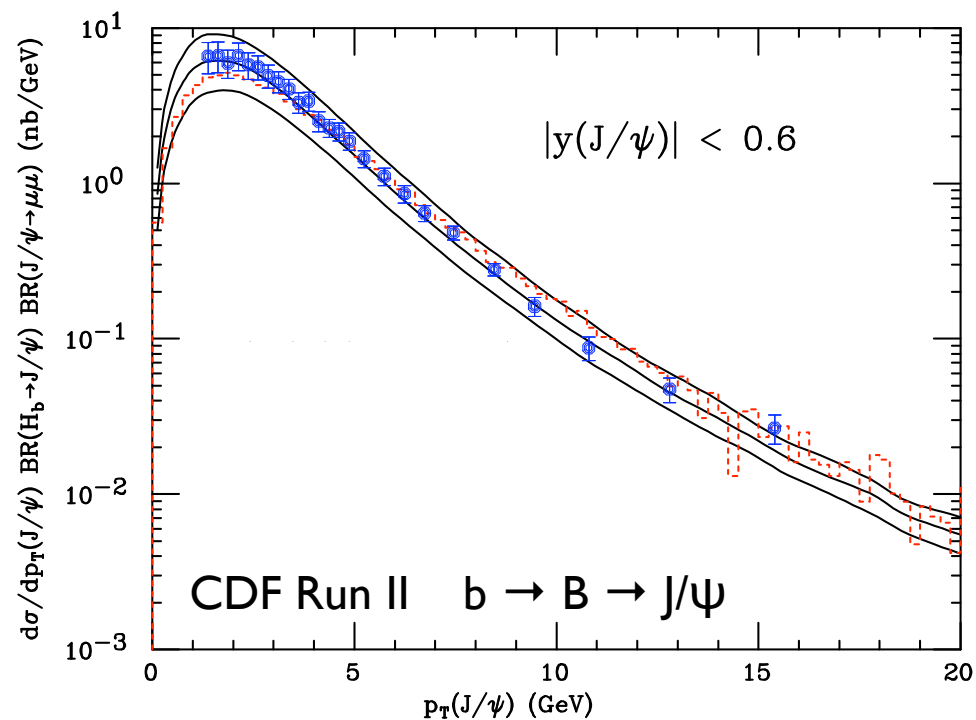
# Bottom production @: Run I vs Run 2

‘Before’



‘Factor of 3’ excess

‘After’

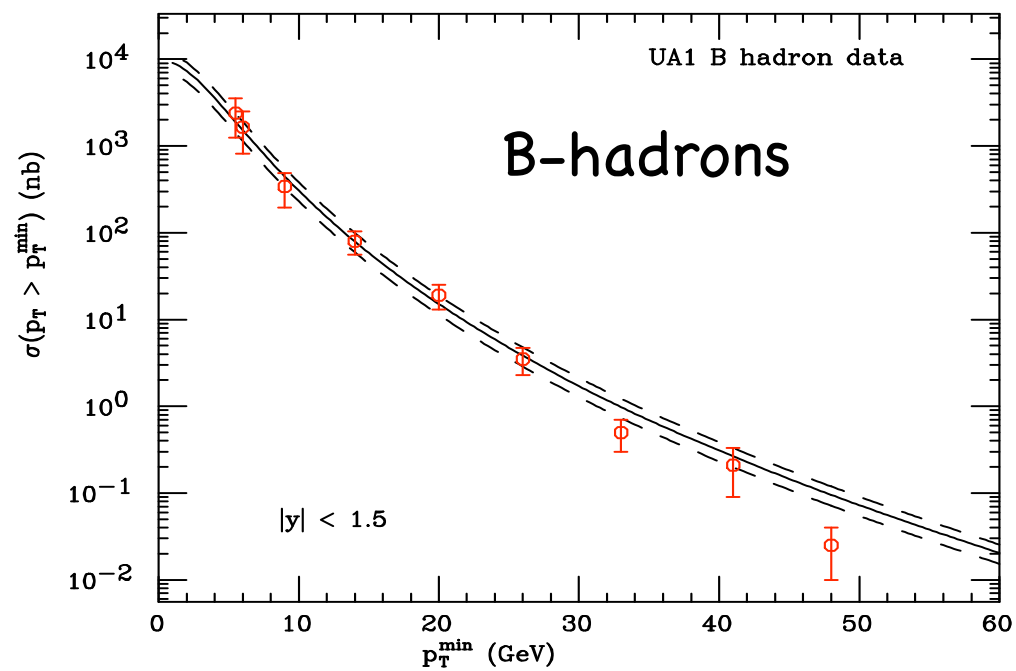
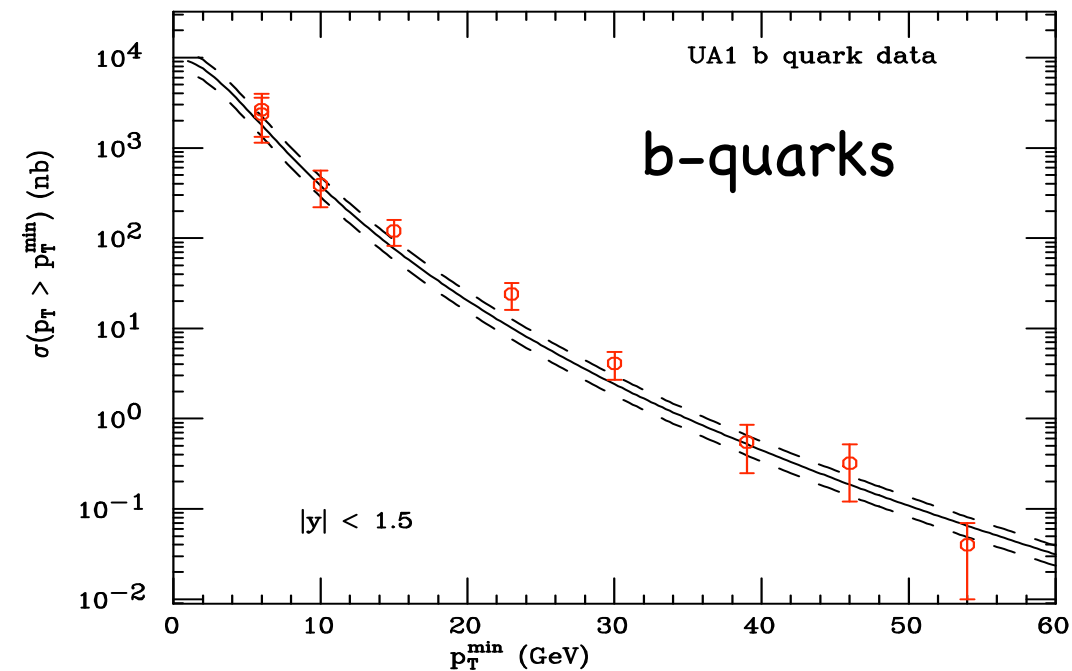
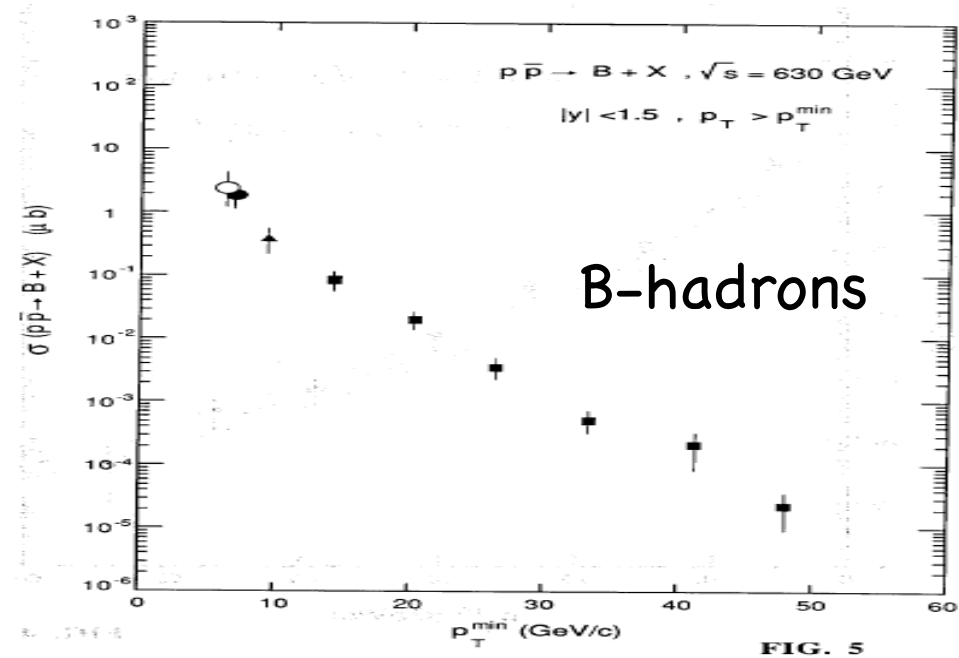
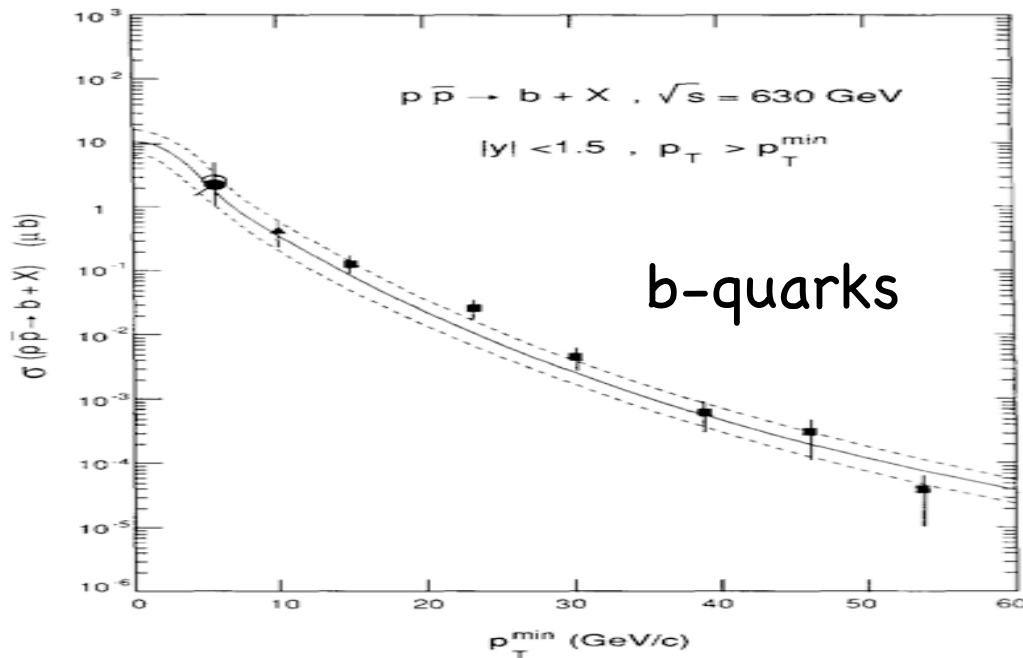


Perfect agreement

Key improvement:

$b \rightarrow B$  non-perturbative fragmentation properly extracted from LEP data within the FONLL framework

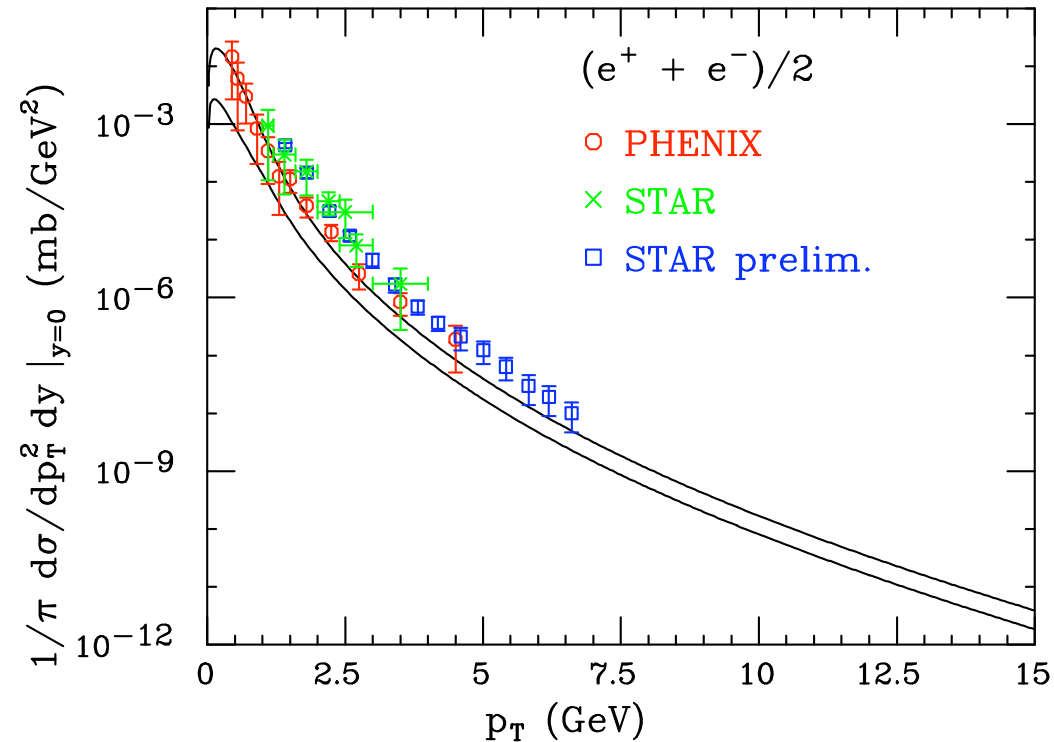
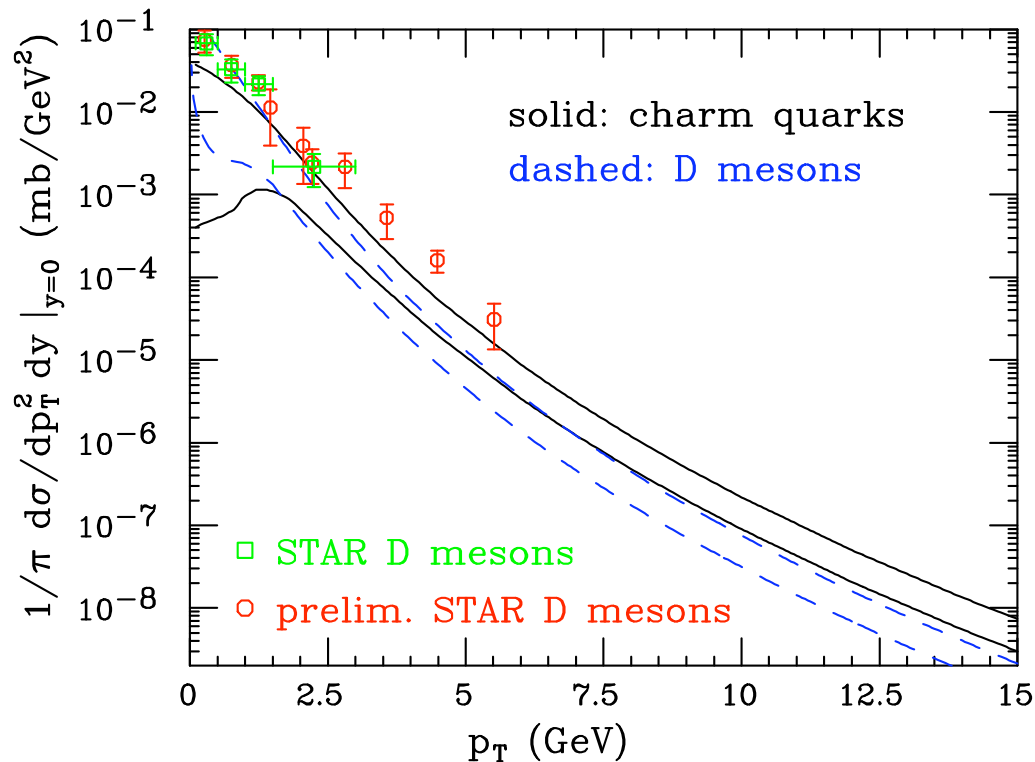
# What about the old UAI data?



**No artificial inflation of theoretical prediction to 'fit' the Tevatron data**

# Charm and bottom production @ RHIC: first data

The matched-resummed pQCD + NP FF formalism can be applied at RHIC

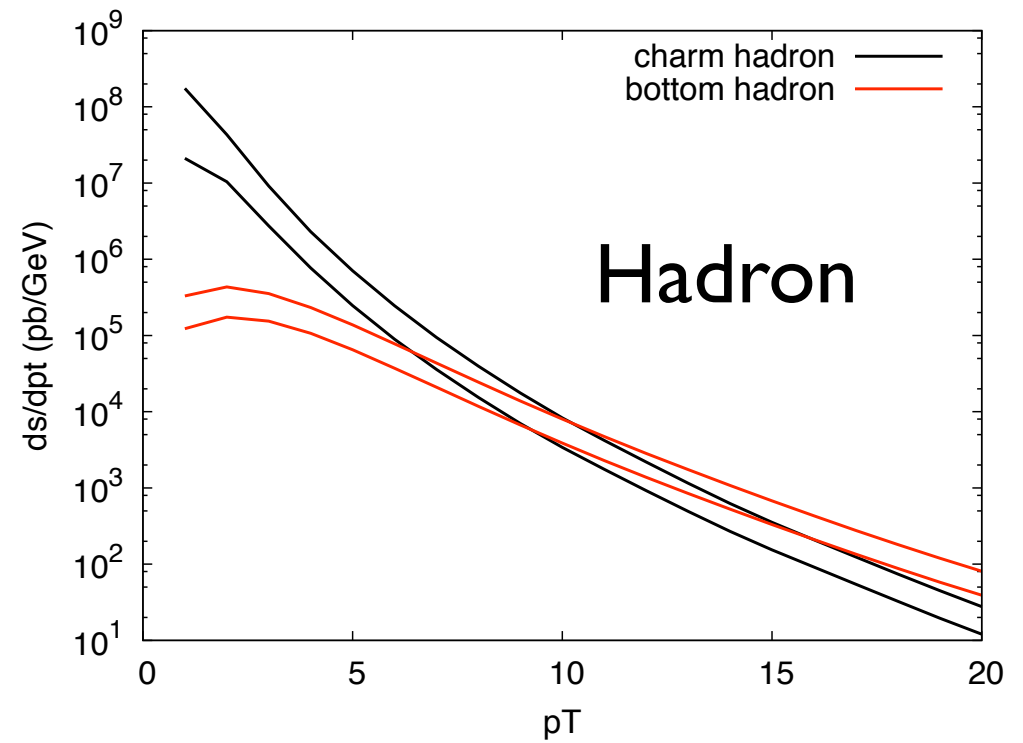
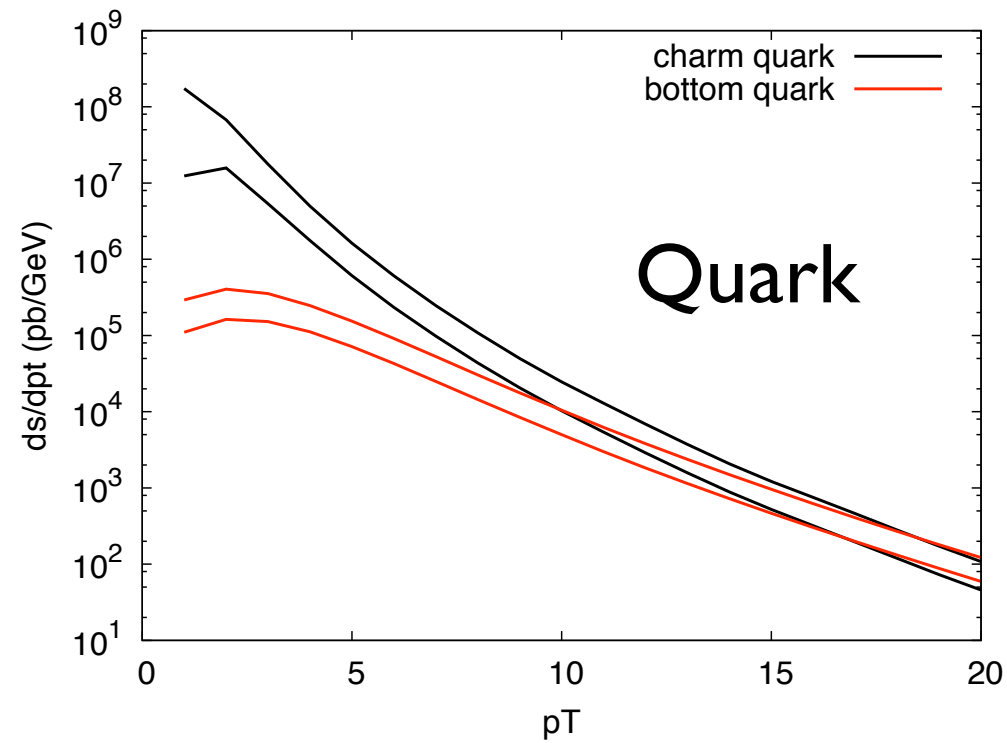


Fair agreement. In line with other measurements.

Note, though, the large theoretical uncertainties at low transverse momentum, especially for charm.

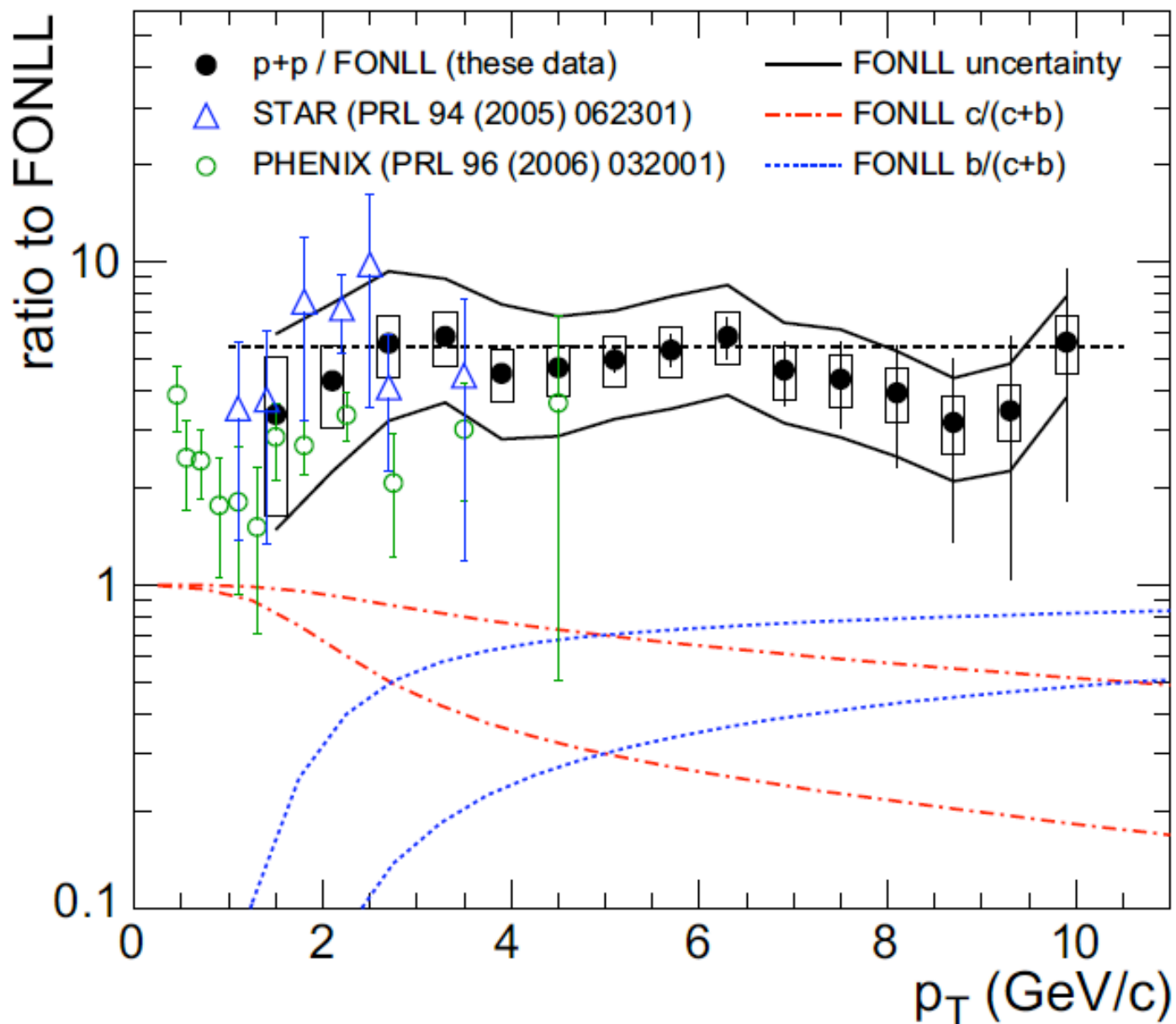
But, what happens with STAR measurements at large  $p_T$  ([nucl-ex/0607012](#))?

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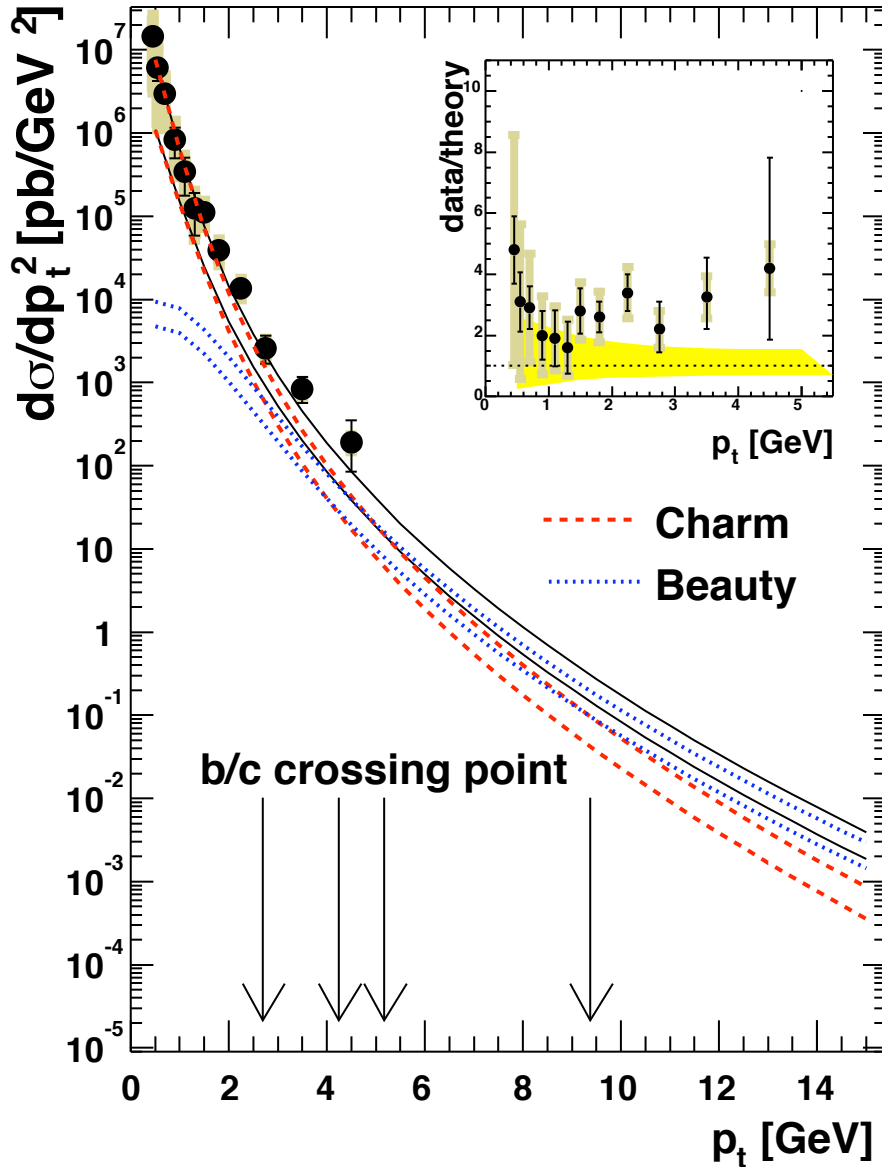
# Charm and bottom production @ RHIC

Plot from STAR, nucl-ex/0607012



A factor of five excess would seem to be a bit too large, if compared to measurements in other experiments

# Relative Charm and Bottom contributions @ RHIC

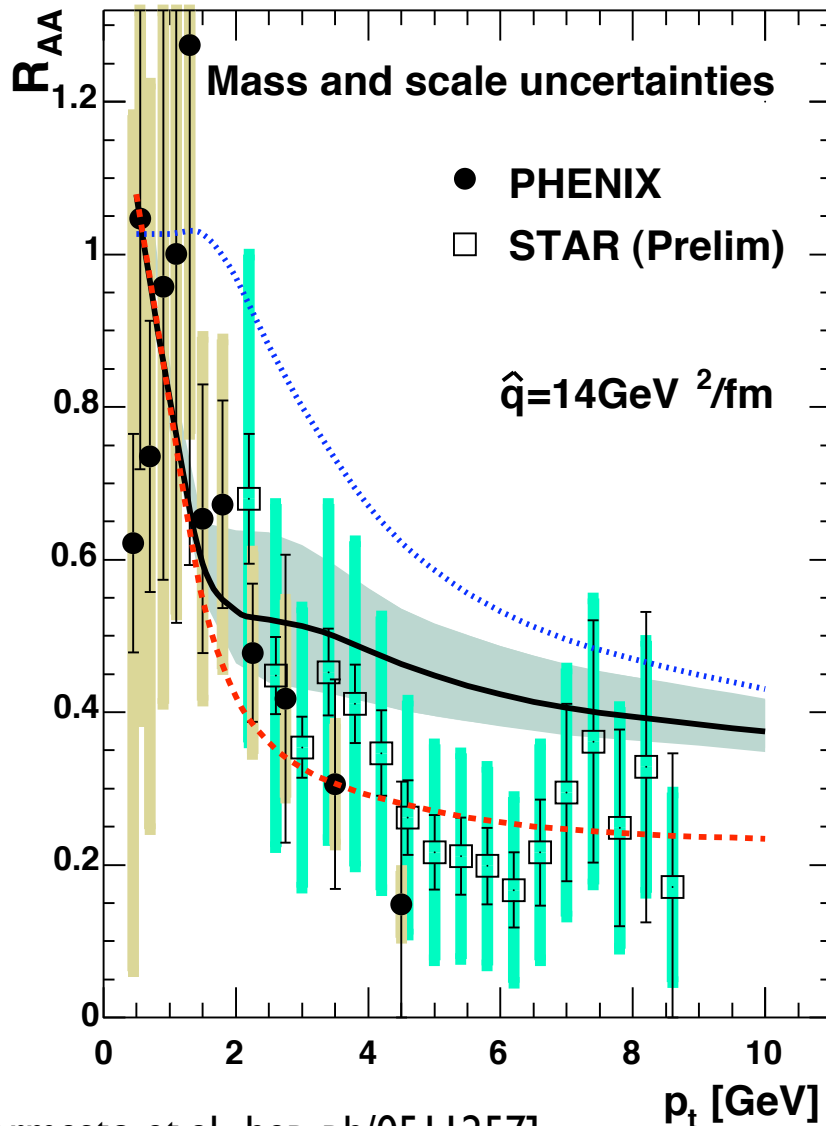


The slope of the charm and bottom contribution is fairly similar: the crossing point easily moves, though the relative contributions are less affected by uncertainties

**NB.** Especially for bottom the transverse momentum is small: all the uncertainties previously mentioned can apply

# R\_AA for Charm and bottom @ RHIC

The charm and bottom spectra translate into R\_AA via the application of quenching weights



[Armesto et al., hep-ph/0511257]

The uncertainty on the charm and bottom relative contribution reflects on an uncertainty of order 0.1 on R\_AA

R\_AA looks too high. However, remember the very large perturbative uncertainty on charm: the NNLO prediction could be quite larger.

**Observation: if you normalize charm to the data R\_AA comes out about right**