Nuclear Modification to Parton Evolution and Onset of Parton Saturation

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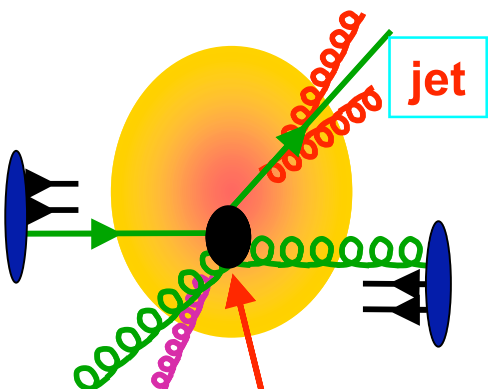
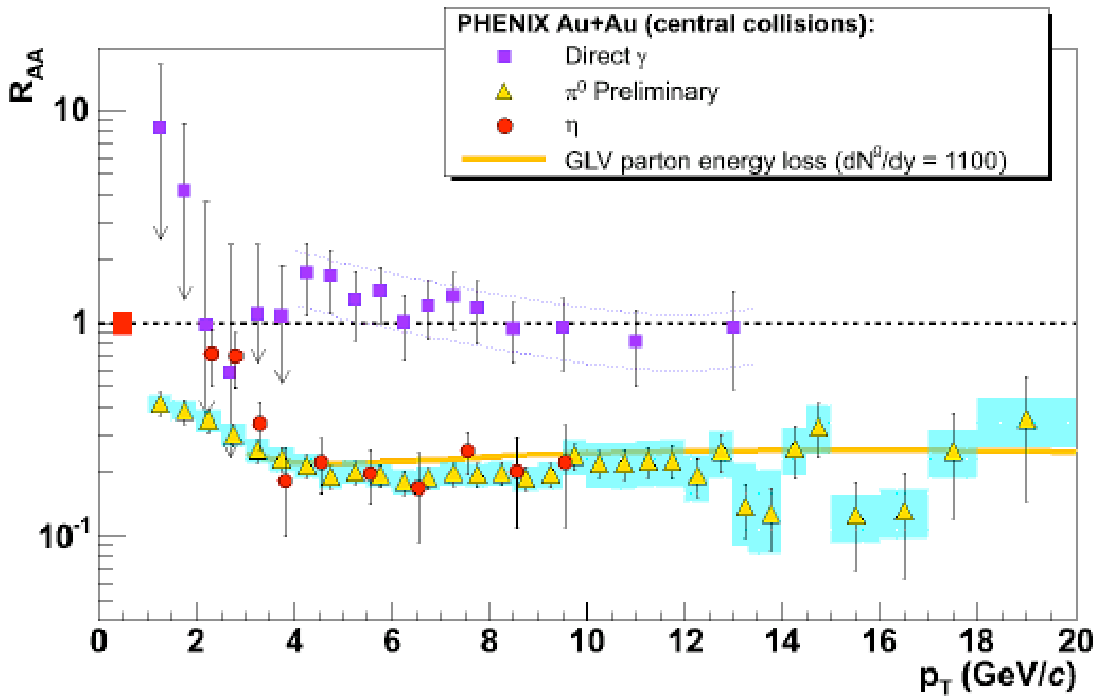
Evidence of Hot Dense Medium

$$R_{AA} = \frac{(d\sigma / dp_T)_{AA}}{N_{coll}\,(d\sigma / dp_T)_{NN}}$$

- ❑ No suppression for direct photons
- ❑ Strong suppression for leading pions

A new hot and dense matter was produced





Phys. Rev. Lett. 96, 2023(2006)

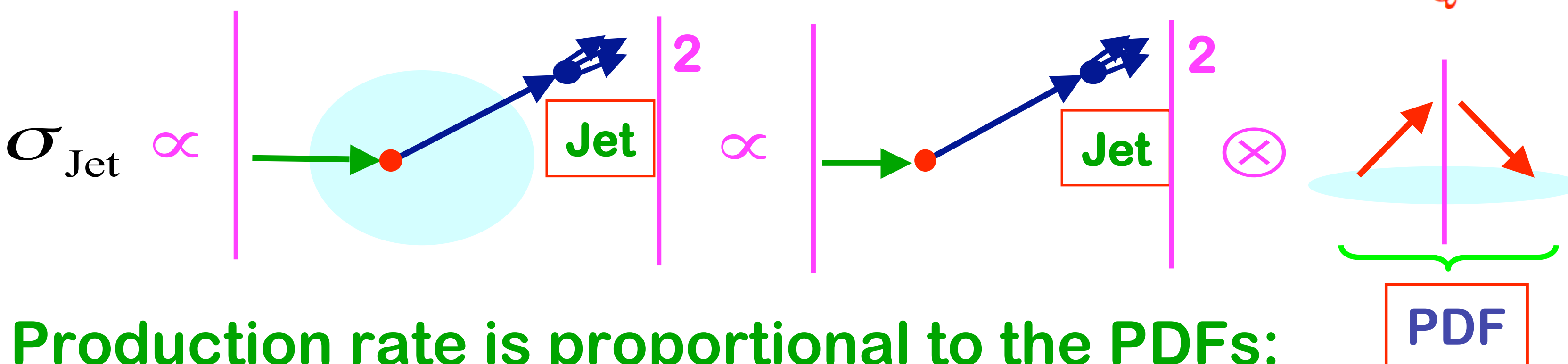
Medium properties:

- ➢ Calibration of single hard scattering
- ➢ Multiple scattering in medium

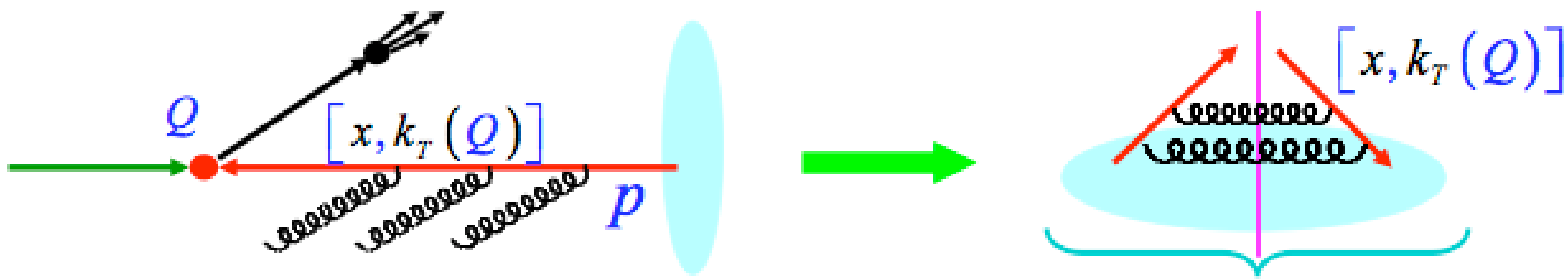
Single Hard Scattering

Calibration of the hard probes

- **Non-perturbative dynamics is effectively frozen:** $\frac{1}{Q} \ll \text{fm}$



- **Production rate is proportional to the PDFs:**



$\varphi_{f/A}(x, k_T(Q), Q)$

- **QCD collinear factorization:**

Valid only if $k_T(Q) \ll xp \sim Q$ →

Need a new approach if

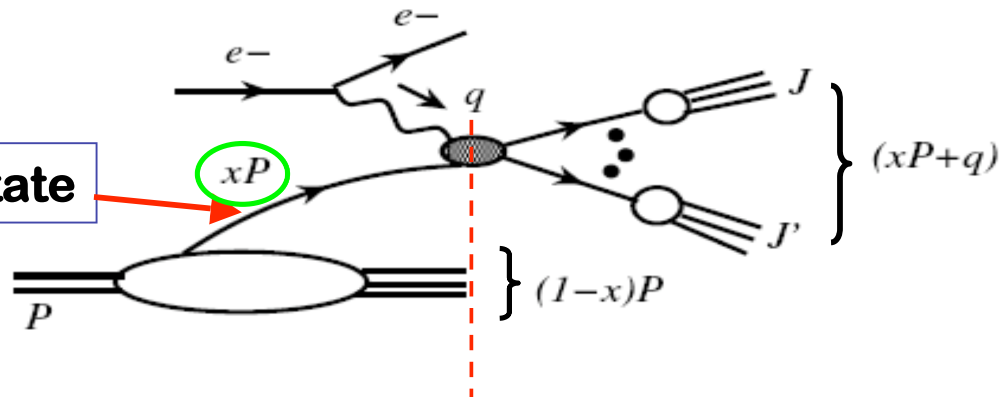
$Q \sim xp \approx k_T(Q) \equiv Q_s$

QCD Collinear Factorization

Deep Inelastic Scattering (DIS) as an example

- **Time evolution:**

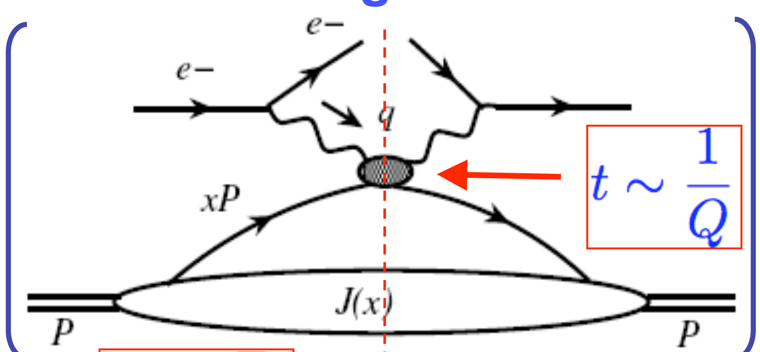
Long-lived parton state



Time: **"Past"** **Now** **"Future"**

- **Inclusive cross section – summing over all final states**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}$$



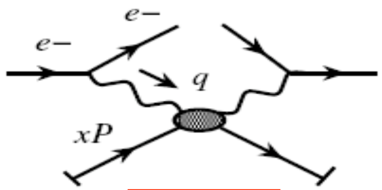
$t \sim \frac{1}{Q}$

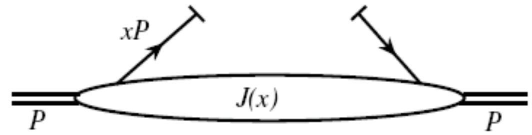
$t \sim R$

Not IR safe

Interactions between the "past" and "now" are suppressed!

Predictive Power of PQCD – Factorization

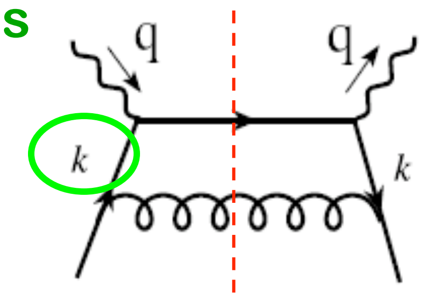
$\sigma_{tot}^{DIS} \sim$ 

$\otimes$ 

$+ \, O\left(\frac{1}{QR}\right)$

Now **Past** **Connection**

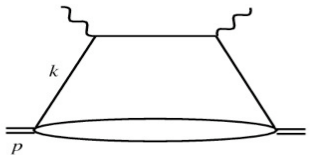
- **Now:**
  Short-distance part is Infrared-Safe, and pQCD calculable
- **Past:**
  Long-distance part – parton distribution functions
- **Partonic diagram ≠ short-distance part:**
  Could have both short- and long-distance pieces



- **Parton distribution functions (PDFs):**
  absorb all long-distance pieces of partonic diagrams – the past
  process independent – predictive power
- **Factorization – prescription to separate short- from long-distance**
  short-distance and long-distance are separately gauge invariant

Factorization at the Leading Power

- LO = Parton Model



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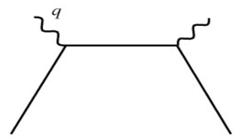

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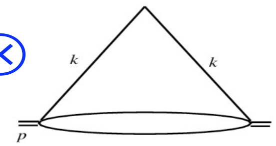
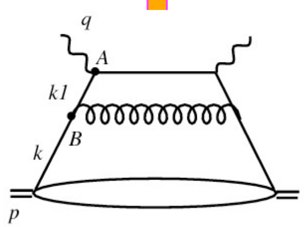
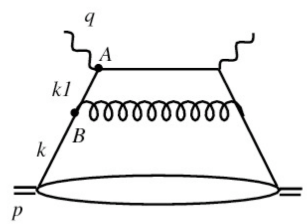


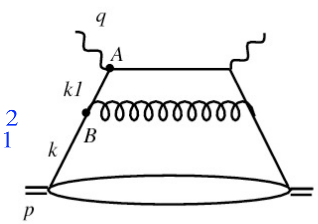
- NLO: diagram has both long- and short-distance physics

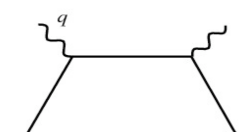
  Factorization: separation of short- from long-distance

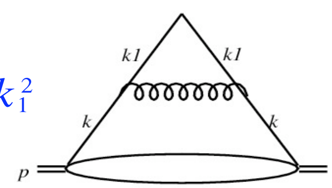
$\propto \int_0^{\sim Q^2} \frac{dk_1^2}{k_1^2}$ Dominated by $\begin{cases} k_1^2 \to 0 \\ t_{AB} \to \infty \end{cases}$ CO divergence

$\int_0^{\sim Q^2} dk_1^2$


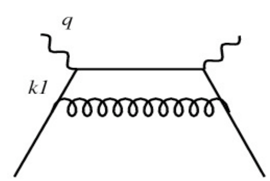
$= \int_0^{\mu^2} dk_1^2$


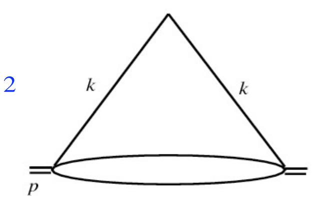
$+ \int_{\mu^2}^{\sim Q^2} dk_1^2$


$C^{(0)} \otimes \varphi^{(1)} \longrightarrow$ =


$k_1^2 \approx 0$
$\otimes \int_0^{\mu^2} dk_1^2$


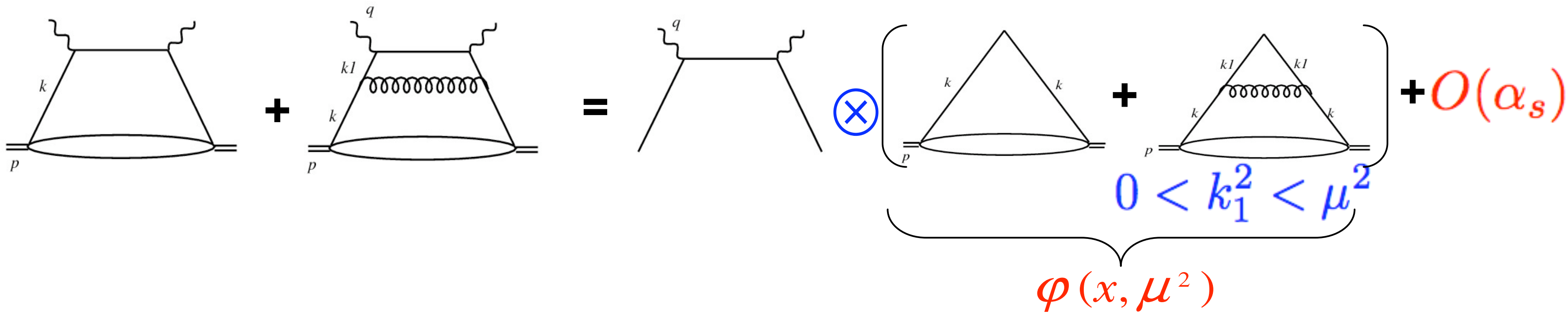
LO + evolution

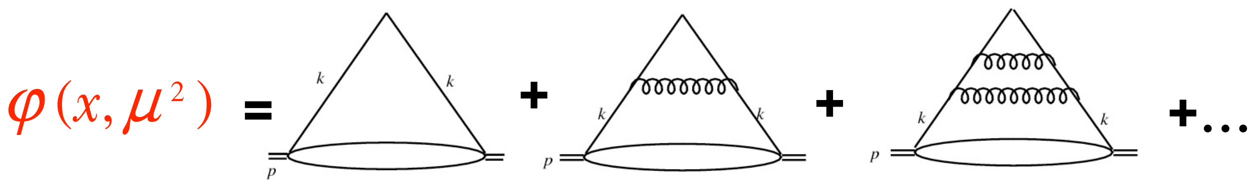
$C^{(1)} \otimes \varphi^{(0)} \longrightarrow$ $+ \int_{\mu^2}^{\sim Q^2} dk_1^2$


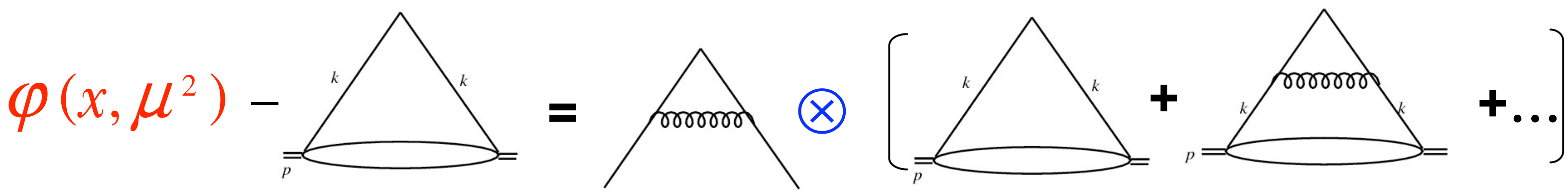
$\otimes \int_0^{k_1^2} dk^2$


NLO

Scale Dependence of Parton Distributions





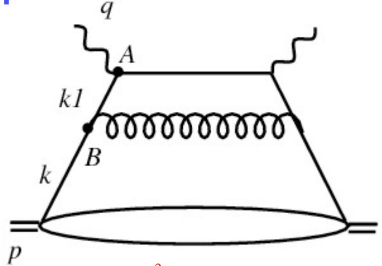


DGLAP Equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} \varphi_i(x, \mu^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu^2)$$

DGLAP is NOT Enough

- PDFs defined in DGLAP evolution do not remove all the long distance piece(collinear divergences) in partonic scattering amplitude

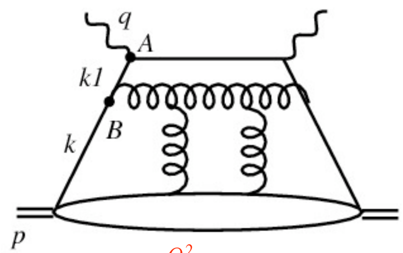


$$\int_0^{\sim Q^2} \frac{dk_1^2}{k_1^2}$$

$$k_1^2 \to 0$$

$$t_{AB} \to \infty$$

+

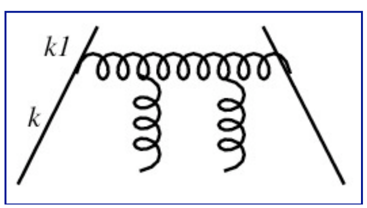


$$\int_0^{\sim Q^2} \frac{dk_1^2}{(k_1^2)^2}$$

$$k_1^2 \to 0$$

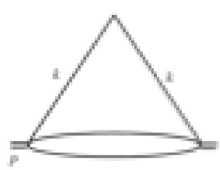
$$t_{AB} \to \infty$$

+…



Should be a part of PDF

$\varphi(x, \mu^2)$ =



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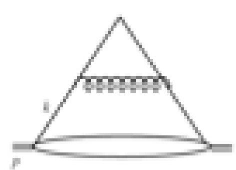

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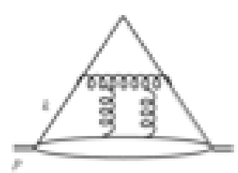


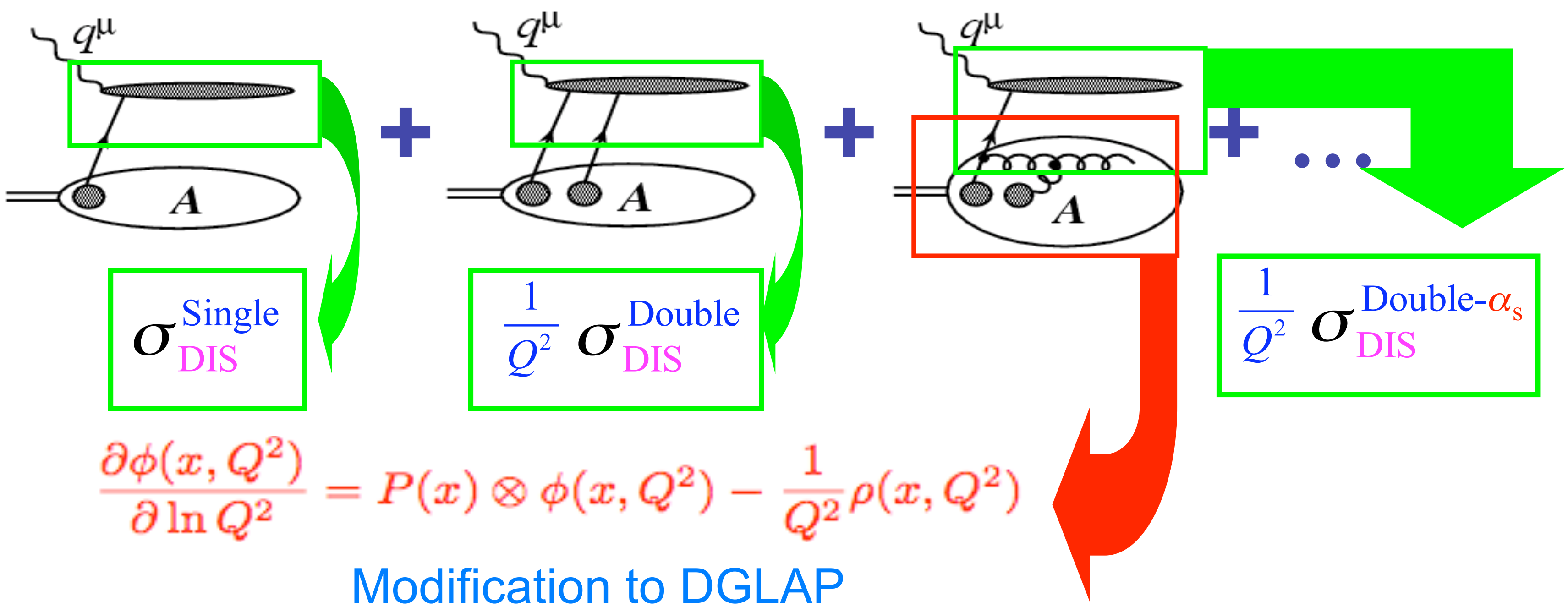
+…

When should we consider this

- Low *x*: size of a hard probe might be larger than a Lorentz contracted hadron

$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2R\left(\frac{m}{p}\right) \longrightarrow x \ll x_c \equiv 0.1$$

- Low *x* parton can interact coherently among themselves if there are more than one at a given impact parameter



$$\frac{\partial \phi(x,Q^2)}{\partial \ln Q^2} = P(x) \otimes \phi(x,Q^2) - \frac{1}{Q^2}\rho(x,Q^2)$$

Modification to DGLAP

Modification to DGLAP Evolution

$$\frac{\partial \phi(x,Q^2)}{\partial \ln Q^2} = P(x) \otimes \phi(x,Q^2) - \frac{1}{Q^2}\rho(x,Q^2)$$

- Power corrections at very small *x*:

Mueller-Qiu, 1986

$$\rho\left(x,Q^2\right) \propto \alpha_s \left[g\left(x,Q^2\right)^2\right]$$

- AGK cutting rule was used, and valid when *x* is very small
- Hard to be generated to all powers in $1/Q^2$

- If x is not too small, and distribution is very steep:

This talk
Kang-Qiu, 2006

$$\rho\left(x,Q^2\right) \propto \alpha_s \left[x\frac{\partial}{\partial x}\varphi\left(x,Q^2\right)\right]$$

- No AGK cutting rule was used
- Leading pole approximation was used – leading power in $A^{1/3}$
- Can be generated to all powers in $1/Q^2$

- Two contributions complement to each other

Modification can be very important

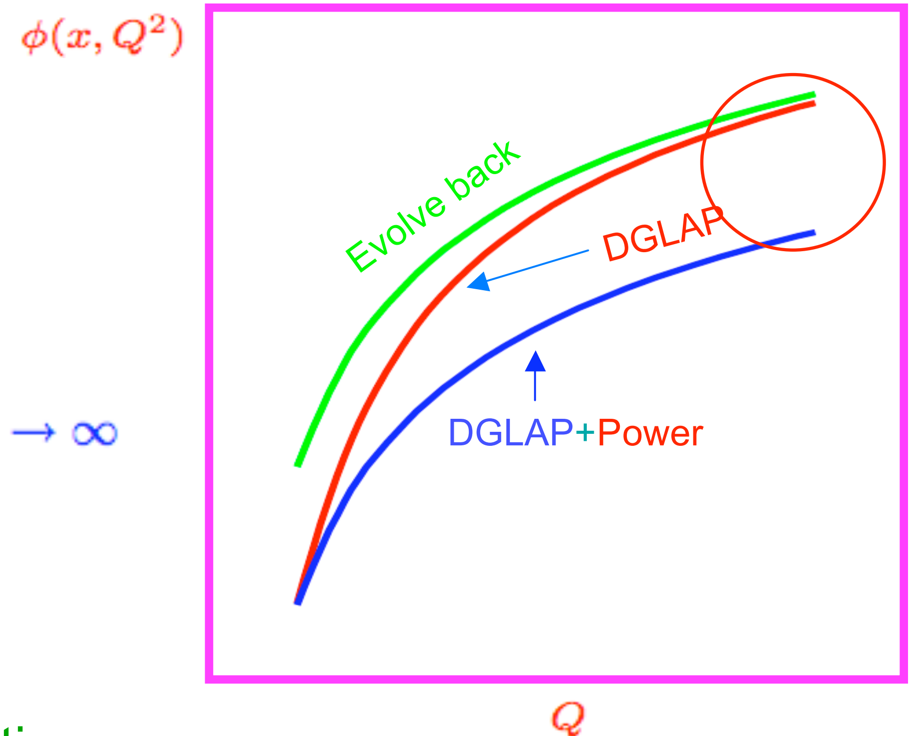
$$\frac{\partial \phi(x,Q^2)}{\partial \ln Q^2} = P(x) \otimes \phi(x,Q^2) - \frac{1}{Q^2}\rho(x,Q^2)$$

- Power corrections to the evolution of PDFs:

  does not vanish as $1/Q^2$ to the physical cross sections

$$\int_{Q_0^2}^{Q^2} \frac{d\ln\mu^2}{\mu^2} = \frac{1}{Q_0^2} - \frac{1}{Q^2} \to \frac{1}{Q_0^2} \quad as \quad Q \to \infty$$

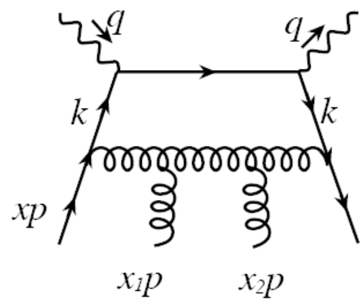
- Large effect to can be built up to PDFs:

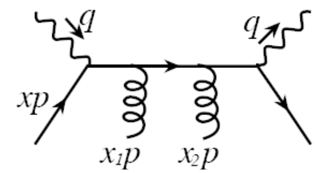


A big difference in parton distribution when evolved from low Q^2 to high Q^2 (blue line vs red line), or from high Q^2 to low Q^2 (green line vs red line)

First Order Power Correction

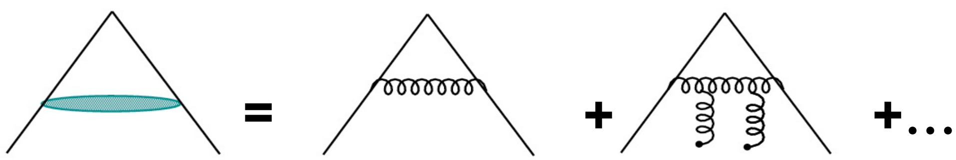
- One-loop diagram:



  - $k_T^2 \sim Q^2 \Rightarrow O(\alpha_s)$ to 

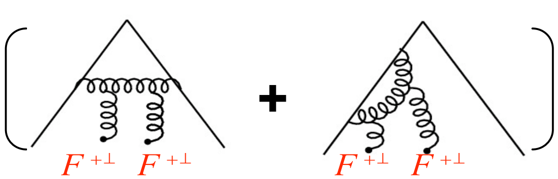
  - $k_T^2 \ll Q^2 \Rightarrow$ Modification to DGLAP

- Modified $q \rightarrow q$ evolution kennel:

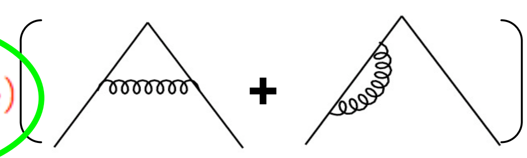


+ **Virtual diagrams**

- Leading pole approximation $\Leftrightarrow \int dx_1 dx_2$ by poles – less diagrams:



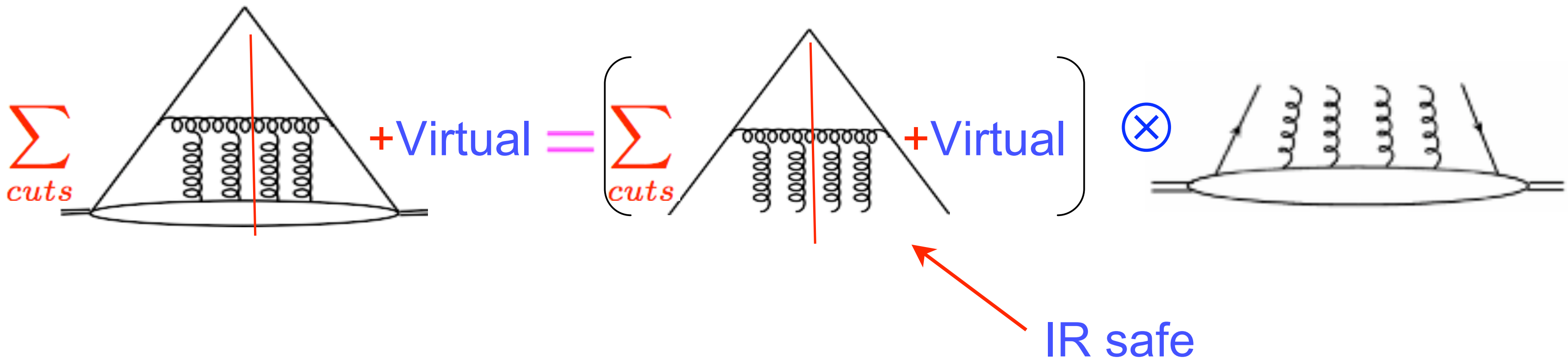
$\sim [< F^{+\perp} F_{\perp}^{+} > (-1)] \frac{d}{dy}\delta(y - y_B)$



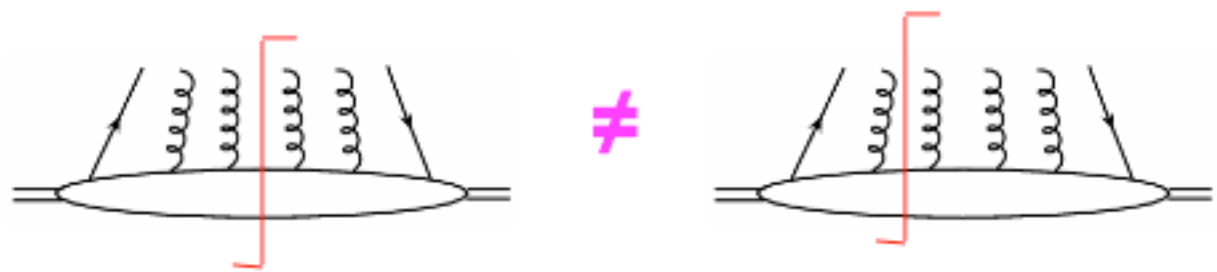
Leading twist evolution kernel

Collinear Expansion is Necessary

- Within collinear factorization:



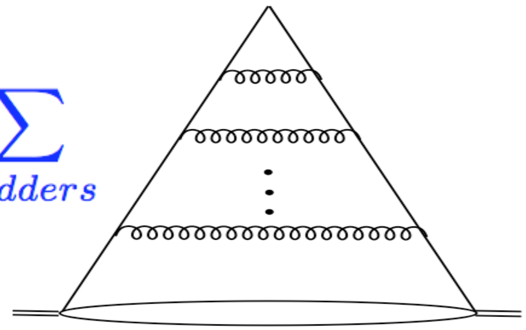
Different cuts for matrix elements of partons with k_T are not equal:



Modified Ladder Diagrams and Higher Powers

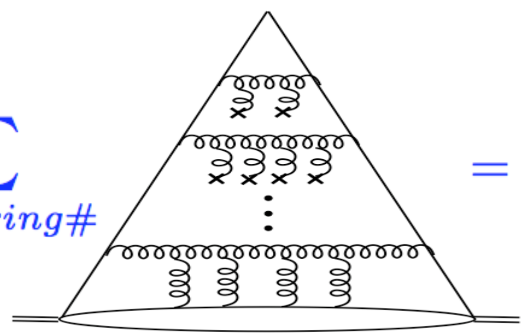
- Leading contribution to DGLAP

$$\phi(x,\mu^2)^{DGLAP} = \sum_{ladders}$$

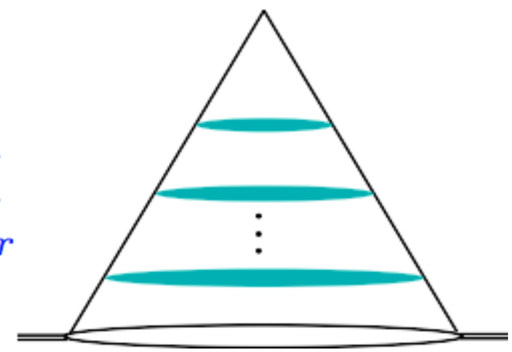


- Leading-pole - Leading $A^{1/3}$ term - less diagrams

$$\phi(x,\mu^2) = \sum_{ladders} \sum_{scattering\#}$$



$$= \sum_{ladder}$$



Same ladder structure

→ Modified DGLAP has the same functional form as DGLAP

Modified DGLAP Equation

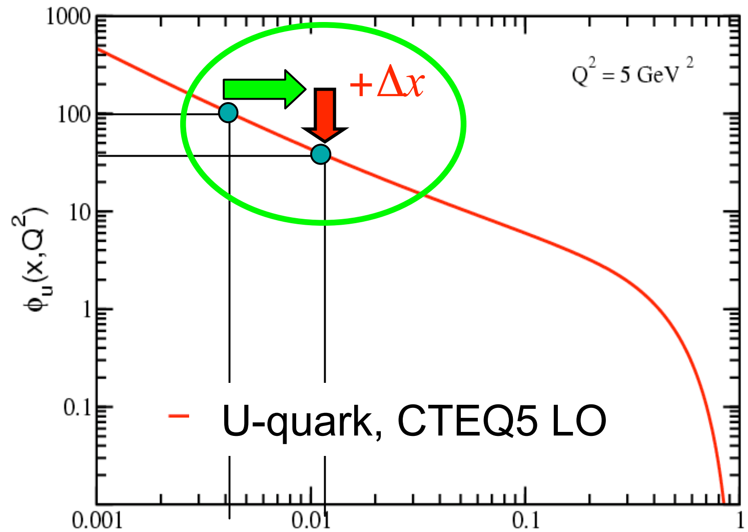
- Modified DGLAP equation (same form as normal DGLAP equation)

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} P_{qq}\hat{T}_{\Delta'} & P_{qg}\hat{T}_{\Delta} \\ \sum_{i=1}^{2n_f} P_{gq}\hat{T}_{\Delta} & P_{gg}\hat{T}_{\Delta'} \end{pmatrix} \otimes \begin{pmatrix} q(y,Q^2) \\ g(y,Q^2) \end{pmatrix}$$

$\hat{T}_{\Delta}$ Translation operator

$$\Delta = \xi^2(A^{1/3}-1)/Q^2 \quad \Delta' = \frac{C_A}{C_F}\cdot\Delta \qquad \xi^2 = \frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2} < p|\hat{F}^2(\lambda_i)|p >$$

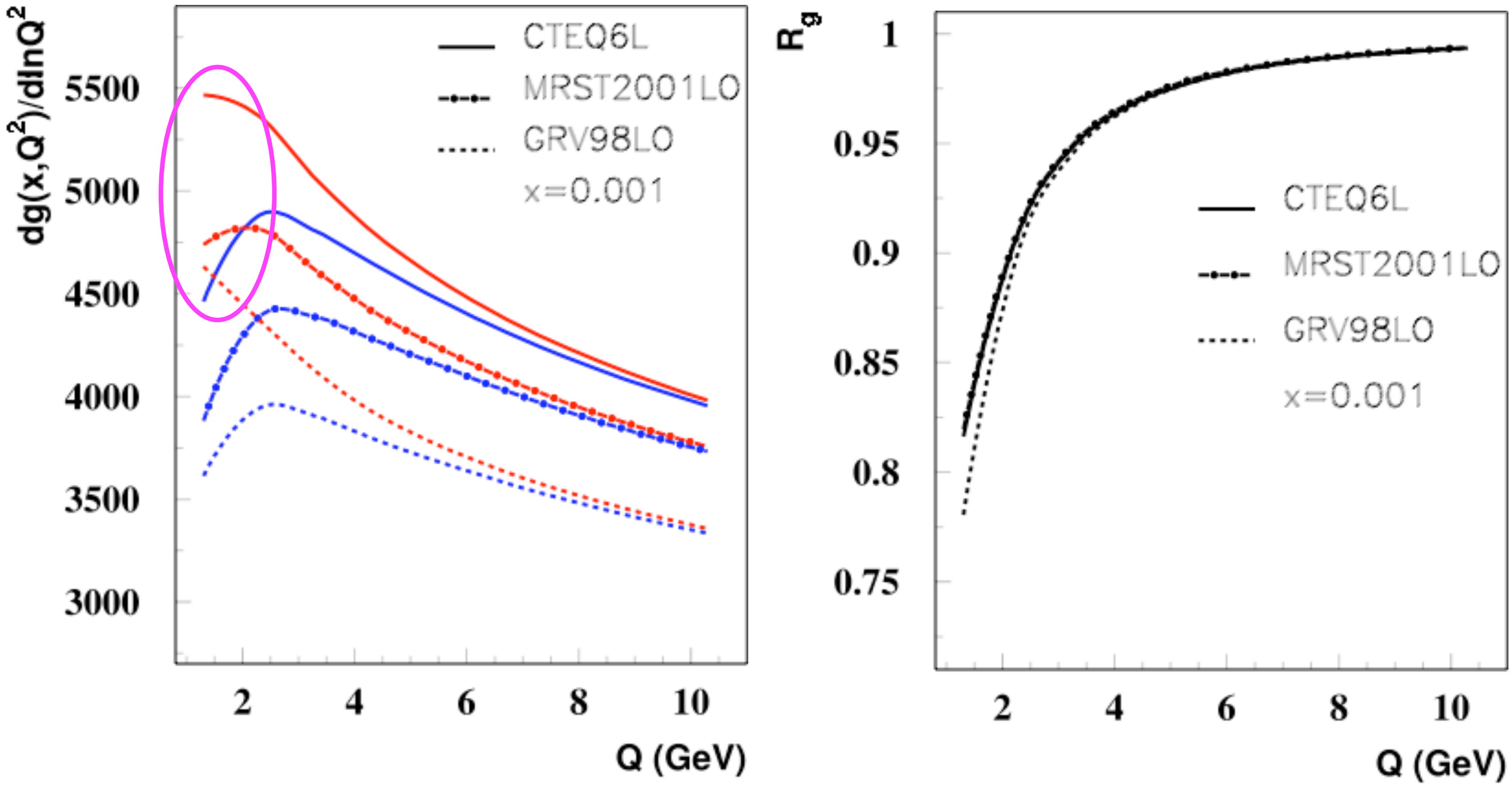
- **Resum all orders of leading pole coherent power corrections to nPDFs ⇔ Shift in momentum fraction y in the distributions**
- **Negative slope for $x<x_c$=0.1 in x space ⇔ Power corrections slow down the evolution**
- **Valence quark number conservation**



$$\frac{\partial}{\partial \ln Q^2}\int_0^1 dx q_v(x,Q^2) = \frac{\alpha_s(Q)}{2\pi}\int_0^1 dz P_{qq}(z)\int_0^1 dy q_v(y(1+\Delta'),Q^2) = 0$$

Numerical Results for Evolution Slopes

- Gluon evolution slope for A=197



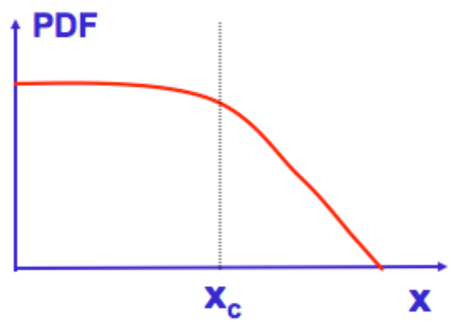
- Gluon distribution continues to evolve, but evolves much slow
- No saturation in pQCD factorization formalism

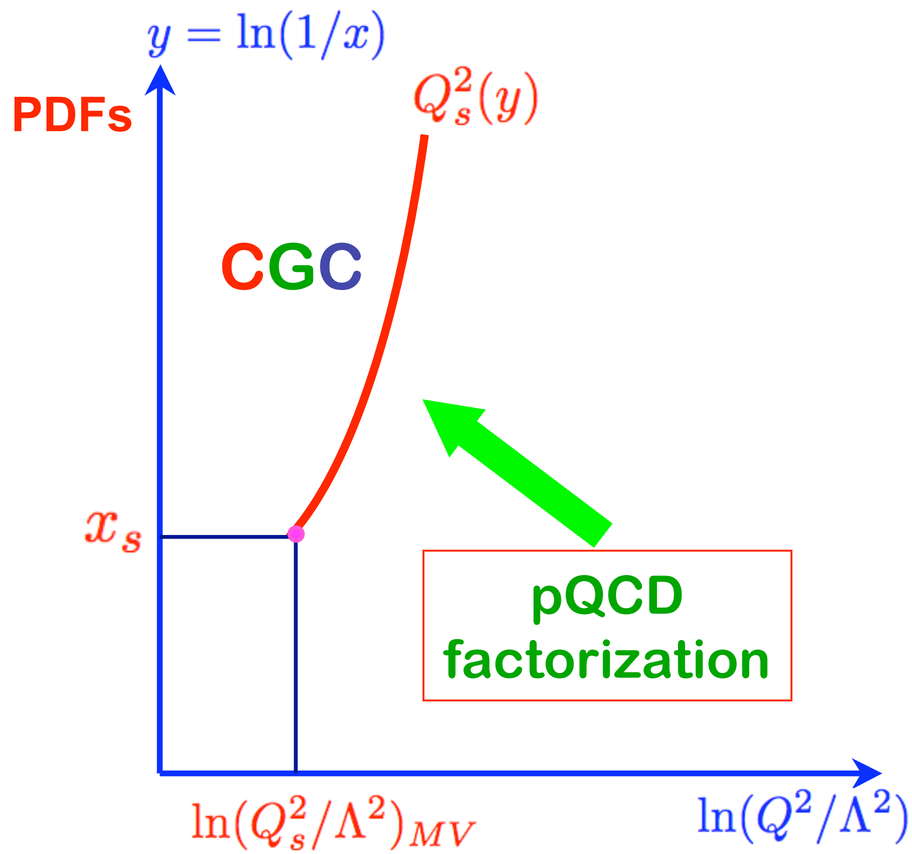
Approach Saturation from Perturbative Region

- ❑ Saturation – where collinear factorization breaks:

$$Q \sim xp \sim k_T(Q) = Q_s$$

- ❑ If x-dependence of PDF saturates, this type of power correction should have no effect.





- ❑ With the power corrections – improved predictive power of factorization approach, we should be able to get closer to the saturation region from the perturbative side

Conclusions

- **Global fitting of nPDF → Correct nPDF → can calibrate single scattering**

  Global fitting of nPDF needs to include both power corrections to hard parts as well as the evolution kernels

- **Leading-pole power corrections to DGLAP are important**

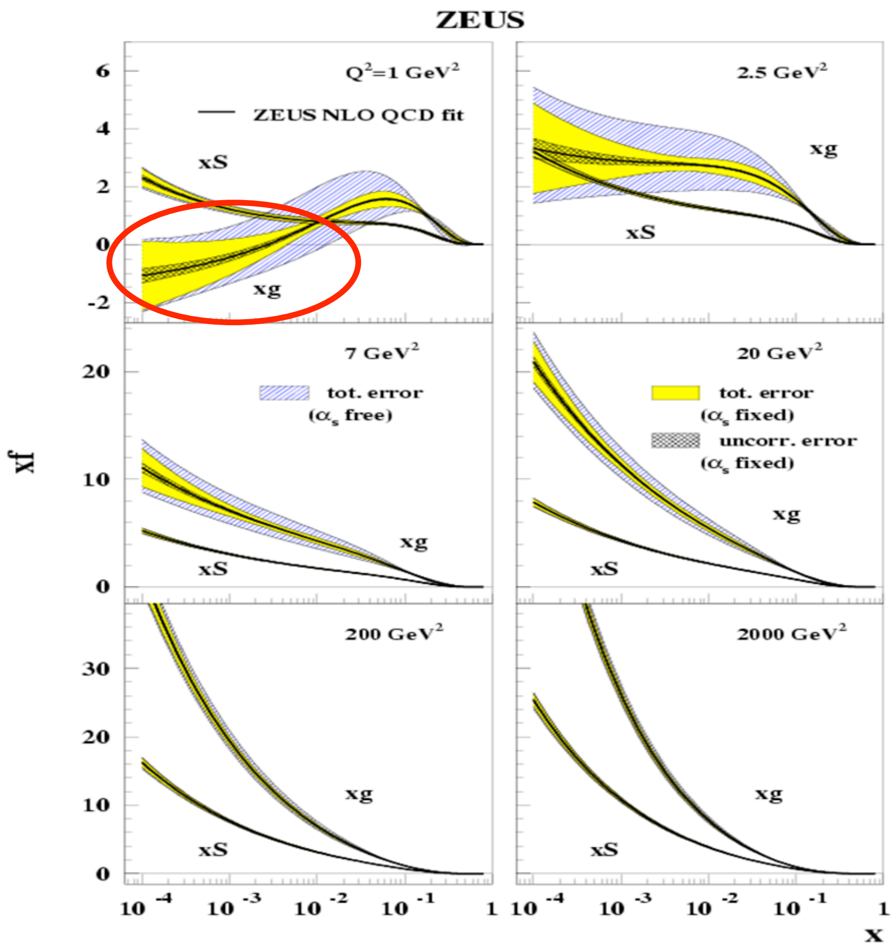
- **Leading-pole power corrections vanish for saturated nPDFs**

- **Power corrections improve the predictive power of pQCD, which enables us to get closer to the saturation region from perturbative side**

Backup

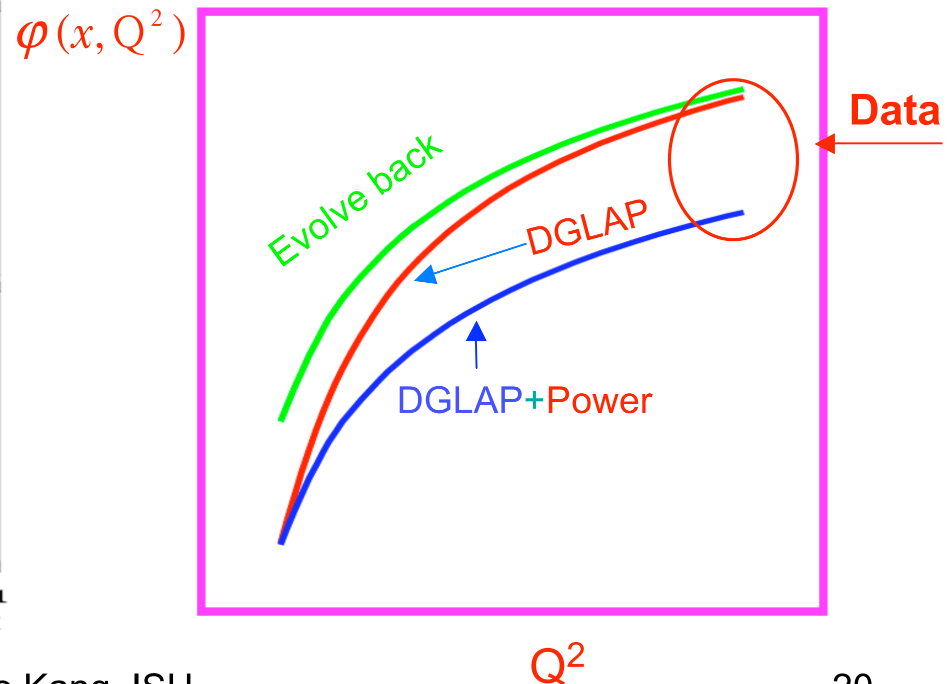
Negative Gluon Distribution at Low Q

- NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution



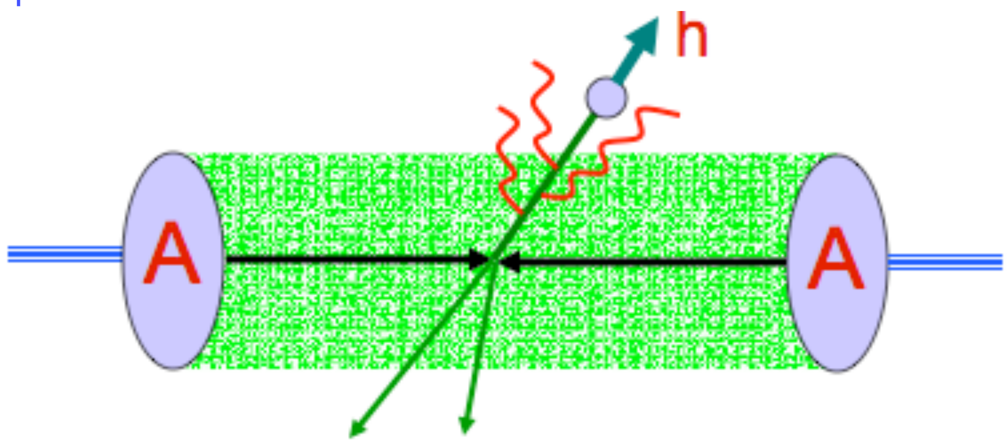
Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

No! Power corrections slow down the small-x evolution



Multiple Scattering Induced Energy Loss

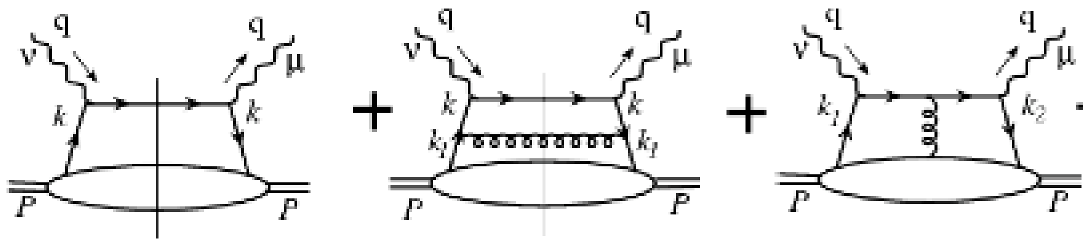
- Multiple scattering with the medium leads to energy loss
- Reduction of leading hadron momentum leads to suppression at high p_T



- Production rate is determined by single scattering
  - It's very important to calibrate single scattering

Long-lived parton states

- **Feynman diagram representation:**

$W^{\mu\nu} \propto$  $+ \dots$

- **Perturbative pinched poles:**

$$\int d^4k \; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$

- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

$$\int \frac{dx}{x} d^2k_T \; \mathrm{H}(Q,k^2=0) \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})$$

Nonperturbative matrix element

Short-distance

Model for the Correlation Functions

- **Matrix elements:**

$$\left\langle P_A \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(y^-) \left[\prod_{i=1}^{N} \int \tilde{F}^2(0) \right] \right| P_A \right\rangle$$

- **Approximation:**

Nucleus is made of a group of loosely bound nucleons

$$|P_A\rangle \propto \prod_{i=1}^{A} |p\rangle \quad \text{with } p = \frac{P_A}{A}$$

$$\left\langle P_A \left| \hat{O}_0 \prod_{i=1}^{N} \hat{O}_i \right| P_A \right\rangle \propto A \left\langle p \left| \hat{O}_0 \right| p \right\rangle \prod_{i=1}^{N} \left\langle p \left| \hat{O}_i \right| p \right\rangle$$

- **Reduce the correlation functions to one unknown – a universal matrix element**

$$\left\langle p \left| F^{+\alpha} F_\alpha^{\ +} \right| p \right\rangle$$