Transition Temperature in QCD

with physical light and strange quark masses

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- Introduction
  Thermodynamics on lines of constant physics, improved actions
- The transition temperature on $N_\tau = 4,\ 6$ lattices
  extrapolation to physical quark masses and continuum extrapolation
- Equation of state on $N_\tau = 4,\ 6$ lattices
  quark mass and cut-off dependence, crossover-region
- Conclusions

†based on:

1) N. H. Christ, M. Cheng, S. Datta, J. Van der Heide, C. Jung, O. Kaczmarek, F. Karsch, E. Laermann, R. D. Mawhinney, C. Miao, K. Petrov, P. Petreczky, C. Schmidt and T. Umeda, Phys. Rev. D74, 054507 (2006)

2) RBC-Bielefeld collaboration, in preparation

Introduction

Goal: QCD thermodynamics with realistic quark masses and controlled extrapolation to the continuum limit T_c, EoS, $\mu_q > 0$, ...

- use an improved staggered fermion action that removes $\mathcal{O}(a^2)$ errors in bulk thermodynamic quantities and reduces flavor symmetry breaking inherent to the staggered formulation

  RBC-Bielefeld choice: p4-action + 3-link smearing (p4fat3)

  MILC: Naik-action + (3,5,7)-link smearing (asqtad);

  Wuppertal: standard staggered + exponentiated 3-link smearing (stout)

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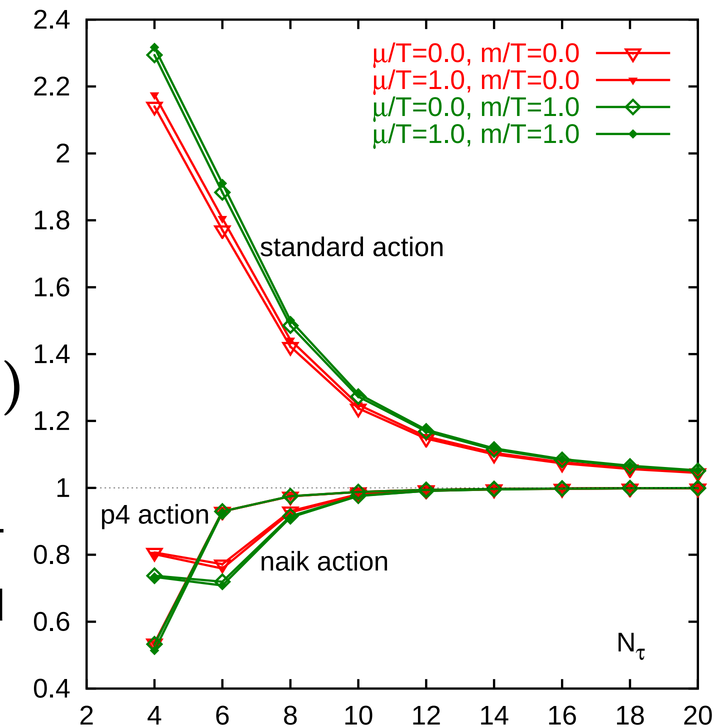
- p4-action: smooth high-T behavior for bulk thermodynamics on lattice with temporal extent N_τ

$$p(N_\tau)/T^4 = p_{SB}^{cont}/T^4 + \mathcal{O}(N_\tau^{-4})$$

- p4&Naik: small cut-off dependence of renormalized Polyakov loops (see K. Petrov) and quark number susceptibilities



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- use the newly developed RHMC algorithm to remove 'step-size errors' in the numerical simulation
- perform simulations with (3-4) different light quark masses corresponding to $150\ \mathrm{MeV} \lesssim m_\pi \lesssim 500\ \mathrm{MeV}$ at 2 different values of the lattice cut-off controlled by the spatial lattice size $N_\tau = 4,\ 6$ to perform the chiral and continuum extrapolation

previous results with p4-action:
2-flavor QCD: $N_\tau = 4,\ m_\pi \simeq 770$ MeV

Thermodynamics on QCDOC and apeNEXT

US/RBRC QCDOC

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BI – apeNEXT

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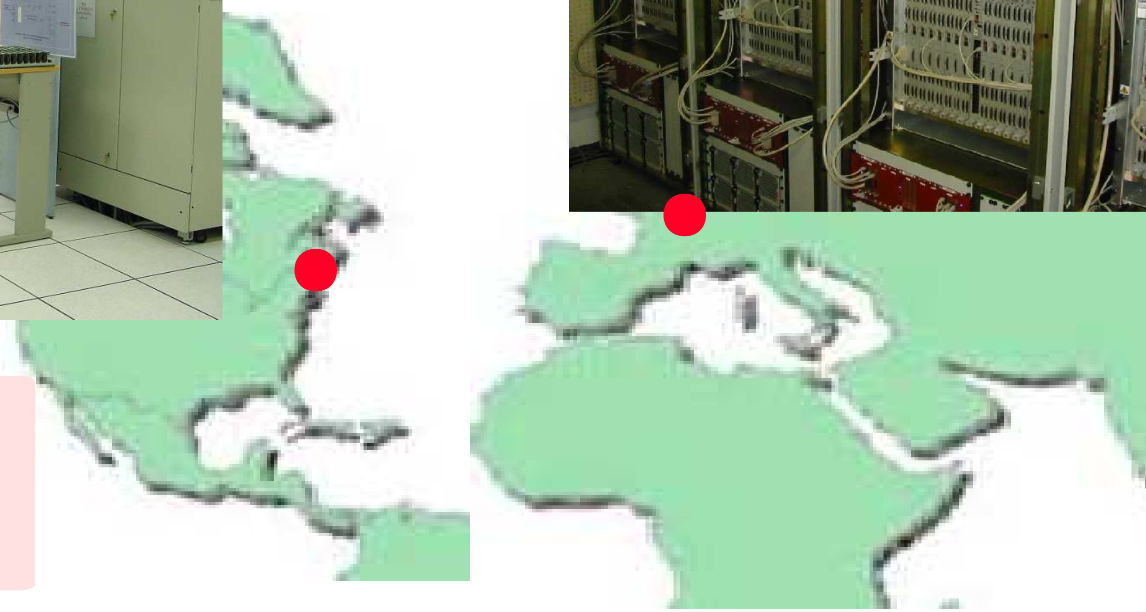
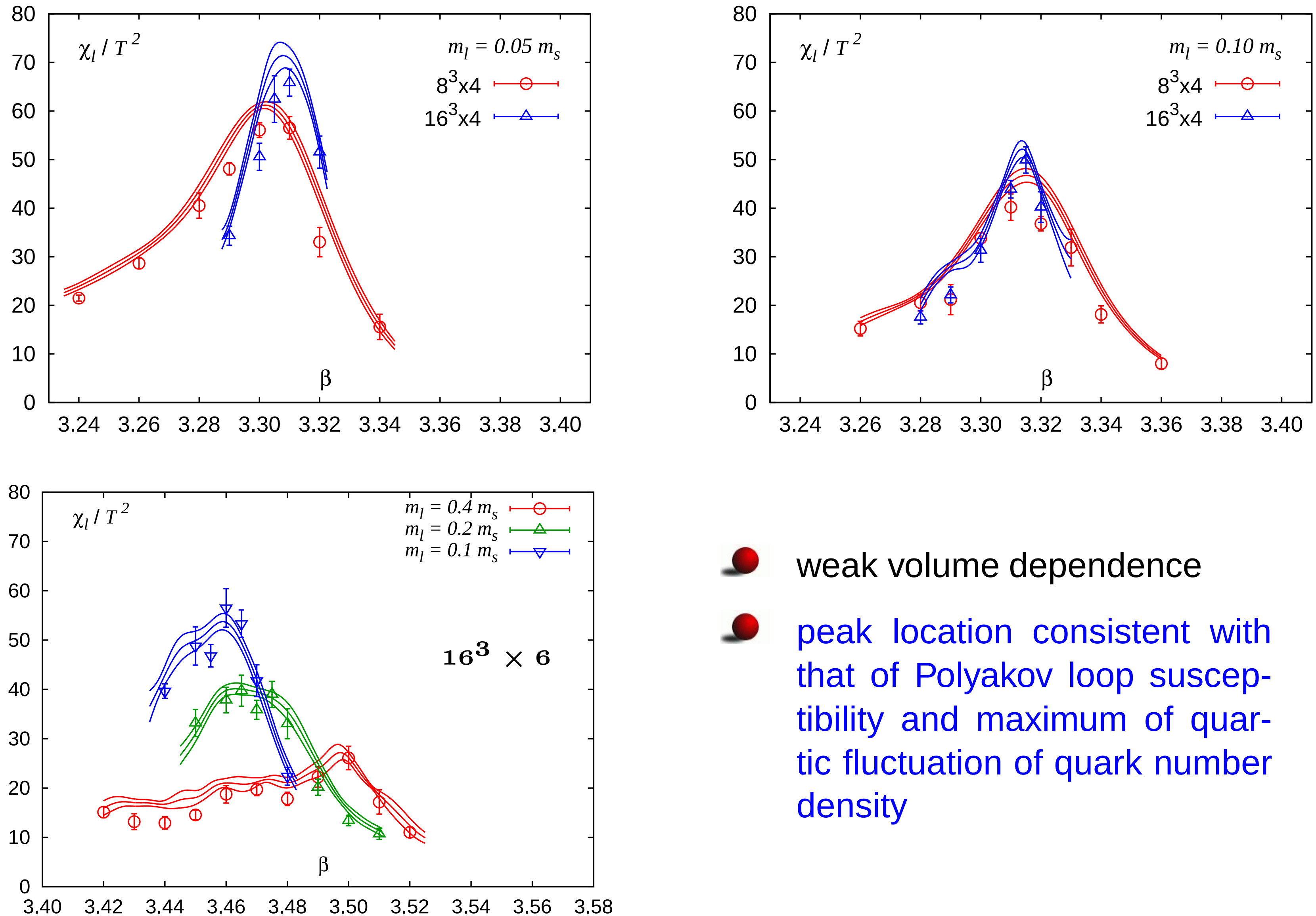


- critical temperature
- equation of state
- finite density QCD

Transition temperature

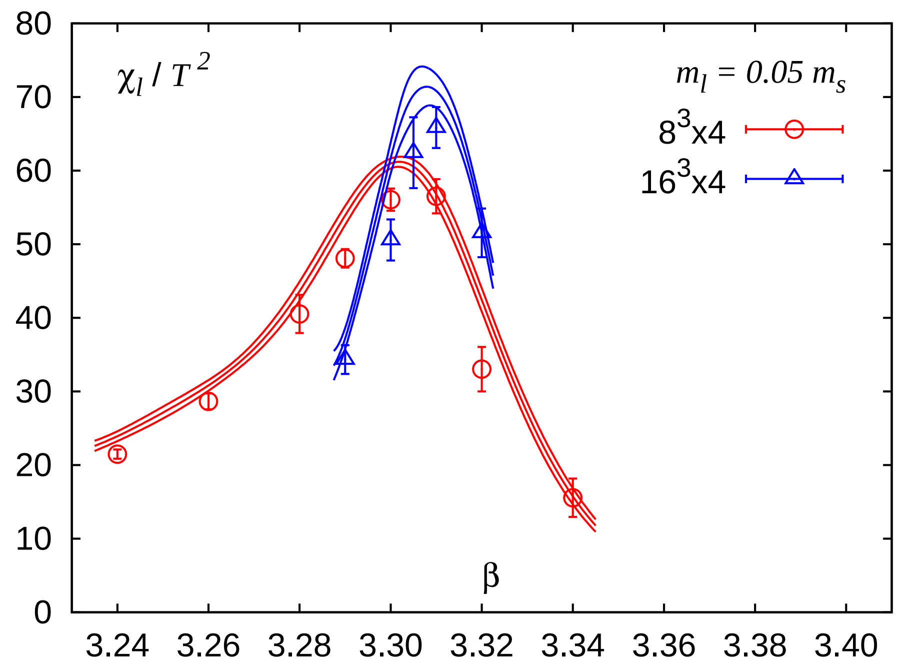
- simulations at several temperatures (β-values) in a narrow temperature range to insure the applicability of Ferrenberg-Swendsen reweighting; combines information from several independent simulations
- determine location of the transition from various susceptibilities: (disconnected part of the) light and strange quark chiral susceptibility; Polyakov loop and quark number susceptibility,... requires large statistics; several ten thousand trajectories differences in location of transition are small
- runs on different size volumes; confirm small finite volume effects; find little influence on location of transition
- overall scale setting using $T = 0$ potential parameter; find weak cut-off dependence, well understood in the pure gauge theory
- verify that T_c determination is consistent with crossover region in the energy density (ongoing)

Chiral susceptibility, $N_\tau = 4,\ 6$



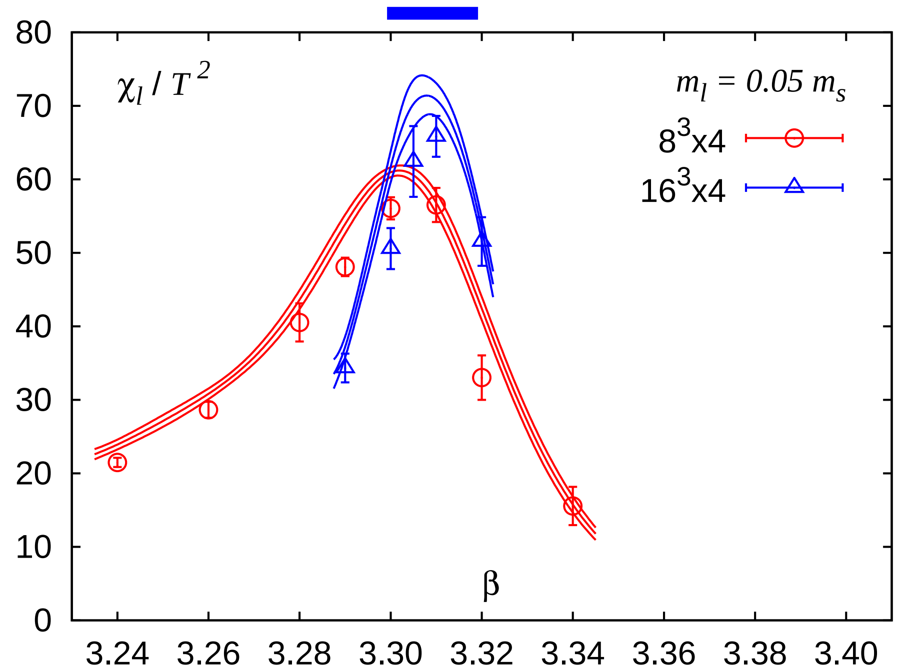
- weak volume dependence
- peak location consistent with that of Polyakov loop susceptibility and maximum of quartic fluctuation of quark number density

Chiral susceptibility, $N_\tau = 4$



Chiral susceptibility, $N_\tau = 4$

2.5% error band $\Leftrightarrow$ $\mathbf{5}$ MeV

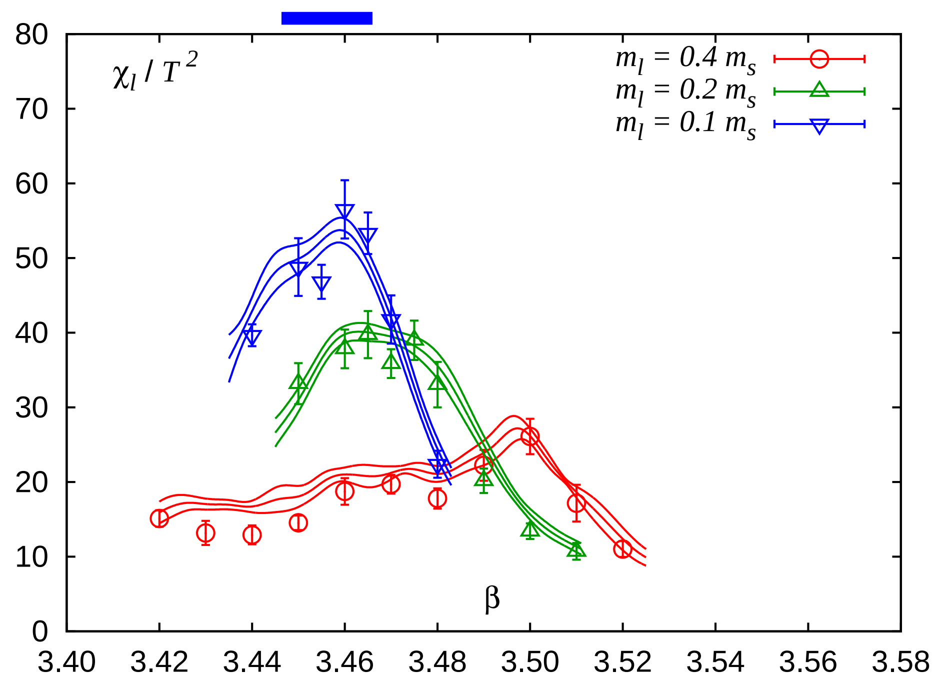


data sample for
smallest quark mass
on $\mathbf{16^3} \times \mathbf{4}$ lattice

β	no. of conf.
3.2900	38960
3.3000	40570
3.3050	32950
3.3100	42300
3.3200	39050

200,000/10 trajectories enter Ferrenberg-Swendsen sample

Chiral susceptibility, $N_\tau = 6$



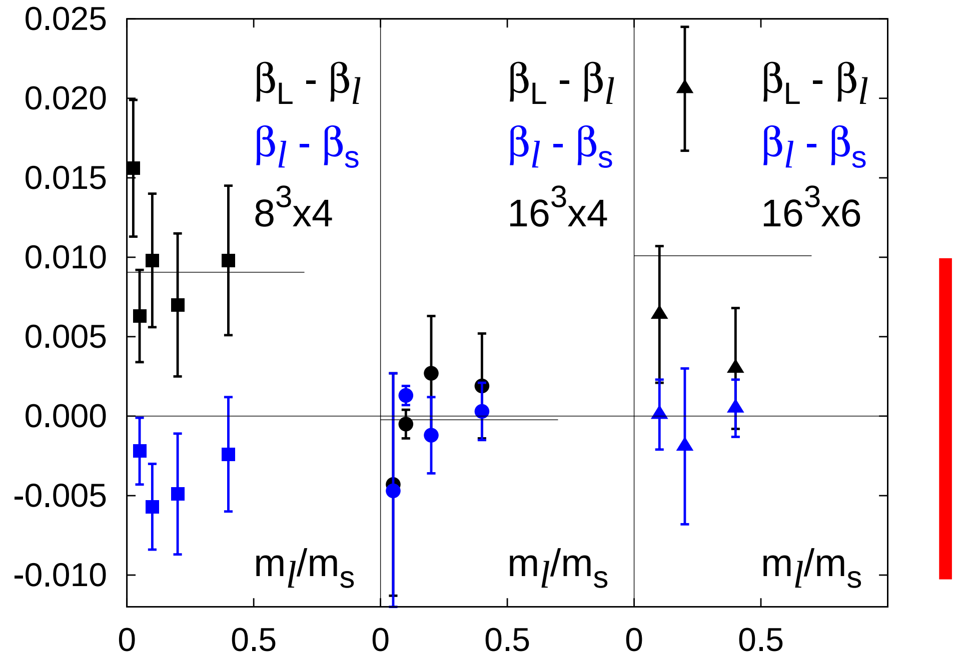
data sample for
smallest quark mass
on $16^3 \times 6$ lattice

β	no. of conf.
3.4400	25850
3.4500	38680
3.4550	40030
3.4600	60000
3.4650	40030
3.4700	30000
3.4800	30000

260,000/10 trajectories enter Ferrenberg-Swendsen sample

Ambiguities in locating the crossover point

differences of pseudo-critical couplings locating peaks in light (β_l), strange (β_s) and Polyakov loop (β_L) susceptibilities

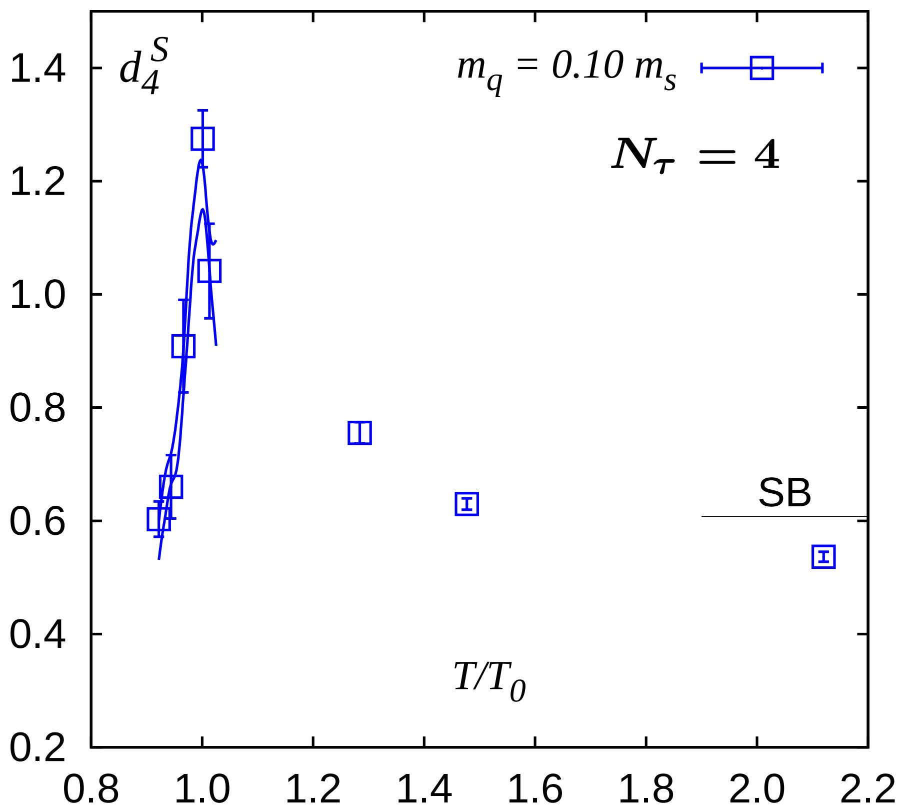


2.5% ($N_\tau = 4$) or 4% ($N_\tau = 6$)
error band $\Leftrightarrow$ 5 or 8 MeV

differences in the location of pseudo-critical couplings are taken into account as systematic error

Strangeness fluctuations

- Strangeness fluctuations provide yet another completely different obersvable to locate the crossover point
- quartic strangeness fluctuations are strongly peaked in the transition region



$$d_4^s = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial(\mu_s/T)^4}$$

T_c determined from peak in chiral susceptibility

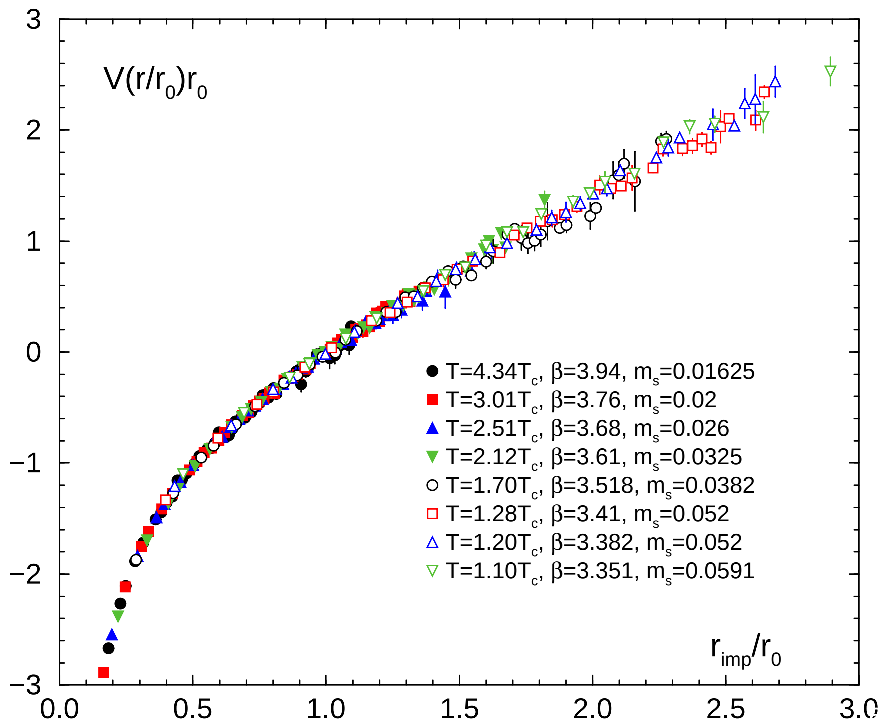
differences of pseudo-critical couplings deduced from peak in strangeness and Polyakov loop fluctuations
$\beta_c(\chi_L) - \beta_c(\chi_s) = 0.001(2)$

similar for light quark number fluctuations:
$\beta_c(\chi_L) - \beta_c(\chi_{u,d}) = 0.005(3)$

$T = 0$ scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T_c = 1/N_\tau a(\beta_c)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\Big|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 4

i.e. from the transition region on $N_\tau = 4$ lattices to that on $N_\tau = 16$ lattices !!

we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy

A. Gray et al, Phys. Rev. D72 (2005) 094507

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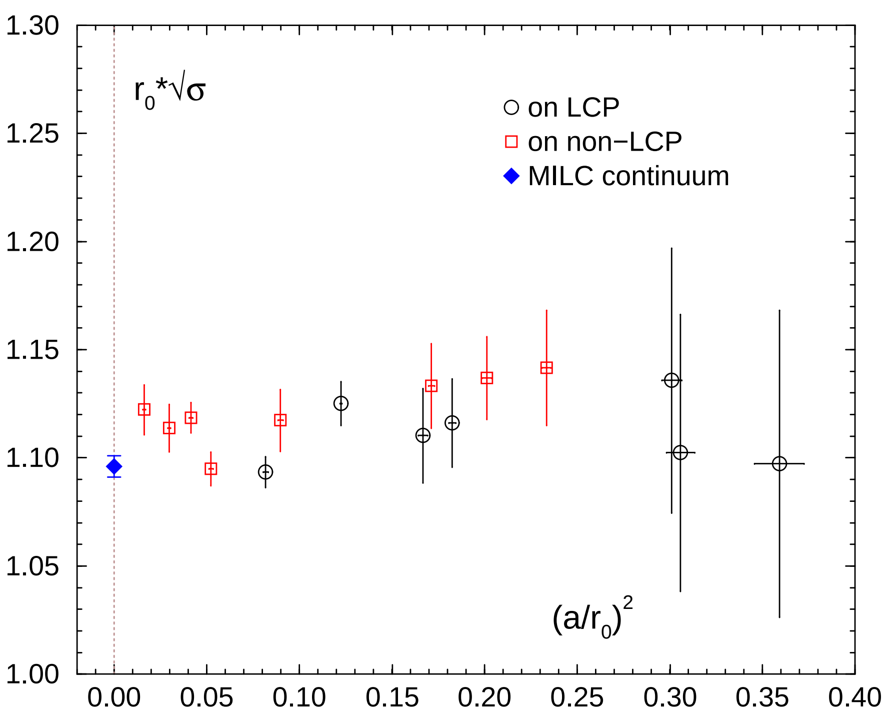

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$r_0\sqrt{\sigma}$ fluctuates by about 3% in this interval
no hint for large $\mathcal{O}(a^2)$ corrections

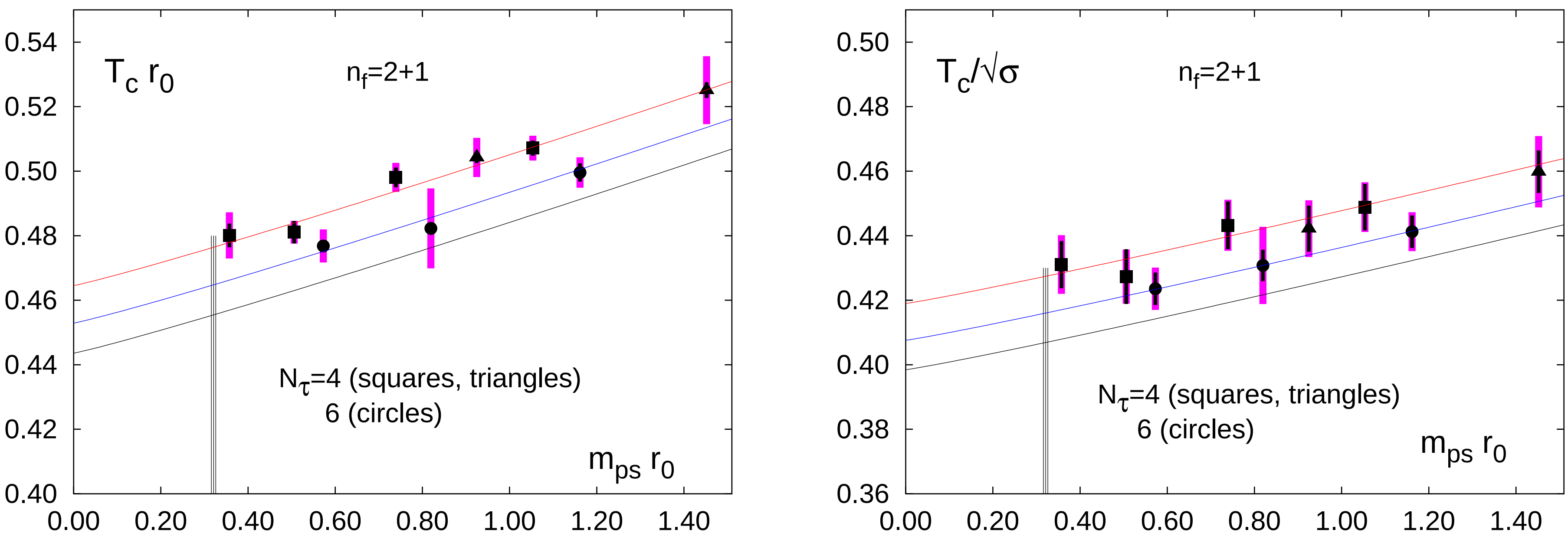
we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy
A. Gray et al, Phys. Rev. D72 (2005) 094507

$\Rightarrow \quad T_c r_0,\ T_c/\sqrt{\sigma}$

extrapolation to chiral and continuum limit

$$(r_0 T_c)_{N_\tau} = (r_0 T_c)_{cont.} + b\,(m_{PS} r_0)^d + c/N_\tau^2$$

(d=1.08 (O(4), 2nd ord.), d=2 (1st ord.))



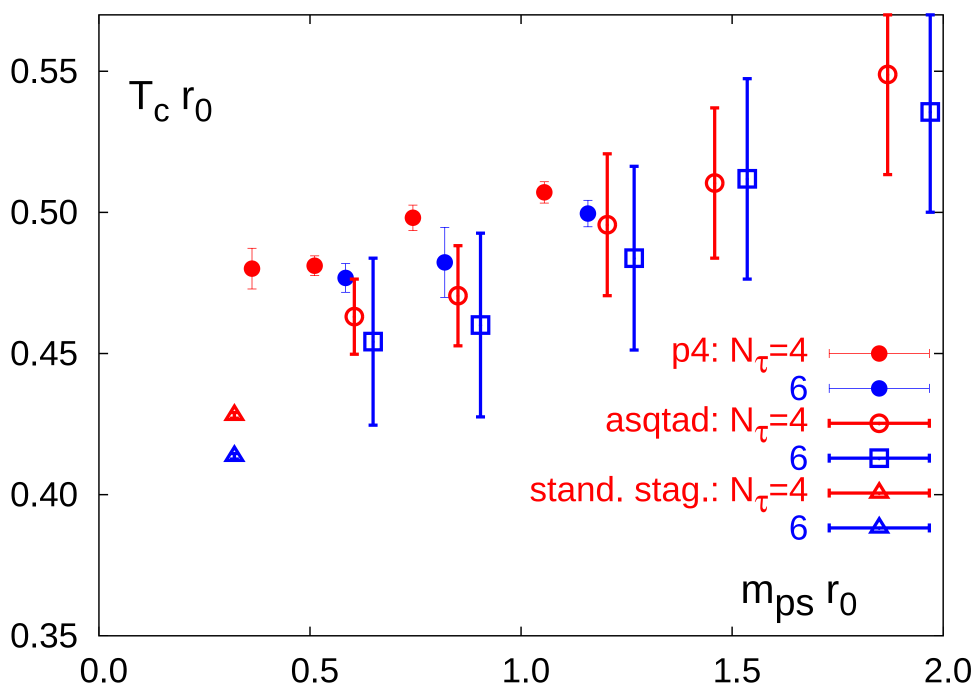
$\Rightarrow \quad r_0 T_c = 0.456(7)^{+3}_{-1} \quad , \quad T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$ at phys. point

$\Rightarrow T_c = 192(7)(4)$ MeV

(1st error: stat. error on β_c and r_0; 2nd error: N_τ^{-2} extrapolation)

Transition temperature: $N_\tau = 4, 6$

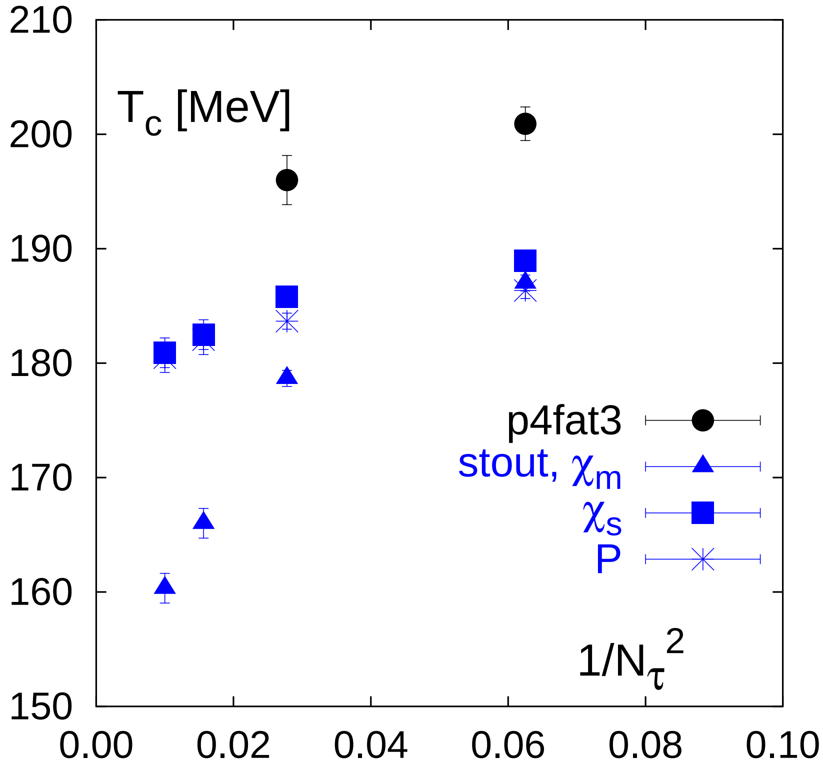
- RBC-Bielefeld (p4fat3 (p4)) vs. MILC (asqtad (Naik)) and Wuppertal (stout (stand. staggered))
- asqtad results for $N_\tau = 4$ and 6 agree with p4 results within statistical errors; (C.Bernard et al., PR D71, 034504 (2005))
- results obtained with stout action for $N_\tau = 4$ and 6 are about 15% lower; β_c from $N_\tau = 8, 10$ covers $(151 - 176)$ MeV; (Y. Aoki et al., hep-lat/0609068)



MILC data for $T_c r_1$ rescaled with $r_0/r_1 = 1.4795$

extrapolations to phys. point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- results for $N_\tau = 4, 6$ differ by 15% but show similar cut-off dependence
- stout results for different observables no longer consistent with each other for $N_\tau = 8, 10$



overall scale set with $r_0 = 0.469$ fm

Calculating the EoS on lines of constant physics (LCP)

- The pressure

$$\frac{p}{T^4}\Big|_{\beta_0}^{\beta} = N_\tau^4 \int_{\beta_0}^{\beta} d\beta' \Big[\frac{1}{N_\sigma^3 N_t} (\langle S_g\rangle_0 - \langle S_g\rangle_T)$$

$$- \left(2(\langle\bar\psi\psi\rangle_{l0} - \langle\bar\psi\psi\rangle_{lT}) + \frac{\hat m_s}{\hat m_l}(\langle\bar\psi\psi\rangle_{s0} - \langle\bar\psi\psi\rangle_{sT}) \right) \left(\frac{\partial \hat m_l}{\partial\beta'}\right)_{\hat m_s/\hat m_l}$$

$$-\hat m_l \left(\langle\bar\psi\psi\rangle_{s0} - \langle\bar\psi\psi\rangle_{sT}\right) \left(\frac{\partial \hat m_s/\hat m_l}{\partial\beta'}\right)_{\hat m_l} \Big]$$

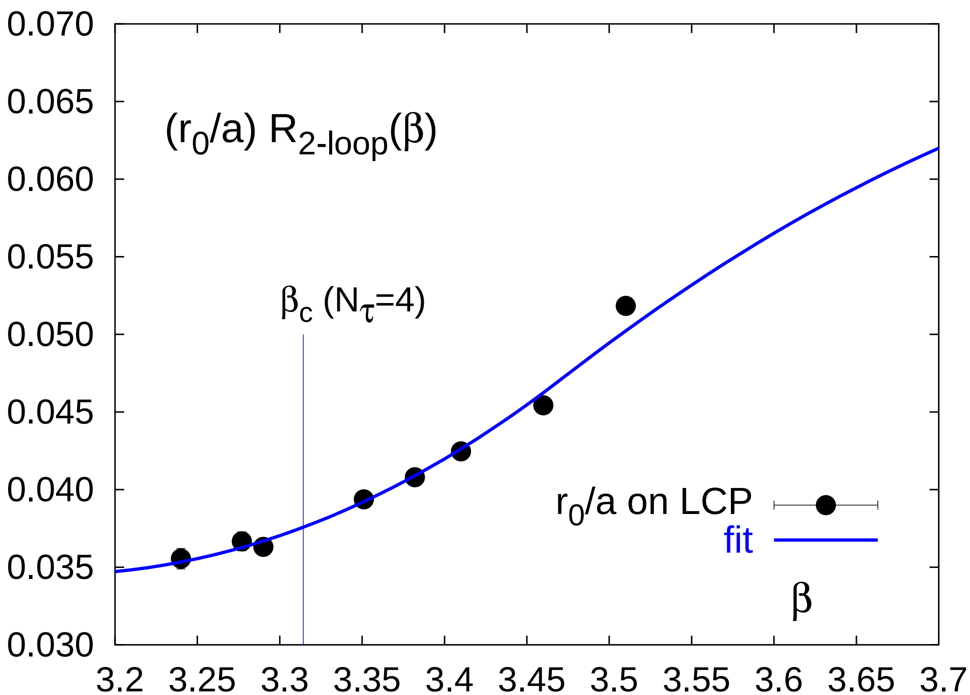
- The interaction measure for $N_f = 2+1$

$$\frac{\epsilon-3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)_{LCP} \frac{\partial p/T^4}{\partial\beta}$$

$$= \left(\frac{\epsilon-3p}{T^4}\right)_{gluon} + \left(\frac{\epsilon-3p}{T^4}\right)_{fermion} + \left(\frac{\epsilon-3p}{T^4}\right)_{\hat m_s/\hat m_l}$$

r_0/a on LCP

- a precise knowledge of $r0/a$ is mandatory to fix the temperature scale and the β-function, $(a\mathrm{d}\beta/\mathrm{d}a)_{LCP}$
- RG-inspired ansatz for $\beta > 3.47$; rational polynomial ansatz for $\beta < 3.47$
- particularly important in the transition region on coarse lattices such as $N_\tau = 4,\ 6$



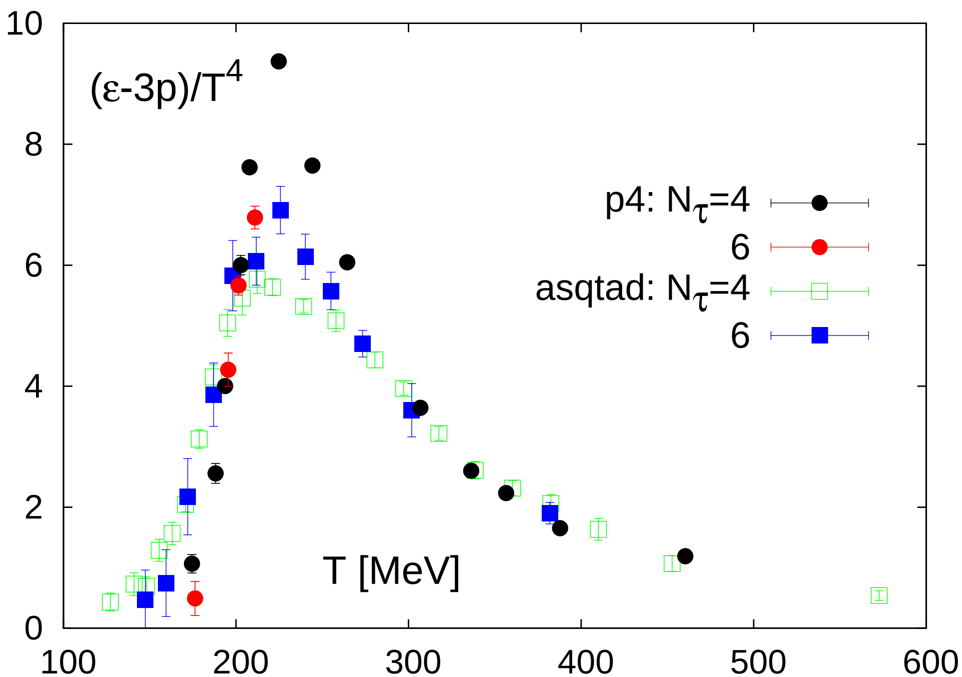
NOTE:

$r_0 T = (r_0/a)/N_\tau$ defines the temperature scale irrespective of any ambiguities in locating T_c.

RBC-Bielefeld, preliminary

$(\epsilon - 3p)/T^4$ on LCP

- Using an RG-inspired 2-loop β-function underestimates $(\epsilon - 3p)/T^4$ in the transition region and stretches the temperature interval in the low temperature regime artifically, i.e. makes the transition region look broader than it is.



RBC-Bielefeld, preliminary

differences in the transition region partly arise from differences in the β-functions used in the crossover region

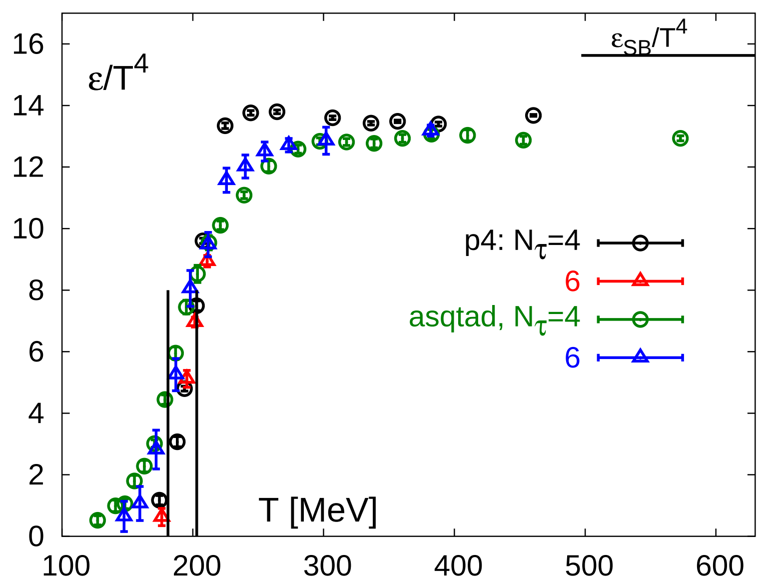
- overall good agreement

Note:
T-scale is not dependent on T_c determination

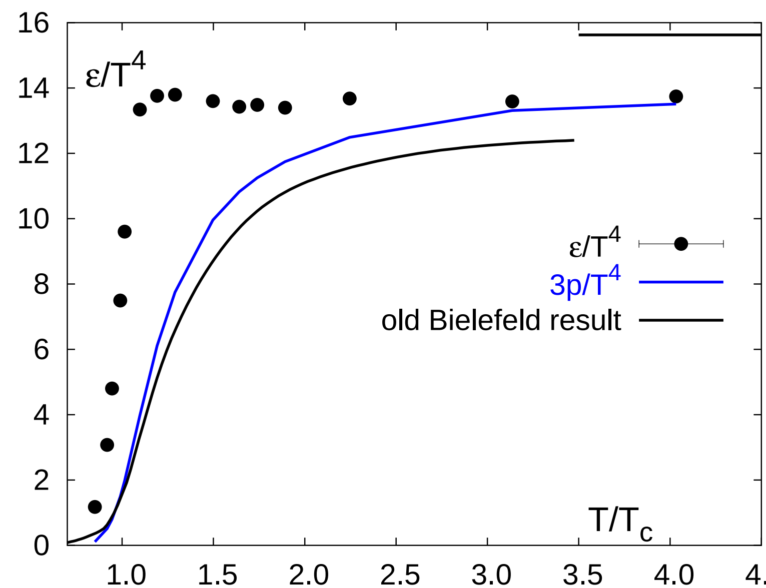
asqtad data:
C. Bernard et al., hep-lat/0610017 (Lattice'06)

Energy density and pressure $N_\tau = 4,\ 6$

- RBC-Bielefeld vs. MILC: the RBC-Bi energy density on $N_\tau = 4$ lattices rises more steeply;
  direct consequence of the use of a non-perturbative β-function directly deduced from calculated r_0/a values
- overall good agreement for $N_\tau = 4,\ 6$,
  Note: T-scale does not depend on T_c determination!!



band marks $T = (192 \pm 11)$ MeV
RBC-Bielefeld, preliminary



pressure increased slightly with smaller quark mass

Conclusions

- We studied the thermodynamics of QCD with a realistic quark mass spectrum and performed extrapolations to the physical point
- using an improved staggered fermion action (p4fat3) we find from an analysis at 2 different lattice spacings and for several values of the quark mass consistent crossover temperatures from several observables
- At the physical point of (2+1)-flavor QCD we obtain the continuum extrapolated value:

$$T_c = 192(7)(4)\text{MeV}$$

- the analysis of ϵ/T^4 shows a rapid crossover at this temperature;
- this analysis of bulk thermodynamics is consistent with an analysis based on the asqtad action performed by MILC
details in the transition region still have to be resolved
$\Rightarrow$ hotQCD-collaboration (see poster by R. Soltz)