Unified description of deconfined QCD equation of state

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Quark Matter, Shanghai, Nov 15, 2006

Mao dun

矛盾

Mao dun

máo

“spear”

矛盾

dùn

“shield”

a) “contradiction”
b) “powerful”
c) “spear and shield”

?

Mao dun

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“spear”

矛盾

dùn

“shield”

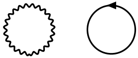
a) “contradiction” ✓
b) ~~“powerful”~~
c) ~~“spear and shield”~~

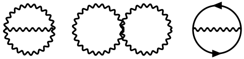
?

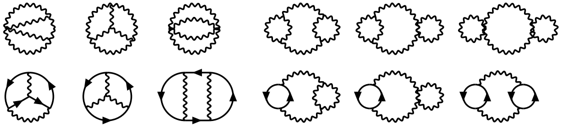
The problem

Thermodynamic potential:

$$\Omega = \frac{-1}{\beta V} \ln \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\mathcal{S}_{\mathrm{QCD}}}$$

g^0 

g^2 

g^4 

which of these diagrams cause trouble?

(+ ghost diagrams)

massless QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{\psi} \not{D} \psi$$

$$\mathcal{S}_{\mathrm{QCD}} = \int_0^\beta d\tau \int d^3x \, (\mathcal{L} + \mu \bar{\psi} \gamma_0 \psi)$$

pressure

$$p_{\mathrm{QCD}}(T, \mu, g) = -\frac{\Omega}{V}$$

dimensional reduction:

$$p_{\mathrm{QCD}} = p_E + p_M + p_G$$

$$= .. + \# g^4 \mu^4 \ln \frac{T}{\bar{\mu}_{\overline{\mathrm{MS}}}}$$

diverges at $T \to 0$

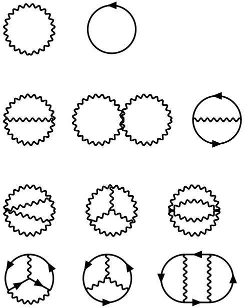
?

The problem

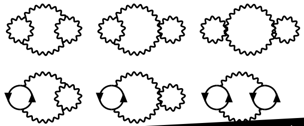
Thermodynamic potential:

$$\Omega = \frac{-1}{\beta V} \ln \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\mathcal{S}_{\mathrm{QCD}}}$$

g^0, g^2, g^4



2GR (two-gluon-reducible)



which o… The usual suspects …cause trouble?

(+ ghost diagrams)

massless QCD Lagrangian

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diverges at $T \to 0$

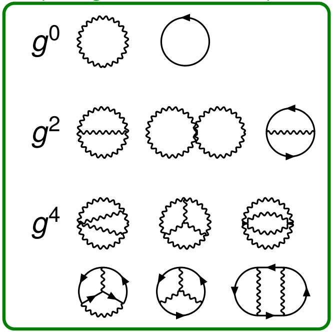
?

The idea: Resum 2GR separately

$$p_{\mathrm{QCD}} = p_{\mathrm{2GI}} + p_{\mathrm{2GR}}^{\mathrm{resummed}} \text{ up to } \mathcal{O}(g^4)$$

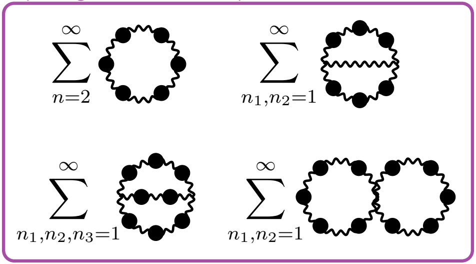
for all T and μ.

2GI
(two-gluon-irreducible)

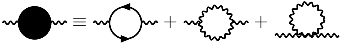


(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)

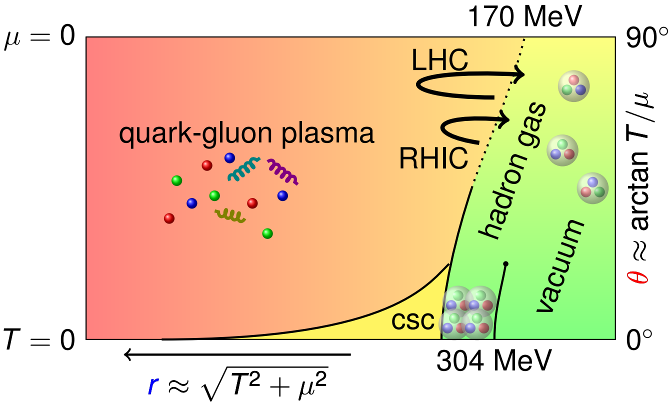


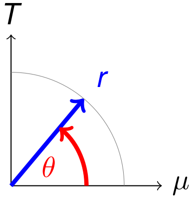
with gluon self-energy



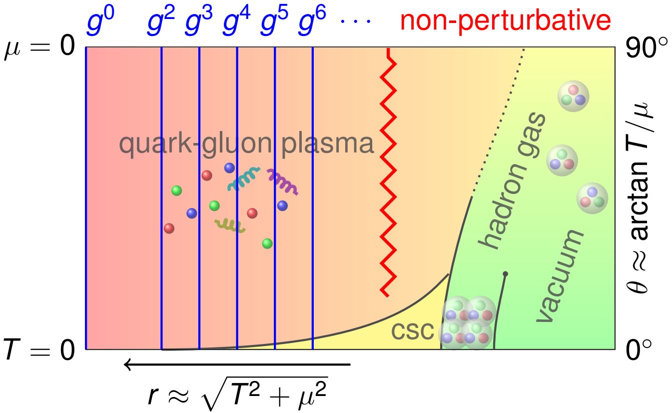
Outline

QCD phase diagram (in polar coordinates)





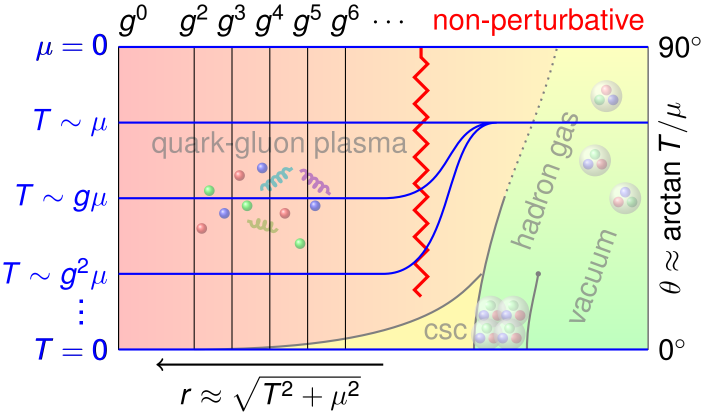
QCD phase diagram (in polar coordinates)



$$\beta(g) \equiv \bar{\mu}\frac{dg}{d\bar{\mu}} = -\frac{2g^3}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) + \mathcal{O}(g^5)$$

Gross, Wilczek, Politzer (1973)

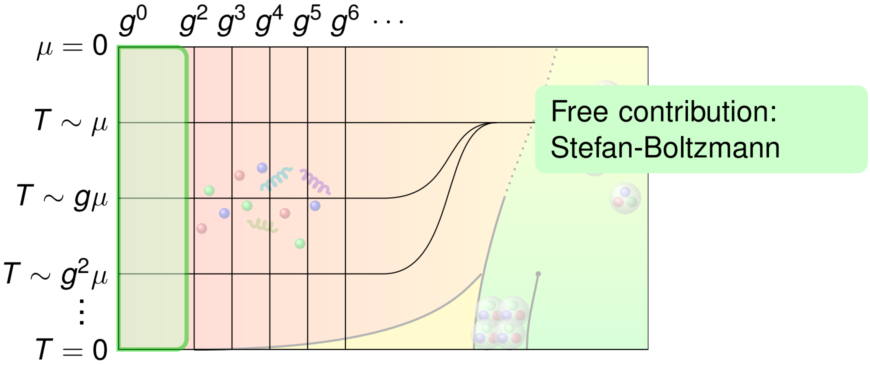
QCD phase diagram (in polar coordinates)



color superconducting gap $\phi \sim T_c \sim \mu g^{-5} \exp \frac{-3\pi^2}{g\sqrt{2}}$

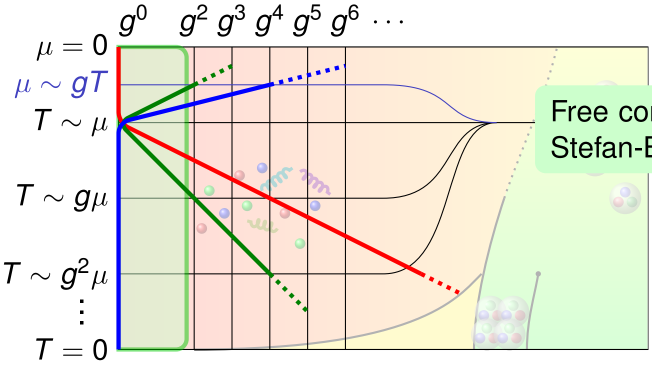
Son (1999)

Previous approaches for the pressure $p(T, \mu, g)$



$$p_{\mathrm{SB}}(T,\mu) = \left(\frac{\pi^2 N_g}{45} + \frac{7\pi^2 N_c N_f}{180}\right) T^4 + \frac{N_c N_f}{6} T^2 \mu^2 + \frac{N_c N_f}{12\pi^2} \mu^4 + \mathcal{O}(g^2)$$

Previous approaches for the pressure $p(T, \mu, g)$



Free contribution: Stefan-Boltzmann

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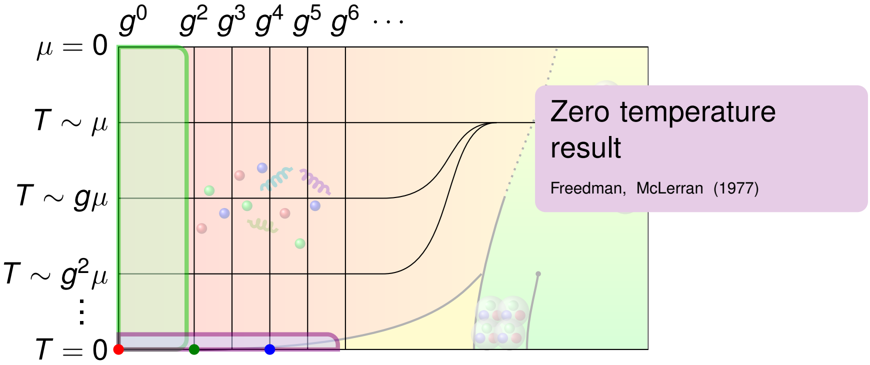
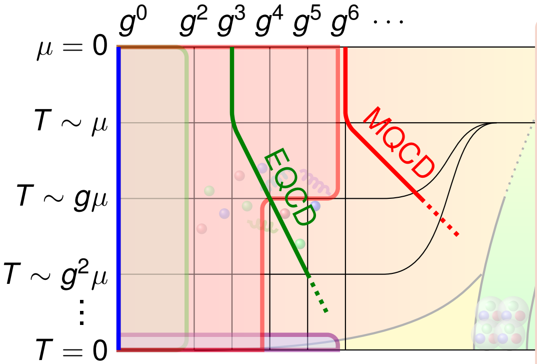


$$p = \frac{\mu^4}{2\pi^2}\left(1 - \frac{g^2}{2\pi^2} - \frac{g^4}{16\pi^4}\left(3.3385 + 4\ln g - \frac{29}{3}\ln\frac{\mu}{\bar{\mu}_{\overline{\rm MS}}}\right) + \mathcal{O}(g^6 \ln g)\right)$$

Previous approaches for the pressure $p(T, \mu, g)$



Perturbation theory

g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
$g^4 \ln g$: Toimela (1983)
g^4: Arnold, Zhai (1994)
g^5: Zhai, Kastening (1995)

Dimensional reduction

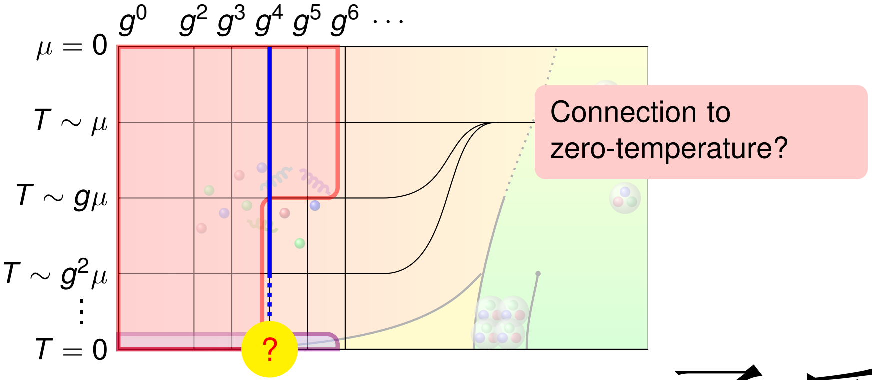
g^5: Braaten, Nieto (1996)
$g^6 \ln g$: Kajantie, Laine Rummukainen, Schröder (2002)
$\mu > 0$ for g^5, $g^6 \ln g$: Vuorinen (2003)
g^6: Di Renzo, Laine, Miccio, Schröder, Torrero (2006) Laine, Schröder, Vuorinen,... (200?)

Separation of scales:

$$T \gg m_E \sim gT \gg m_{\text{mag}} \sim g^2 T$$

$$p = p_E(T, g) + p_M(m_E^2, g_E, \lambda_E, ...) + p_G(g_M, ..)$$

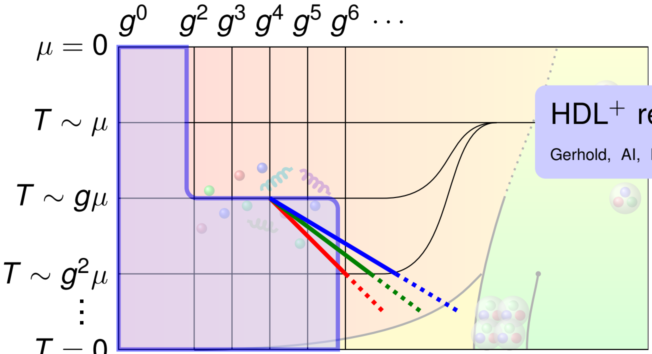
Previous approaches for the pressure $p(T, \mu, g)$



$$p_{\mathrm{DR}} = \ldots + \# g^4 \mu^4 \ln \frac{T}{\bar{\mu}_{\overline{\mathrm{MS}}}} + \ldots$$

矛盾

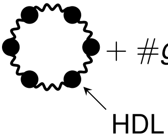
Previous approaches for the pressure $p(T,\mu,g)$



$$
\begin{aligned}
\delta p &\equiv p - p_{SB} \\
&- (p - p_{SB})|_{T=0}
\end{aligned}
$$

HDL$^+$ resummation

Gerhold, AI, Rebhan (2004)

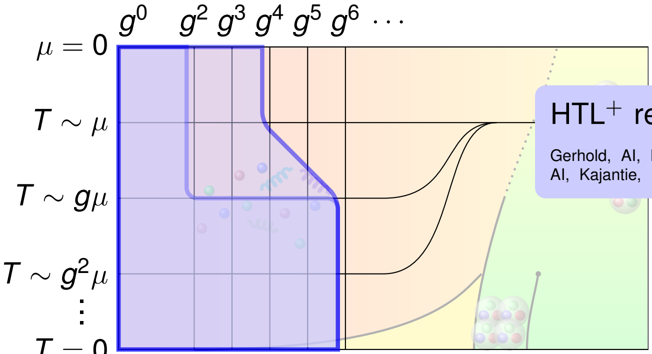


$+ \#g^2 T^2 \mu^2$

$$
\delta p = g^2 T^2 \mu^2 \left\{ \#\ln\frac{T}{g\mu} + \# + \#\left(\frac{T}{g\mu}\right)^{2/3} + \#\left(\frac{T}{g\mu}\right)^{4/3} + \dots \right\}
$$

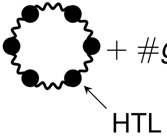
Non-Fermi-liquid behavior in entropy & specific heat

Previous approaches for the pressure $p(T, \mu, g)$



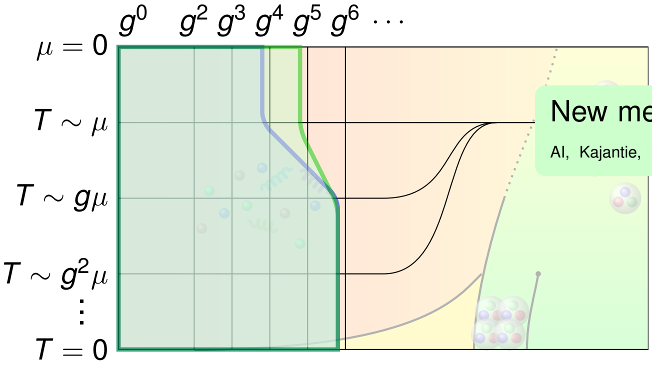
HTL$^+$ resummation

Gerhold, AI, Rebhan (2004)
AI, Kajantie, Rebhan, Vuorinen (2006)



$+ \# g^2 T^2 \mu^2 + \# g^2 T^4$

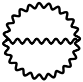
Previous approaches for the pressure $p(T, \mu, g)$

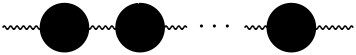


New method: p_{IV}

AI, Kajantie, Rebhan, Vuorinen (2006)

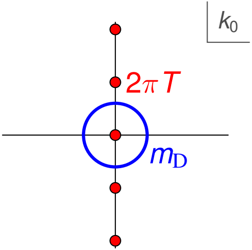
Problem of dimensional reduction





$$\rightarrow \frac{1}{-k_0^2 + \mathbf{k}^2 + \Pi}$$

$2\pi T \gtrsim m_D$

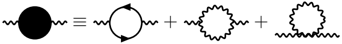


Matsubara sum:

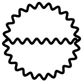
$ik_0 = 0, \pm 2\pi T, \pm 4\pi T, \ldots$

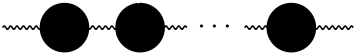
integrate out hard modes, effective 3-D theory for $k_0 = 0$

HTL: $g^2\mu^2 \quad g^2T^2 \quad g^2T^2$

$m_D^2 =$ 

Problem of dimensional reduction





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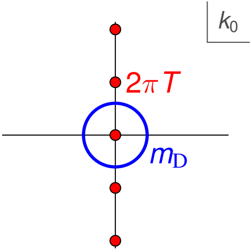
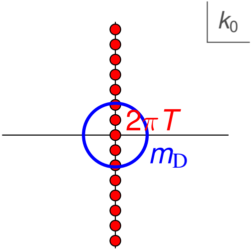


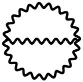
integrate out hard modes, effective 3-D theory for $k_0 = 0$

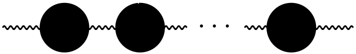
$2\pi T \lesssim m_\mathrm{D} \sim g\mu/\pi$



DR fails, have to use 4D integration $\sum_{\omega_n} \rightarrow \int dk_0 n(k_0)$

Problem of dimensional reduction





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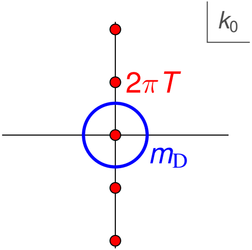

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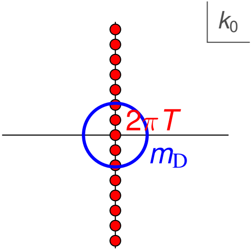

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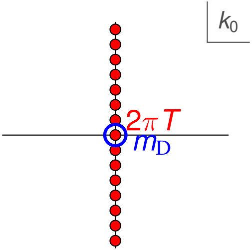
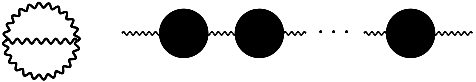


perturbatively can always choose smaller g

Problem of dimensional reduction



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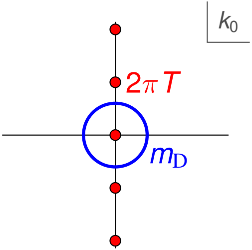

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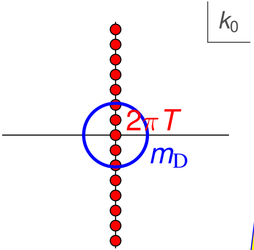

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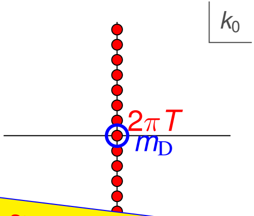
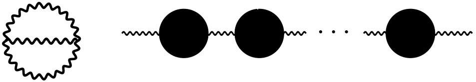


For any given T/μ dimensional reduction is valid, if g is chosen small enough.

smaller g

Problem of dimensional reduction



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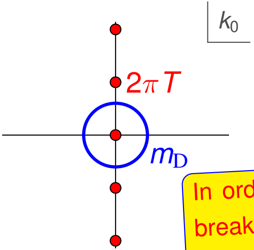
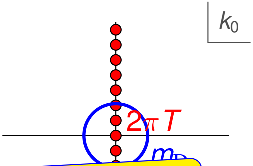


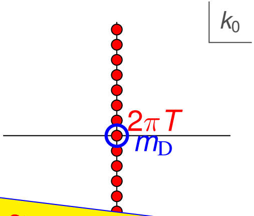
integrate out har
modes, effective
theory for $k_0 = 0$

$2\pi T \lesssim m_D \sim g\mu/\pi$



In order to observe the breakdown of DR, we have to study a parametrically smaller temperature region $T \lesssim g\mu$.

$2\pi T \gtrsim m_D \sim g\mu/\pi$



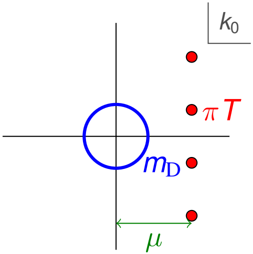
For any given T/μ dimensional reduction is valid, if g is chosen small enough.

smaller g

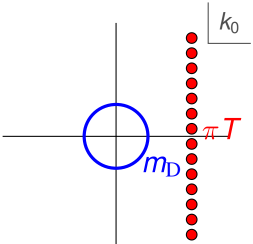
Problem of dimensional reduction

for fermions:

$\pi T \gtrsim m_{\mathrm{D}}$



$\pi T \lesssim m_{\mathrm{D}} \sim g\mu/\pi$



There is no problem with fermions, as μ regulates the IR region.

Matsubara sum for fermions:
$ik_0 = \pm\pi T + i\mu,$
$\pm 3\pi T + i\mu, \ldots$

only need to resum gluon lines:

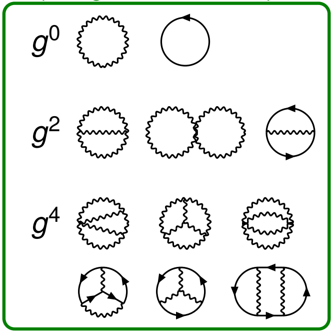
$\Rightarrow$ 2GR

Our solution

$$p_{\text{QCD}} = p_{\text{2GI}} + p_{\text{2GR}}^{\text{resummed}} \text{ up to } \mathcal{O}(g^4)$$

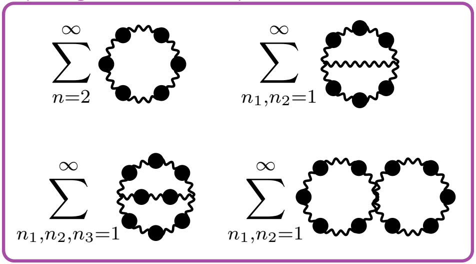
for all T and μ.

2GI
(two-gluon-irreducible)

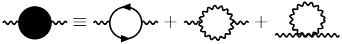


(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)



with gluon self-energy

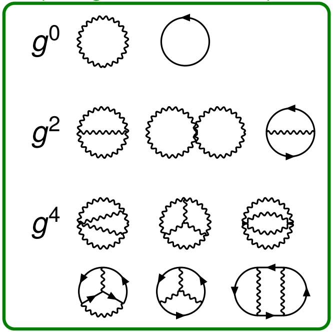


Our solution

$$p_{\rm QCD} = p_{\rm 2GI} + p_{\rm 2GR}^{\rm resummed} \text{ up to } \mathcal{O}(g^4)$$

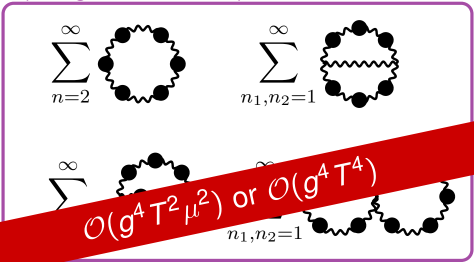
for all T and μ.

2GI
(two-gluon-irreducible)



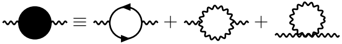
(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)



$\mathcal{O}(g^4 T^2 \mu^2)$ or $\mathcal{O}(g^4 T^4)$

with gluon self-energy

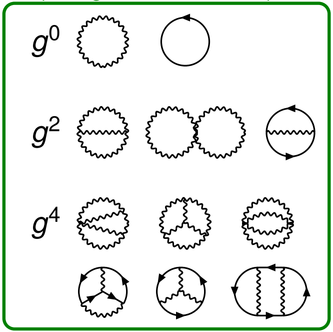


Our solution

$$p_{\mathrm{QCD}} = p_{\mathrm{2GI}} + p_{\mathrm{2GR}}^{\mathrm{resummed}} \text{ up to } \mathcal{O}(g^4)$$

for all T and μ.

2GI
(two-gluon-irreducible)



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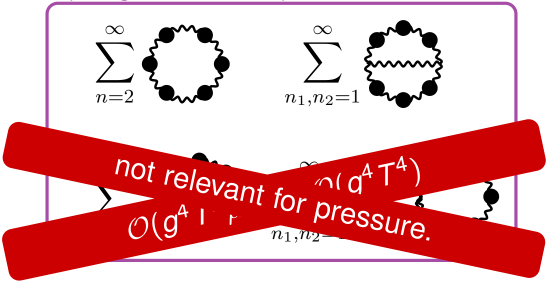

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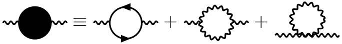

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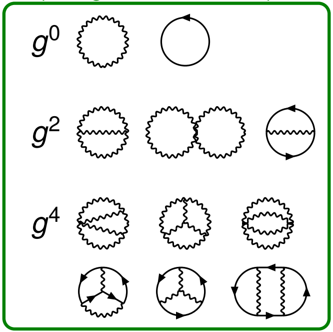

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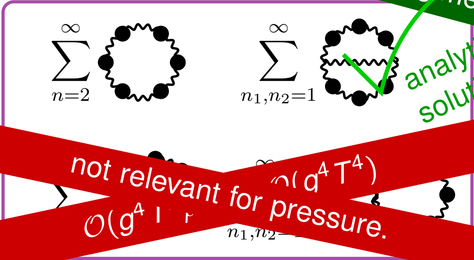


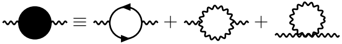
only contributes in DR regime.

analytic solution

not relevant for pressure.

$\mathcal{O}(g^4 T^4)$

with gluon self-energy



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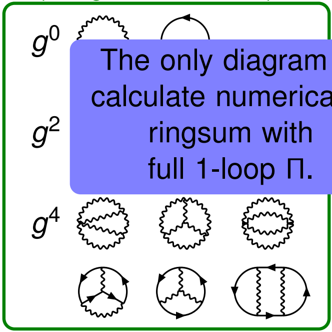
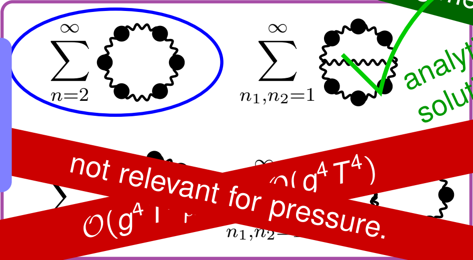


The only diagram to calculate numerically: ringsum with full 1-loop Π.

(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)



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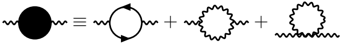
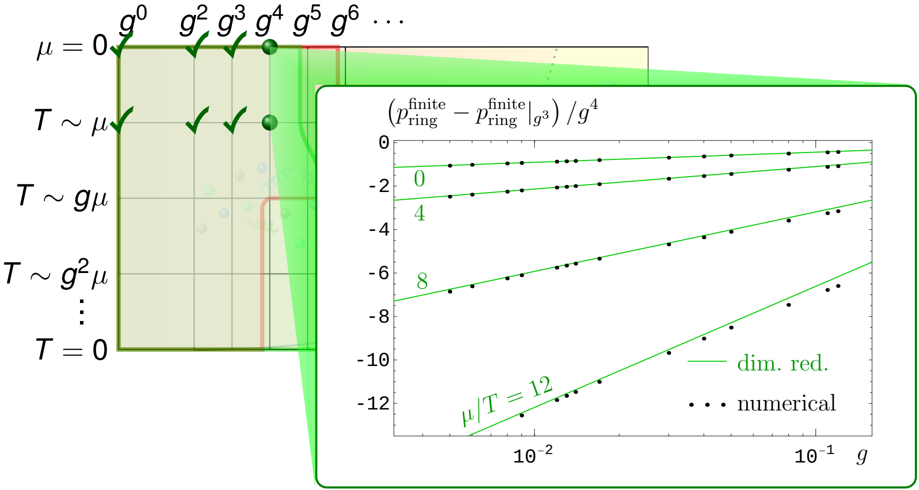
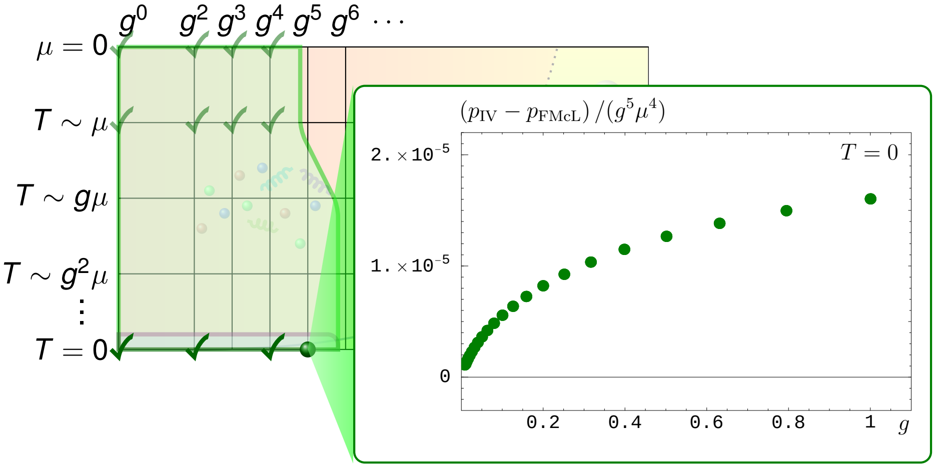


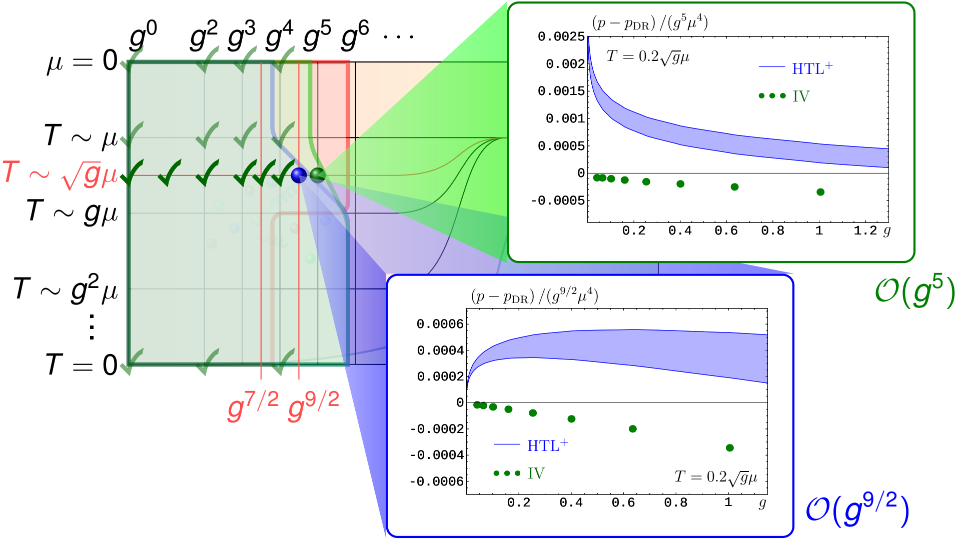
Dimensional reduction OK



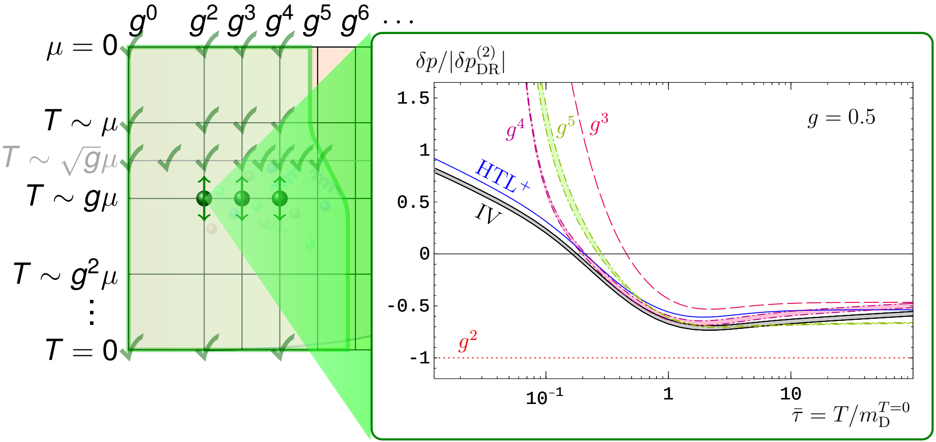
Zero temperature OK



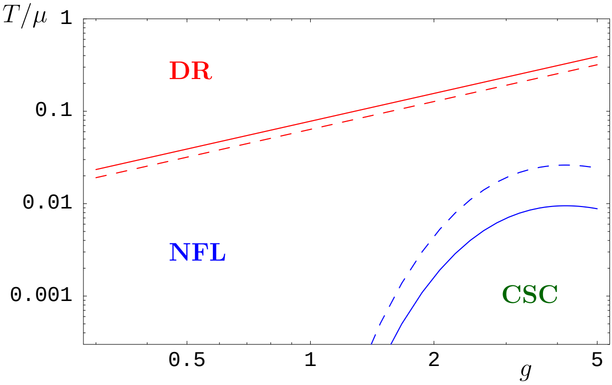
Dimensional reduction at $T \sim \sqrt{g}\mu$ OK



Interaction pressure δp



Dividing line between regimes



non-Fermi liquid (NFL): cooling of neutron stars

Schäfer, Schwenzer PRD70 (2004)

non-Fermi liquid

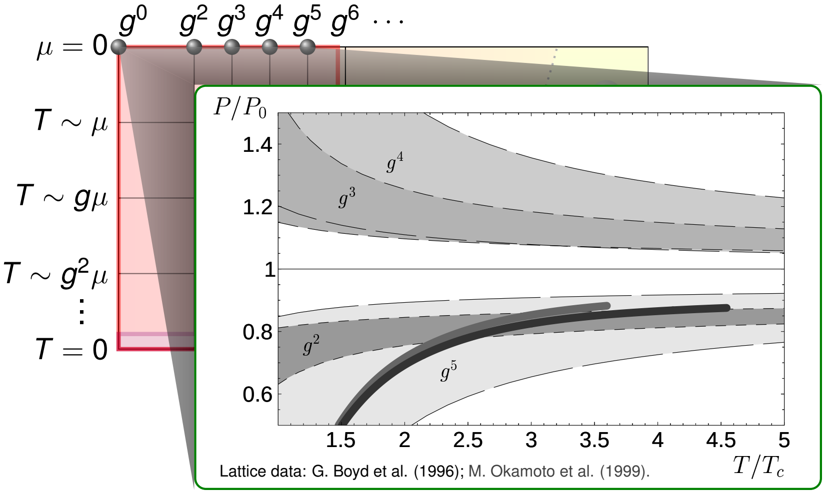
$$\frac{T^{\rm NFL}}{\mu} \approx 0.2\sqrt{\frac{N_f}{2}\frac{g}{\pi}}$$

color superconducting gap

$$\frac{T_c^{\rm SC}}{\mu} \simeq 2\frac{e^{\gamma}}{\pi} e^{-\frac{\pi^2+4}{8}} (4\pi)^4 \left(\frac{2}{N_f}\right)^{5/2} g^{-5} e^{\frac{-3\pi^2}{\sqrt{2}g}}$$

Brown, Liu, Ren (2000); Schmitt, Wang, Rischke (2002)

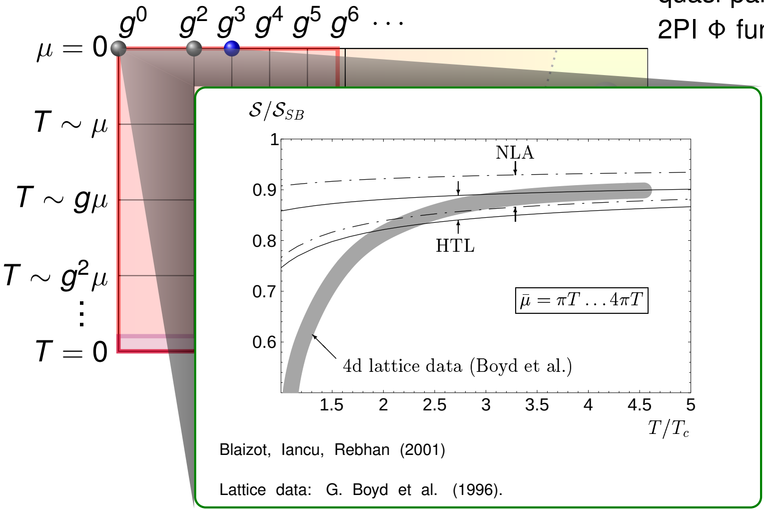
Strict perturbation theory



pure glue QCD

Φ-derivable approximation

quasi-particles from 2PI Φ functional



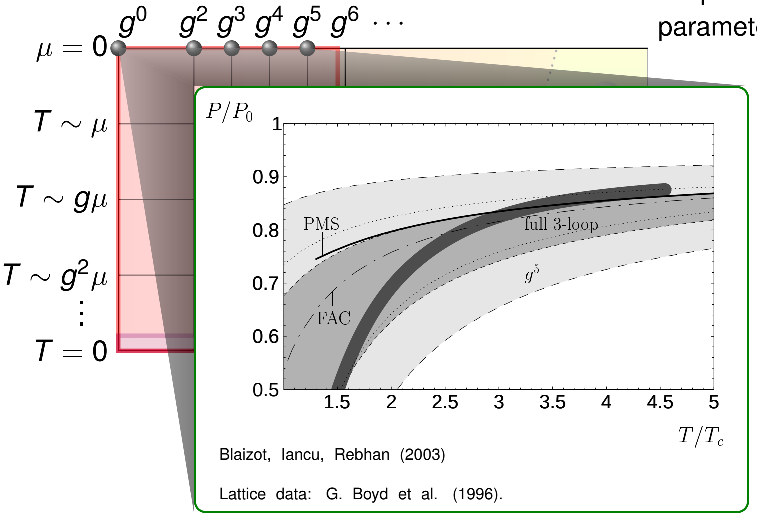
Blaizot, Iancu, Rebhan (2001)

Lattice data: G. Boyd et al. (1996).

pure glue QCD

Improve convergence through effective theory

keep effective theory parameters unexpanded



pure glue QCD

Non-perturbative renormalization group

Effective field theories

Λ

$\sim T$ QCD

$\Lambda_1 \sim \sqrt{g} T$

$\sim gT$ EQCD

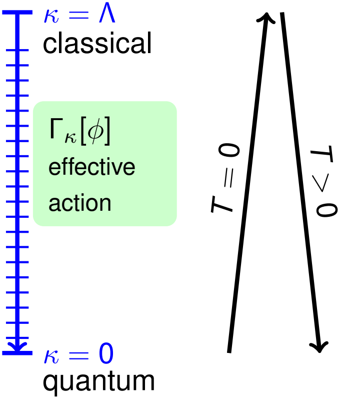
$\Lambda_1 \sim g^{3/2} T$

$\sim g^2 T$ MQCD

Braaten, Nieto (1996)

NP-RG

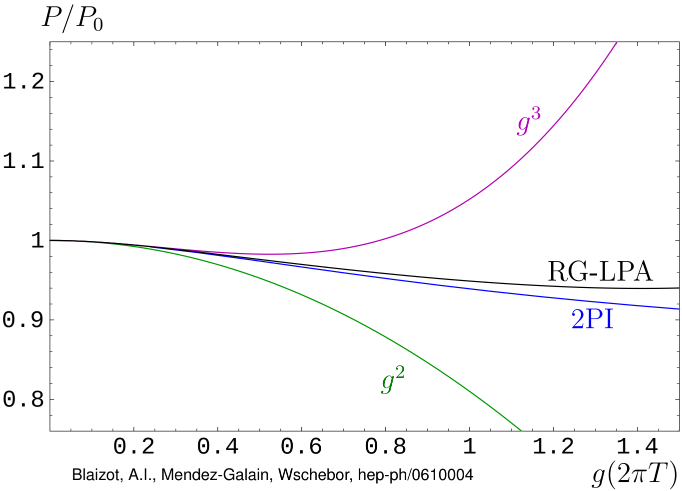
flow



Tetradis, Wetterich (1993)

Non-perturbative renormalization group

Pressure in scalar theory $P = -V_{\kappa=0}(\rho = 0)$



Blaizot, A.I., Mendez-Galain, Wschebor, hep-ph/0610004

Summary

- Dimensional reduction works when $T \gtrsim 0.2 m_D$;
  Non-Fermi-liquid effects important before CSC sets in.
- New approach: Treat 2GI analytically and 2GR numerically.
  Full perturbative understanding for all T and μ up to $O(g^4)$.
- Use 2PI resummation or NP-RG for stronger couplings.

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jī	鸡蛋	dàn
"hen", "chicken"	?	"egg"

a) "paradox"
b) "happiness"
c) "hen's egg"

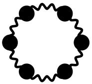
For Further Reading I

Please find all references in

A. Ipp, K. Kajantie, A. Rebhan, A. Vuorinen
The pressure of deconfined QCD for all temperatures and quark chemical potentials
Phys.Rev.D74:045016,2006, hep-ph/0604060

J.-P. Blaizot, A. Ipp, R. Mendez-Galain, N. Wschebor
Perturbation theory and non-perturbative renormalization flow in scalar field theory at finite temperature
hep-ph/0610004

Technical details



$$\sum_{n=2}^{\infty} \quad p_{\rm ring}^{\rm finite} = -\frac{d_A}{2} \oint\!\!\!\!\!\!\sum_P \Big\{ \ln\big[1+\Pi_L(P)/P^2\big] - \Pi_L(P)/P^2 + C_A^2 g^4 T^4/(72(P^2+m^2)^2) + 2\left(\ln\big[1+\Pi_T(P)/P^2\big] - \Pi_T(P)/P^2 + C_A^2 g^4 T^4/(72(P^2+m^2)^2)\right)\Big\}$$

Split off vacuum & regulate UV

$$p_{\rm ring} = p_{\rm ring}^{\rm finite} + p_{\rm ring}^{\rm IR} + p_{\rm ring}^{\rm UV}$$

$$p_{\rm ring}^{\rm IR} \equiv \frac{d_A C_A^2 g^4 T^4}{48} \oint\!\!\!\!\!\!\sum_P \left\{ \frac{1}{(P^2+m^2)^2} - \frac{1}{P^4} \right\}$$

$$p_{\rm ring}^{\rm UV} = \frac{1}{4}(d-1) d_A (\Pi_{\rm UV})^2 \oint\!\!\!\!\!\!\sum_P \frac{1}{P^4}$$

Magnetic mass: $m_{\rm mag} = c_f \frac{3}{32} g^2 C_A T$

$\Pi_T(P) \to \Pi_T(P) + m_{\rm mag}^2$

Avoid singularities