

Heavy Quarks and Quarkonia at finite temperature

New results from the Lattice

Konstantin Petrov

Niels Bohr Institute, Copenhagen, Denmark

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Overview

- Thermodynamics of the static $Q\bar{Q}$ pair in full QCD
with RIKEN-BNL-Columbia-Bielefeld Collaboration
hep-lat/0610041 [to appear in PoS]

- Quarkonia correlators and spectral functions at zero and finite
temperature
with A.Jacovác(Budapest), P.Petreczky (BNL), A.Velytsky (UCLA)
hep-lat/0611017 [to appear in PRD]



Method Description

- Large Scale Simulation on multi-teraflop QCDOC and APE
- P4 + Fat3 fermion action
- Previous studies - Asqtad, $N_t = 4, 6$, 3F and Bielefeld (3F, $N_t = 4$)
- Full QCD, *two* light and *one* strange dynamical quarks
- Physical strange quark mass
- Almost physical quark masses [0.1 strange quark mass]
- Scale setting via zero temperature heavy quark potential
- see talk by Frithjof Karsch

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Free Energies

Determined from Polyakov Loop correlations

$$F_1(|x|, T)/T = -\ln\langle\text{Tr}L(\vec{x})L^\dagger(0)\rangle[\text{McLerran}]$$

- Not really gauge invariant
- However very physical in Coloumb Gauge [Philipsen]
- Gauge invariant at infinite separation of Q and Qbar
- Renormalization required [Karsch et al]
- Assume that at very very short distances medium does not play any role
- Therefore everything is like at $T = 0$
- Do not forget to check

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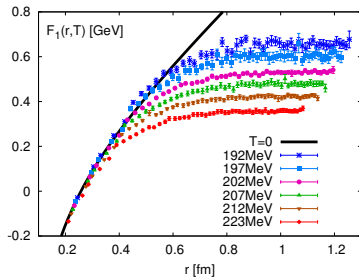
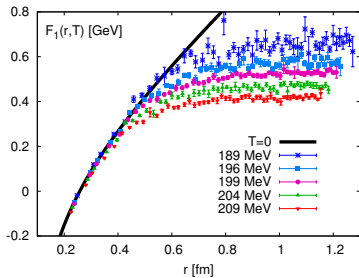
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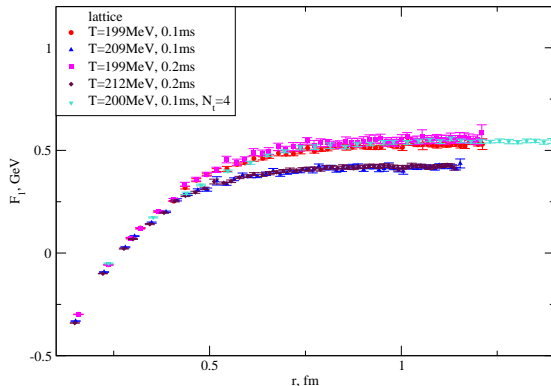
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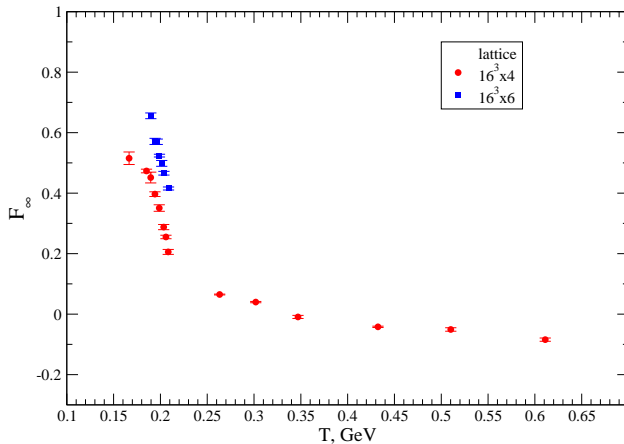
- No medium effects up to $0.3 fm$
- Strong effects at $r > 0.4 fm$

Checks and Balances: Scaling with m_q and N_t

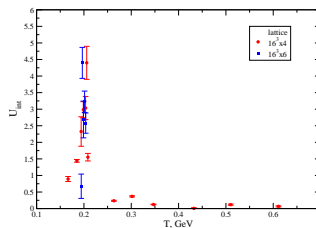
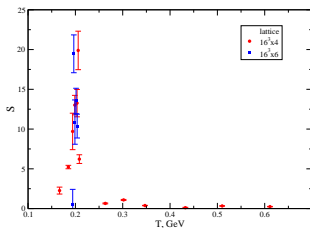


- Small cutoff effects
- Light quark mass scaling is also good

Free energy at infinite separation



Entropy and Internal Energy



- big jump in critical region
- in potential models would mean two or three-fold increase in effective mass
- food for thought.

Renormalized Polyakov Loop

- Very nice order parameter in pure glue
- Still interesting in full QCD
- Loved by model-builders
- Needs renormalization [also see Pisarski way]
- But we define it through already renormalized F_1 [Karsch et al]

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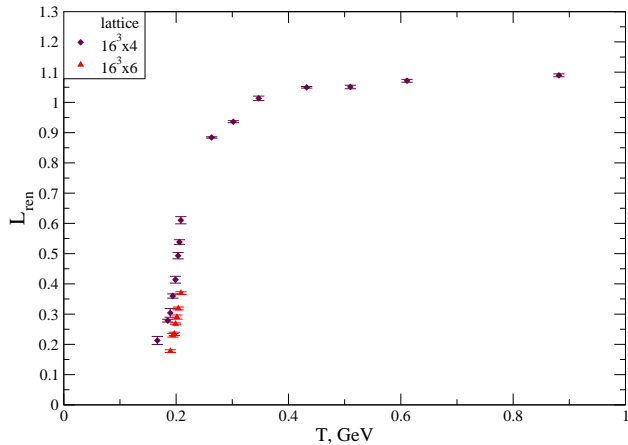
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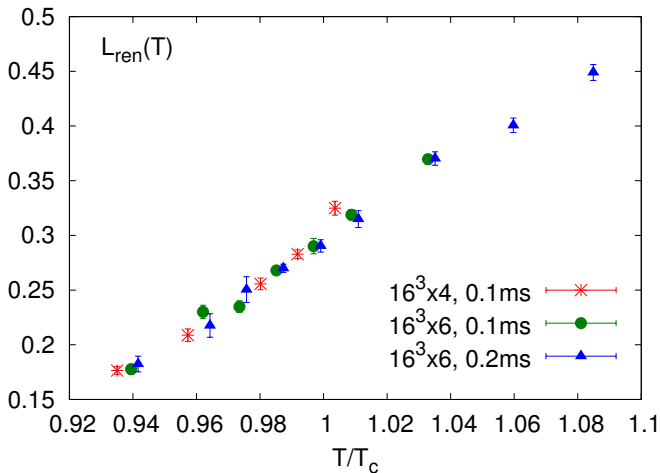
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Renormalized Polyakov Loop in physical units



Renormalized Polyakov Loop Scaling



Reconstruction of the Spectral Function

- $G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma(\omega, T) K(\omega, \tau, T)$
- $O(10)$ data and $O(100)$ degrees of freedom to reconstruct.
- Bayesian technique: find $\sigma(\omega, T)$ that maximizes $P[\sigma | DH]$.
 - D data
 - H prior knowledge: $\sigma(\omega, T) > 0$

New implementation of Maximum Entropy Method:
[[hep-lat/0611017](#)]

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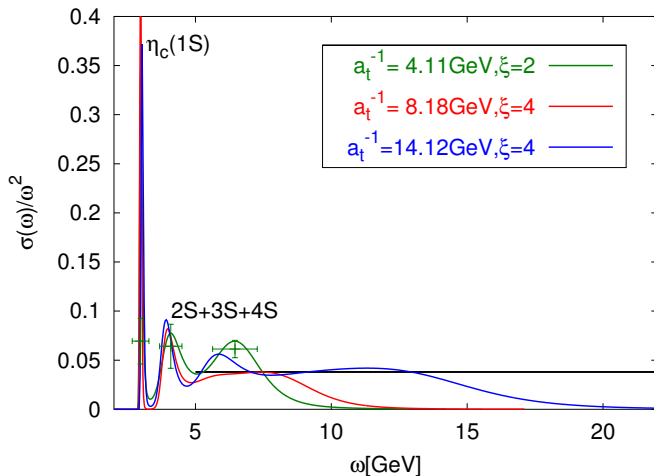
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Charmonium: $T = 0$ 

- visible excited states
- First time we see continuum!

Charmonium: $T > 0$

$$G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

$$G_{recon}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T=0) K(\tau, \omega, T)$$

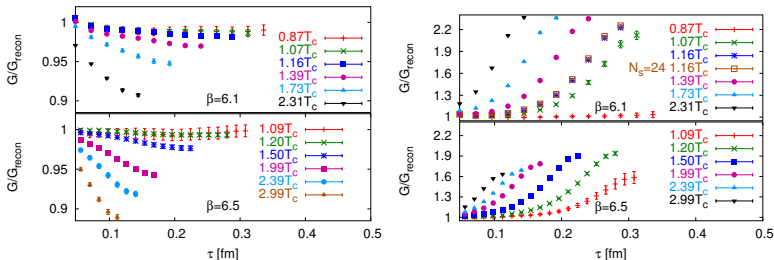
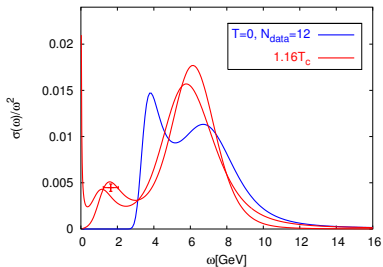
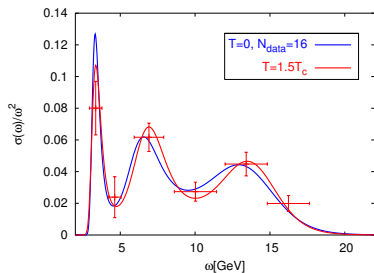


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for (pseudo)scalar channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.



- 1S (left) shows very little dependence on temperature
- hard to accommodate in potential models [see Agnes Mocszy talk]
- 1P is very temperature-dependent

Summary

- Interaction in hot medium gets screened at increasingly short distances
- Scaling is good to very good
- Free energy has a sharp drop at the transition
- Entropy and Internal Energy have sharp peak
- The $1S$ ($\eta_c, J/\psi$) charmonium states **exist** as a resonance at $T \simeq 1.5T_c$.
- Can define and calculate Renormalized Polyakov Loop
- $1P$ (ξ_{c0}, ξ_{c1}) charmonium states **dissolve** at $1.1T_c$.

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β	5.7	5.9	6.1	6.1	6.5
$N_s^2 \times N_t$	$8^3 \times 64$	$16^3 \times 64$	$16^3 \times 64$	$16^3 \times 96$	$24^3 \times 160$
(ξ, ξ_0)	(2,1.655)	(2,1.691)	(2,1.718)	(4,3.211)	(4,3.3166)
r_0/a_s	2.414(8)	3.690(11)	5.207(29)	5.189(21)	8.96(4)
a_t^{-1} [Gev]	1.905	2.913	4.110	8.181	14.12
L_s [fm]	1.66	2.17	1.54	1.54	1.34
configs	2000	1560	930	500	160

Table: Simulation parameters for charmonium at zero temperature.

Correlators and Spectral Functions

Point meson operator

$$J_H(t, \mathbf{x}) = \bar{q}(t, \mathbf{x}) \Gamma_H q(t, \mathbf{x}),$$

where $\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \gamma_\mu \gamma_\nu$.

Meson states in different channels:

Γ	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$ (n=1)	$c\bar{c}$ (n=2)	$b\bar{b}(n=1)$ (n=1)	$b\bar{b}(n=2)$ (n=2)
γ_5	1S_0	0^{-+}	η_c	η_c'	η_b	η_b'
γ_s	3S_1	1^{--}	J/ψ	ψ'	$\Upsilon(1S)$	$\Upsilon(2S)$
$\gamma_s \gamma_{s'}$	1P_1	1^{+-}	h_c		h_b	
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The spectral function

$$\sigma_H(p_0, \vec{p}) = \frac{1}{2\pi} (D_H^>(p_0, \vec{p}) - D_H^<(p_0, \vec{p})) = \frac{1}{\pi} \text{Im} D_H^R(p_0, \vec{p})$$

$$D_H^{>(<)}(p_0, \vec{p}) = \int \frac{d^4 p}{(2\pi)^4} e^{ip \cdot x} D_H^{>(<)}(x_0, \vec{x}) \quad (2)$$

$$D_H^>(x_0, \vec{x}) = \langle J_H(x_0, \vec{x}), J_H(0, \vec{0}) \rangle$$

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The Euclidean propagator

$$G_H(\tau, \vec{p}) = \int d^3 x e^{i\vec{p} \cdot \vec{x}} \langle T_\tau J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle$$

is related to the spectral function through the integral representation

$$G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

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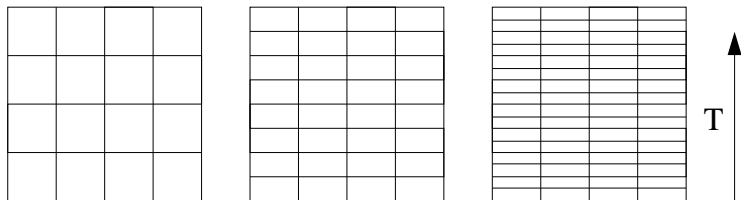
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- Anisotropic lattice $\xi = a_s/a_\tau = 2$ and 4.
- Standard Wilson action in the gauge sector
- Anisotropic clover improved action for heavy fermions
- Quenched approximation

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