A field theoretical model for QCD thermodynamics

Claudia Ratti

ECT, Villazzano (Trento) Italy*

and INFN, Gruppo Collegato di Trento, Povo (Trento) Italy

Can we interpret

Lattice QCD Thermodynamics

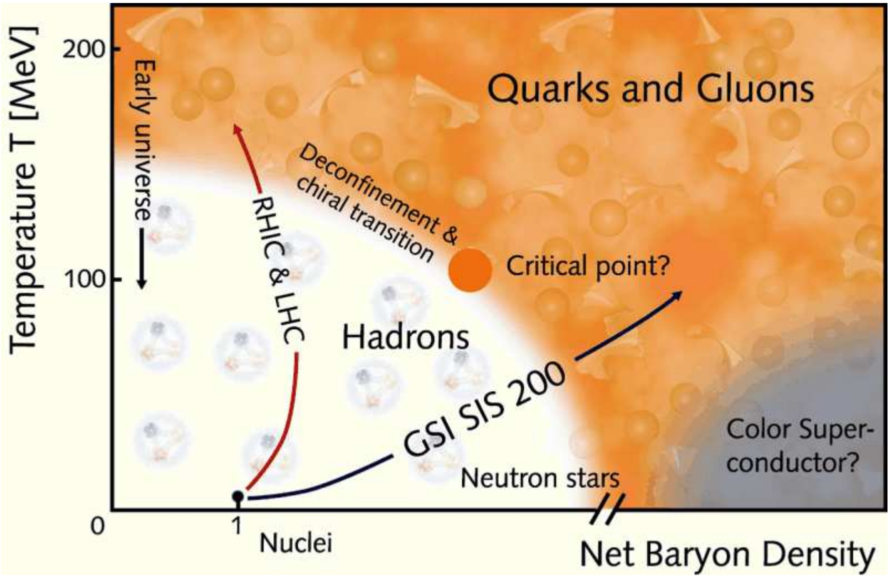
in terms of

QUASIPARTICLE

degrees of freedom?

In collaboration with Simon Rößner, Michael A. Thaler and Wolfram Weise

Introduction



- QCD has a rich phase structure
- Many challenging items:
  - order of the phase transition
  - critical point
  - deconfinement and chiral symmetry
  - color superconductivity at high μ

- Status of lattice QCD thermodynamics:
  - data available in pure gauge sector
  - quarks easily introduced at $\mu = 0$
  - recent lattice data at small μ.

- Taylor expansion method
  - power series of operators in μ/T
  - expansion coefficients at μ/T=0
  - data up to sixth order in μ/T.

Purpose of our work

- MODEL FORMULATION
  - improve NJL model by coupling to Polyakov loop
  - Polyakov loop dynamics fixed in pure gauge QCD
  - parameter fixing in hadronic and pure gauge sectors

Purpose of our work

MODEL FORMULATION

- improve NJL model by coupling to Polyakov loop
- Polyakov loop dynamics fixed in pure gauge QCD
- parameter fixing in hadronic and pure gauge sectors

TESTS OF THE MODEL

- model PREDICTIONS vs lattice data
- different observables
- zero and finite chemical potential

Purpose of our work

MODEL FORMULATION

- improve NJL model by coupling to Polyakov loop
- Polyakov loop dynamics fixed in pure gauge QCD
- parameter fixing in hadronic and pure gauge sectors

TESTS OF THE MODEL

- model PREDICTIONS vs lattice data
- different observables
- zero and finite chemical potential

MODEL PREDICTIONS

- test validity of Taylor expansion: full result versus truncation
- phase diagram and quark mass dependence
- position of critical point and quark mass dependence

PNJL (Polyakov loop extended NJL) model

Starting point: NJL model in temporal background gauge field

$$\mathcal{S}_E(\psi,\psi^\dagger,\phi) = \int_0^{\beta=1/T} d\tau \int d^3x \left[\psi^\dagger\,\partial_\tau\,\psi + \mathcal{H}(\psi,\psi^\dagger,\phi)\right] - \frac{V}{T}\mathcal{V}\left(\Phi,T\right).$$

where:
$$\mathcal{H} = -i\psi^\dagger\left(\vec{\alpha}\cdot\vec{\nabla} + \gamma_4\,m_0 - A_4\right)\psi - \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\,i\gamma_5\vec{\tau}\,\psi\right)^2\right].$$

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D_μ.

$$\Phi\left(x\right) = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta A_4\left(x,\tau\right)d\tau\right)\right] = \frac{1}{3}\mathrm{Tr}\exp\left[\frac{i\mathcal{A}_4^a\lambda_a}{T}\right]$$

Parameters

Λ [GeV]	0.65
G [GeV^{-2}]	10.1
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

P. Meisinger and M. Ogilvie PLB379 (1996)- K. Fukushima PLB591 (2004)-C.R., M. Thaler, W. Weise PRD73 (2006)

Polyakov loop potential

The Polyakov loop is the order parameter related to the $Z(N_c)$ symmetry

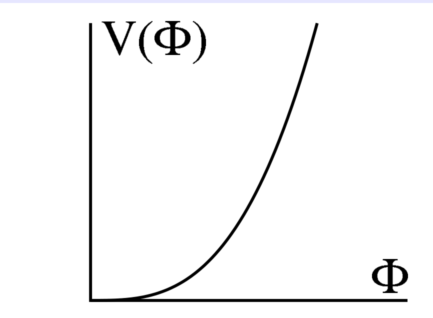
$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - b_4\left(\frac{T_0}{T}\right)^3 \ln[1-6\Phi^*\Phi + 4\left(\Phi^{*3}+\Phi^3\right) - 3(\Phi^*\Phi)^2]$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$$

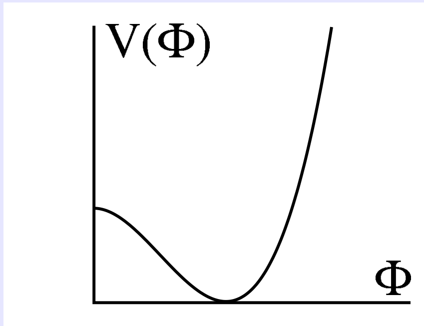
$T < T_0$

- color confinement
- $\langle\Phi\rangle = 0 \longrightarrow Z(3)$ unbroken



$T > T_0$

- color deconfinement
- $\langle\Phi\rangle \neq 0 \longrightarrow Z(3)$ broken

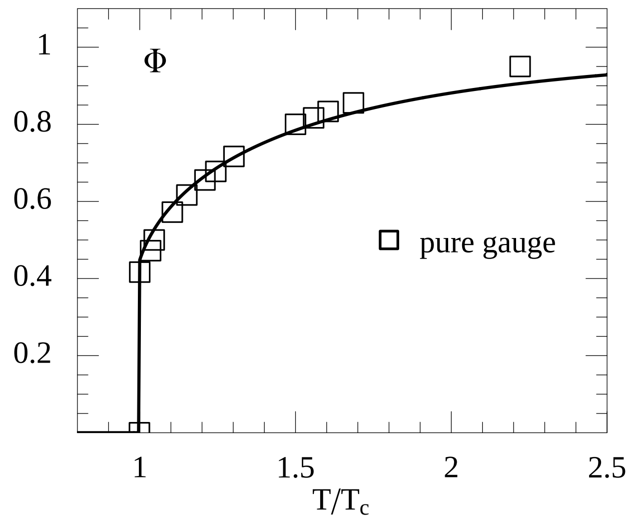


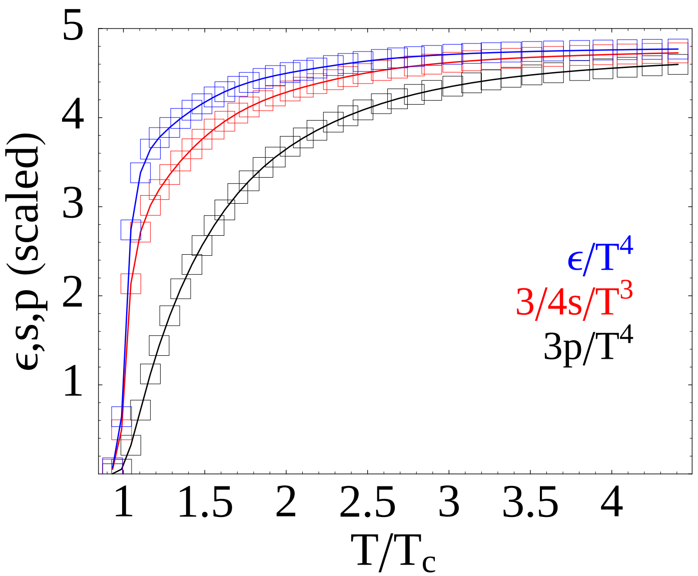
R. Pisarski, PRD62 (2000); K. Fukushima, PLB591 (2004); C.R., M. Thaler, W. Weise PRD73 (2006);

C.R., S. Rößner and W. Weise, hep-ph/0609281.

Fit to Pure Gauge QCD lattice data

- Minimization of $V(\Phi, T)$: Polyakov loop behaviour as a function of T
- Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)





- $p(T) = -V(\Phi(T), T)$
- $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T),T)}{dT}$
- $\epsilon(T) = T\frac{dp}{dT} - p = Ts(T) - p(T)$
- Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

PNJL model at finite temperature and quark chemical potential

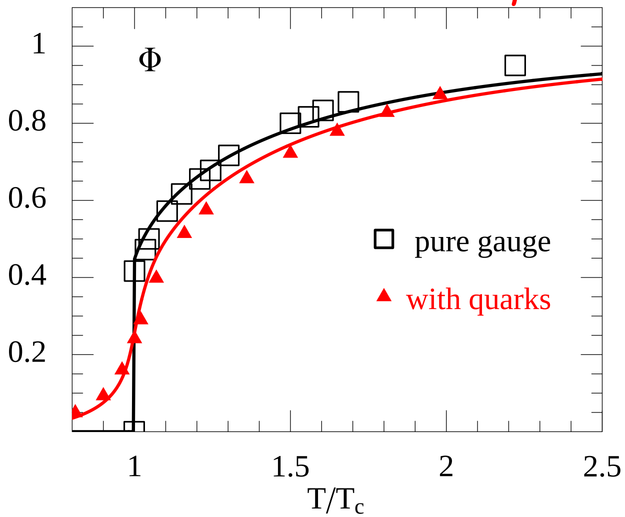
$$\Omega\left(T,\mu,\sigma,\Phi\right) = V\left(\Phi,T\right) + \frac{\sigma^2}{2G}$$
$$-2N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ 3E_p + T\left[\ln\left[1 + 3\Phi \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi^* \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right]\right.\right.$$
$$\left.\left. + \ \ln\left[1 + 3\Phi^* \mathrm{e}^{-(E_p+\mu)/T} + 3\Phi \mathrm{e}^{-2(E_p+\mu)/T} + \mathrm{e}^{-3(E_p+\mu)/T}\right]\right]\right\}$$

with $E_p = \sqrt{\vec{p}^{\,2} + m^2}$ and $m = m_0 - \langle\sigma\rangle = m_0 - 2G\langle\bar{\psi}\psi\rangle$ is the constituent quark mass.

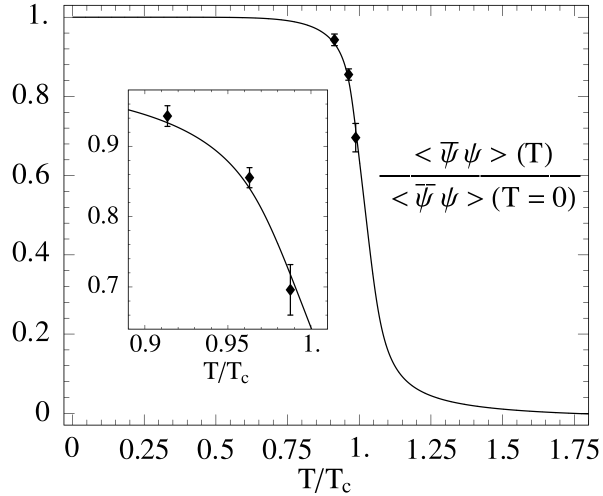
High temperature limit: $\Phi \longrightarrow 1$, $\Phi^* \longrightarrow 1$: we re-obtain the standard NJL formula:

$$\ln\left[1 + 3\Phi^* \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right]$$
$$\downarrow \quad T \rightarrow \infty$$
$$\ln\left[1 + \mathrm{e}^{-(E_p-\mu)/T}\right]^3 = 3\ln\left[1 + \mathrm{e}^{-(E_p-\mu)/T}\right]$$

$\mu = 0$ PREDICTIONS



Lattice data from Kaczmarek *et al.* PLB 543 (2002) and Kaczmarek and Zantow PRD71 (2005).



Lattice data from Boyd *et al.* PLB349 (1995).

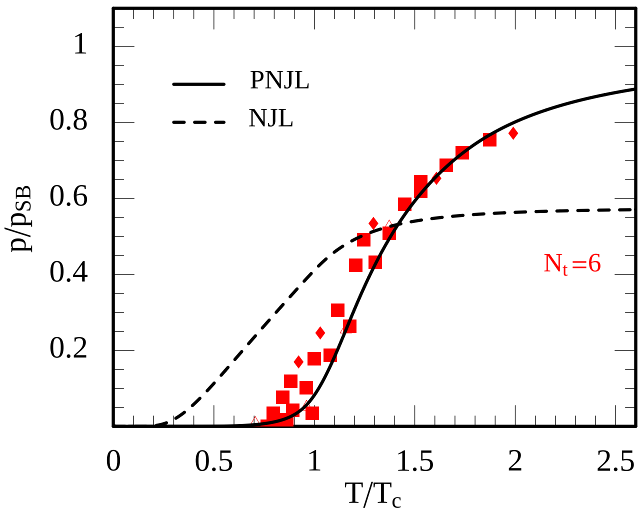
- Scaled pressure as a function of T/T_c

$$\frac{p\,(T,\mu=0)}{T^4} = -\frac{\Omega\,(T,\mu=0)}{T^4}$$

PNJL model results from:

C.R., M. A. Thaler and W. Weise, PRD 73 (2006).

C.R., S. Rößner and W. Weise, hep-ph/0609281.

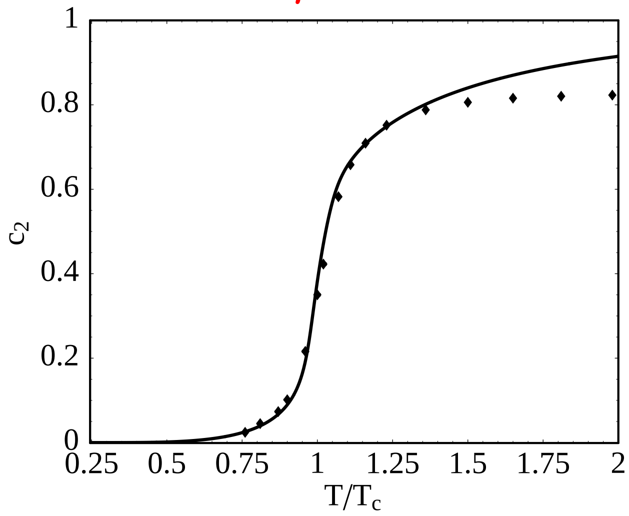


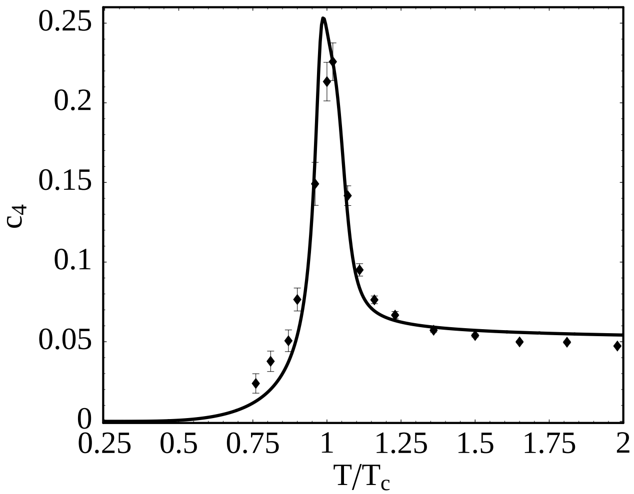
Lattice data from CP-PACS collaboration (2001)

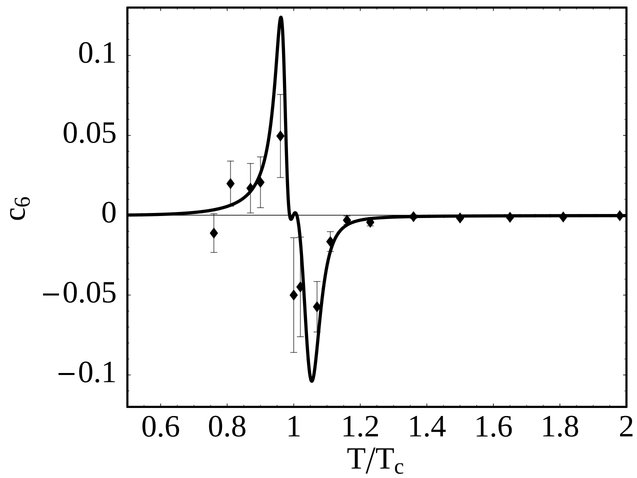
Taylor expansion of pressure around $\mu = 0$

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n ;$$

$$c_n(T) = \frac{1}{n!}\frac{\partial^n\left(p(T,\mu)/T^4\right)}{\partial(\mu/T)^n}\bigg|_{\mu=0}$$

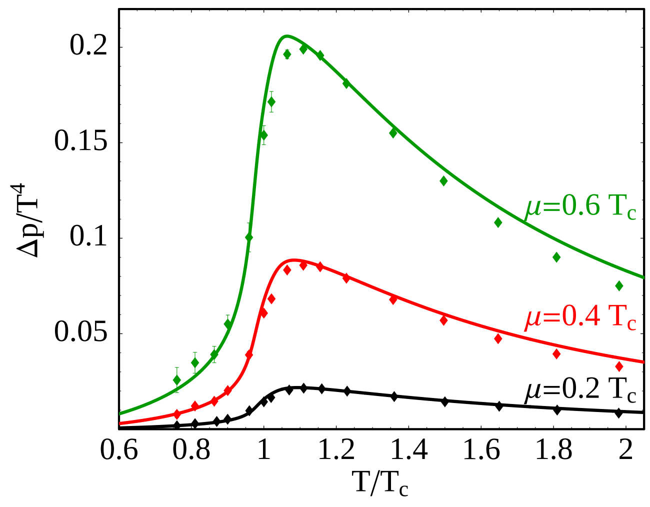


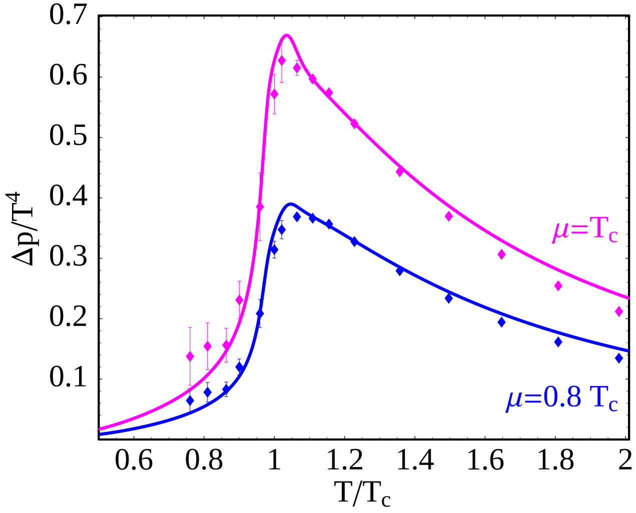




C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, EPJC (2006).
C.R., S. Rößner and W. Weise, hep-ph/0609281. Lattice data from Allton *et al.* (2005).

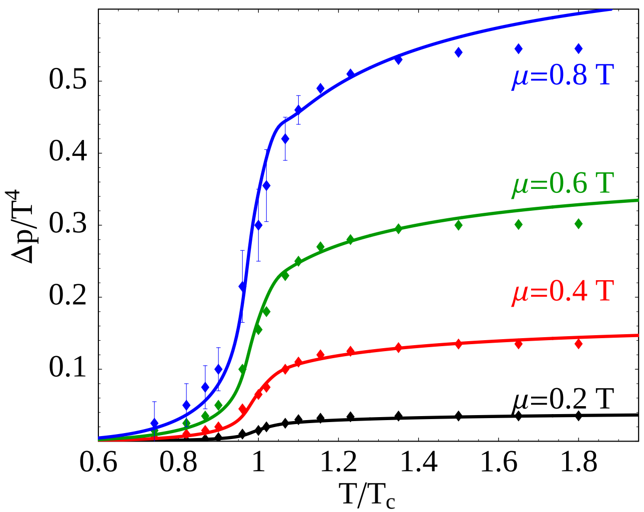
Finite μ PREDICTIONS: pressure





$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n ;$$

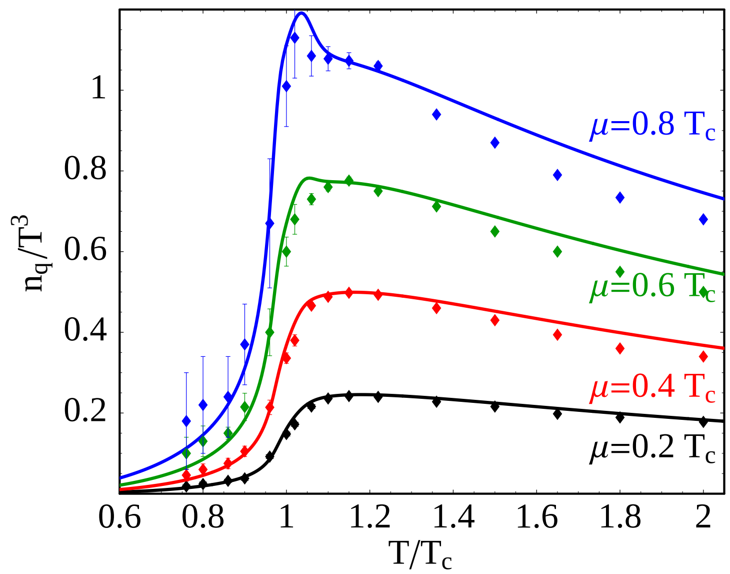
$$c_n(T) = \frac{1}{n!}\frac{\partial^n\left(p(T,\mu)/T^4\right)}{\partial(\mu/T)^n}\bigg|_{\mu=0}$$

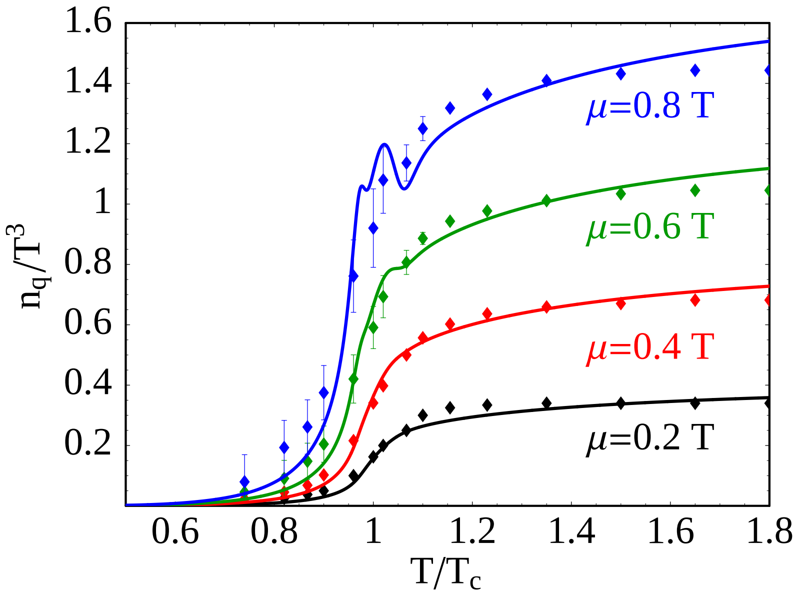


C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, EPJC (2006). Lattice: Allton *et al.* (2005).

Finite μ PREDICTIONS: quark number density

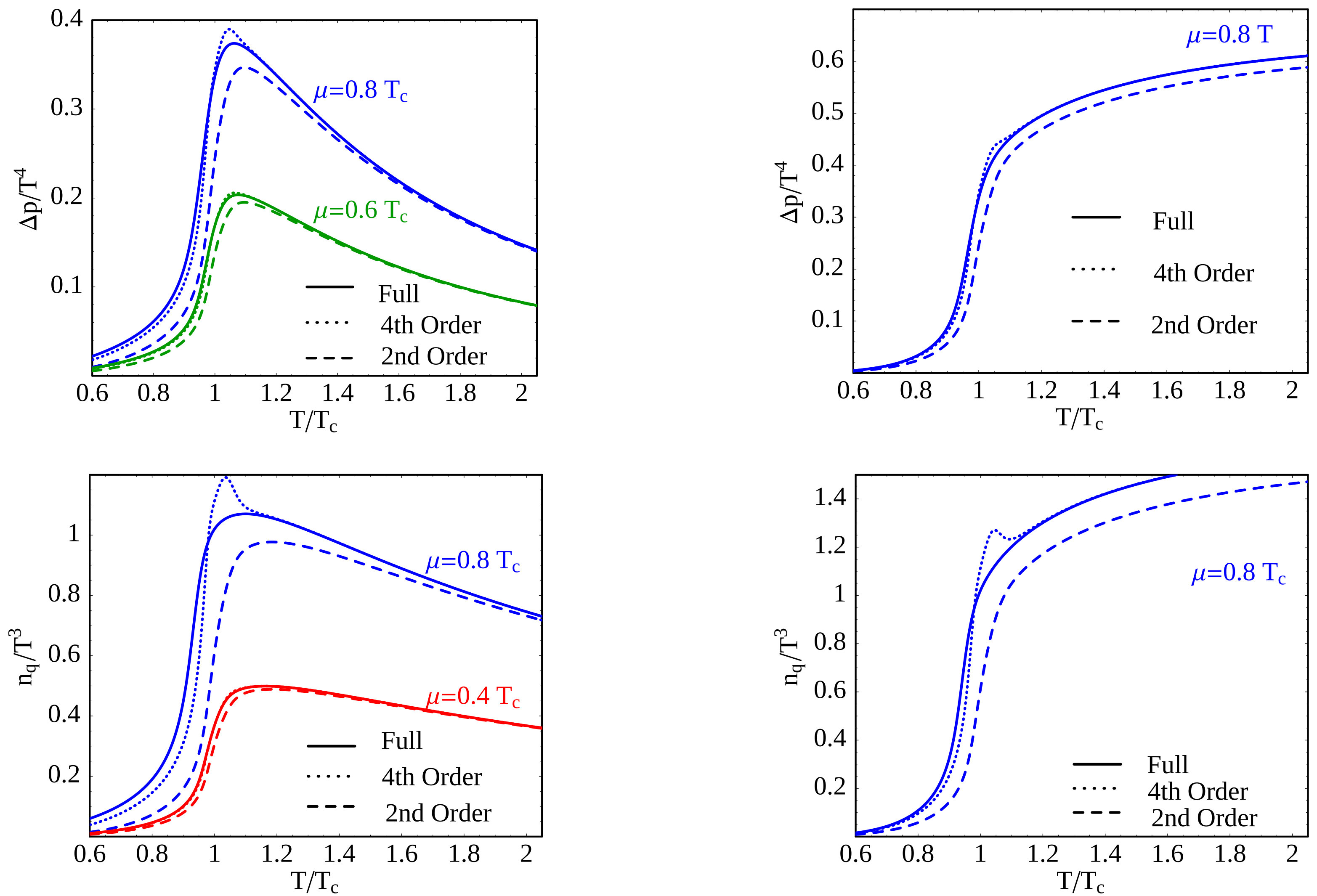
$$\frac{n_q(T,\mu)}{T^3} = \frac{\partial\left(p/T^4\right)}{\partial\mu_q/T} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$$





C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, EPJC (2006). Lattice: Allton *et al.* (2005).

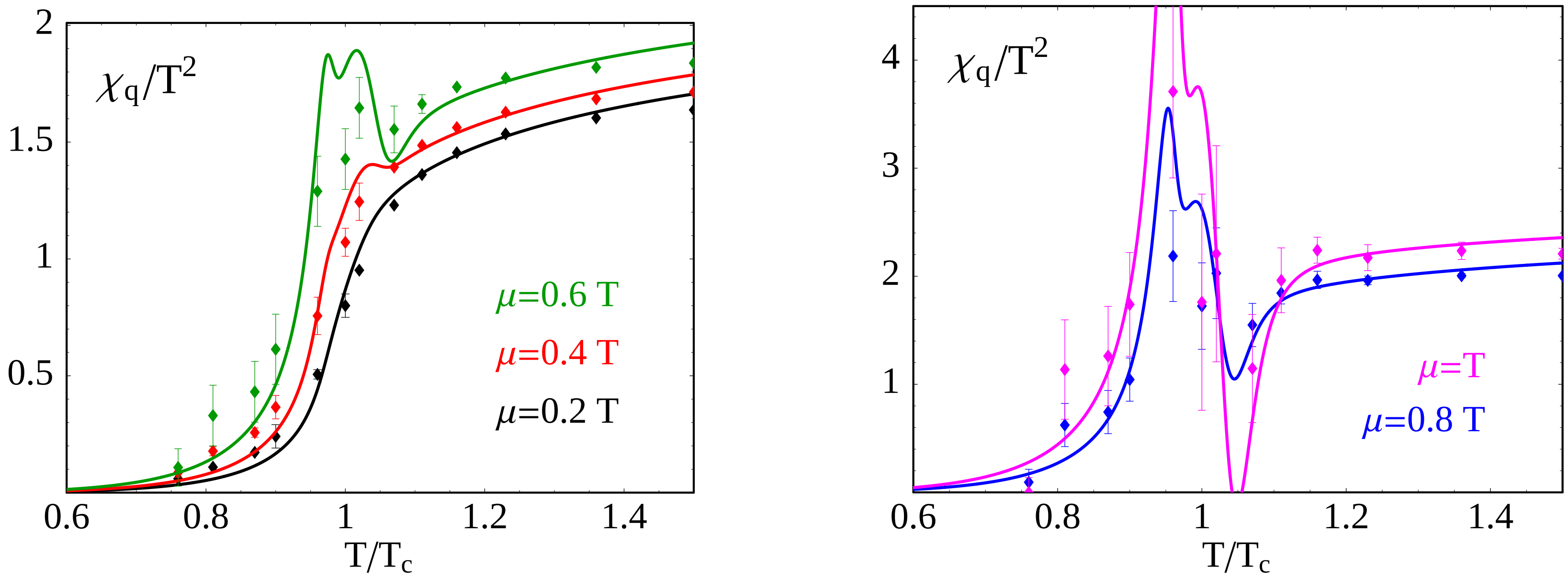
Comparison between Taylor-expanded and full results



C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, EPJC (2006).

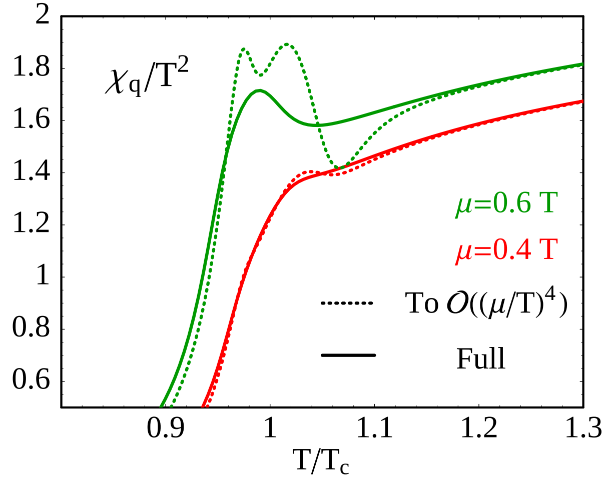
Finite μ PREDICTIONS: quark number susceptibilities

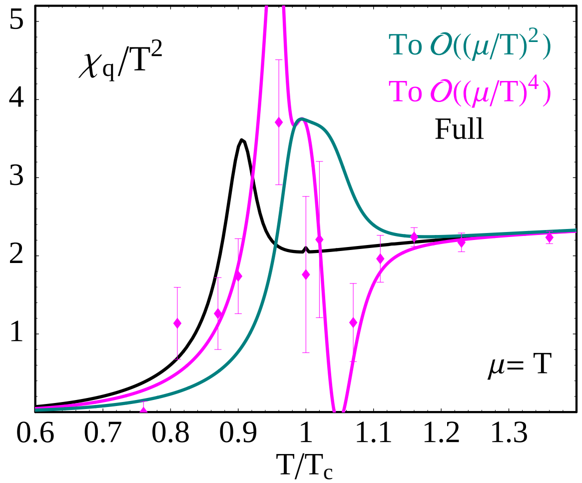
$$\frac{\chi_q(T,\mu)}{T^2} = \frac{\partial\left(n_q/T^3\right)}{\partial\mu_q/T} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$$

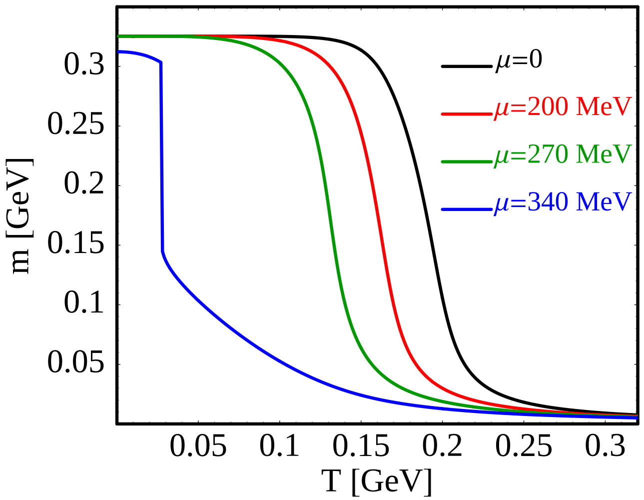


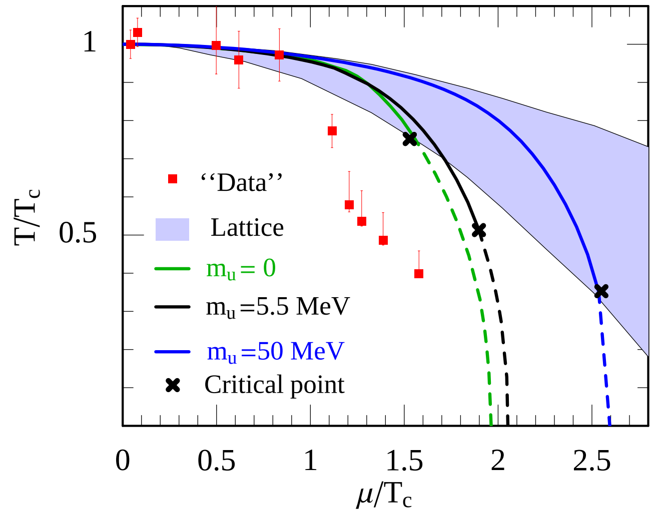
C.R., S. Rößner, W. Weise, forthcoming. Lattice data from Allton *et al.* (2005).

Full vs truncated results









C.R., S. Rößner, W. Weise, forthcoming.

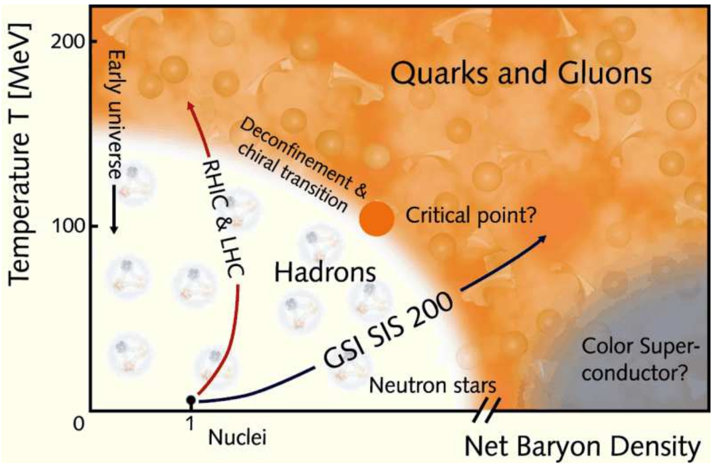
Lattice data from Allton *et al.* (2005).

Therm. fit data from Andronic *et al.* NPA772 (2005).

Conclusions

- The standard NJL model fails in reproducing QCD thermodynamics
- PNJL model: minimal synthesis of features of confinement and chiral symmetry breaking
- A description of QCD thermodynamics with our simple model works very well
- Taylor series converging very quickly at relatively small chemical potentials
- Discrepancy between truncated and full results observed at larger chemical potentials
  - → Quark number susceptibilities

Outlook



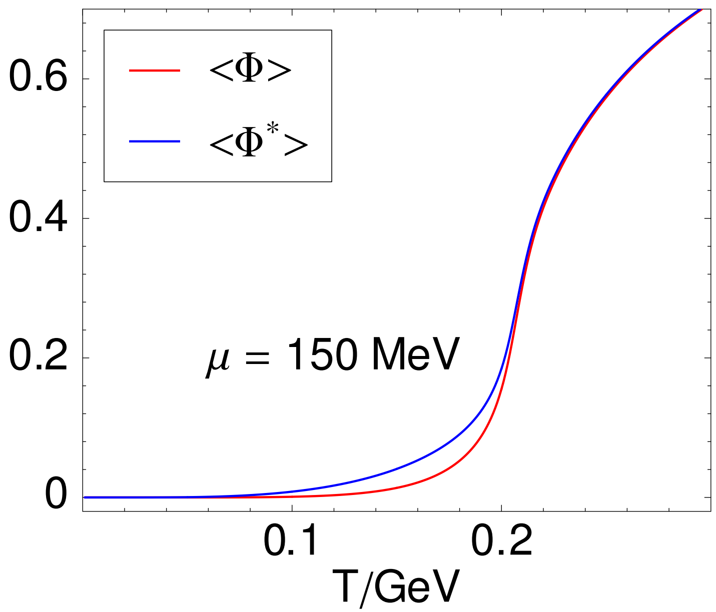
- Exploration of the thermodynamics and phase diagram for $N_f = 2+1$ and $N_f = 3$
- Improvement of approximation: beyond mean field approximation

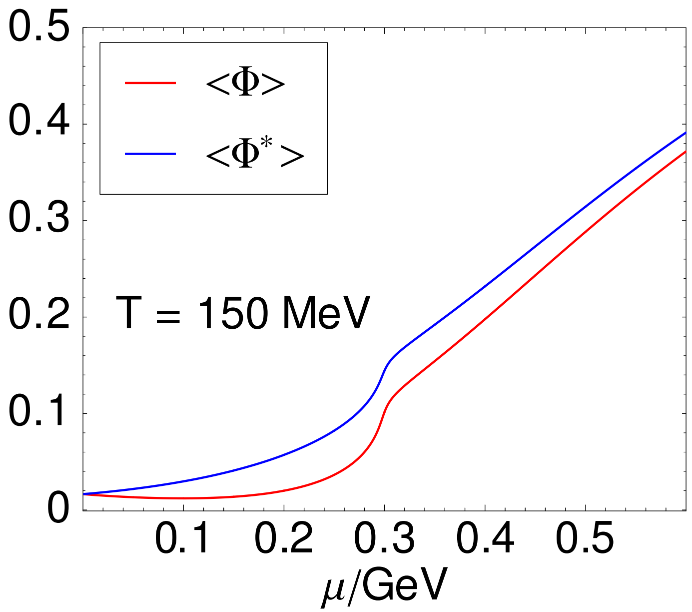
Related works

- First papers on PNJL model
  - P. N. Meisinger and M. C. Ogilvie, PRD52 (1995); PLB379 (1996); PLB407 (1997); PRD65 (2002)
  - K. Fukushima, PRD68 (2003); PLB591 (2004); Y. Hatta and K. Fukushima, PRD69 (2004)
- Other works on coupling chiral quark models to Polyakov loop
  - E. Megias, E. Ruiz Arriola, L.L. Salcedo, PRD74 (2006); JHEP 0601 (2006); hep-ph/0607338
- More recent works
  - S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa, R. Ray, PRD73 (2006); S. Mukherjee, M.G. Mustafa, R. Ray, hep-ph/0609249
  - H. Hansen *et al.*, hep-ph/0609116 $\Rightarrow$ see also poster n. 95
  - Z. Zhang, Y.-X. Liu, hep-ph/0610221
  - K. Fukushima, Y. Hidaka, hep-ph/0610323
  - see also poster n. 105 by C. Sasaki on *Susceptibilities and the Phase Structure of a Chiral Model with the Polyakov Loop*

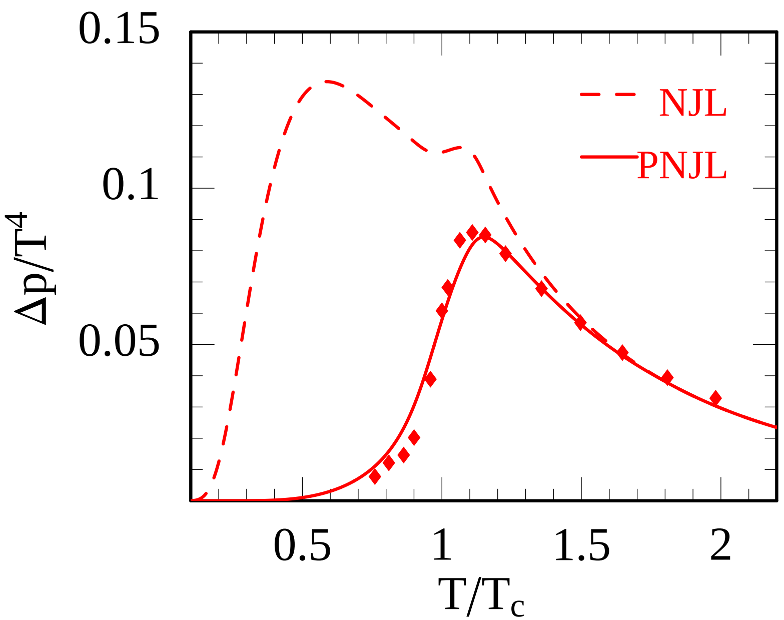
Backup slides

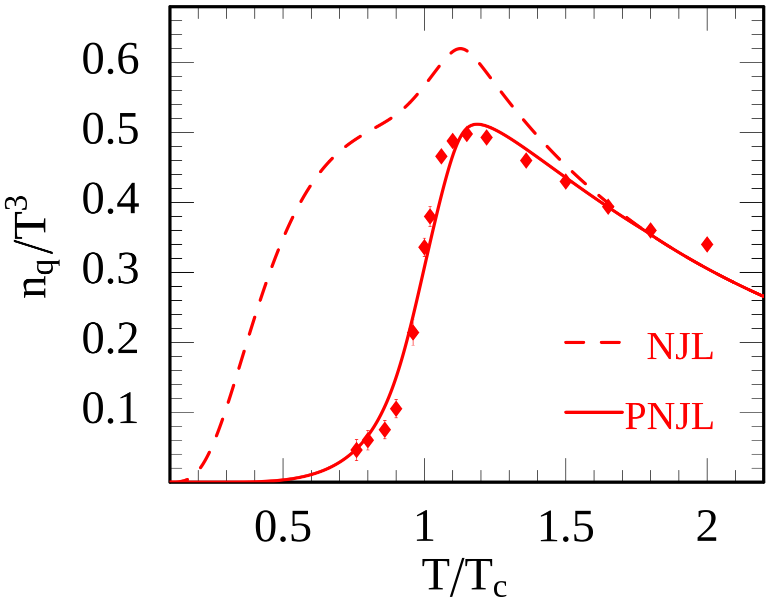
Finite μ results





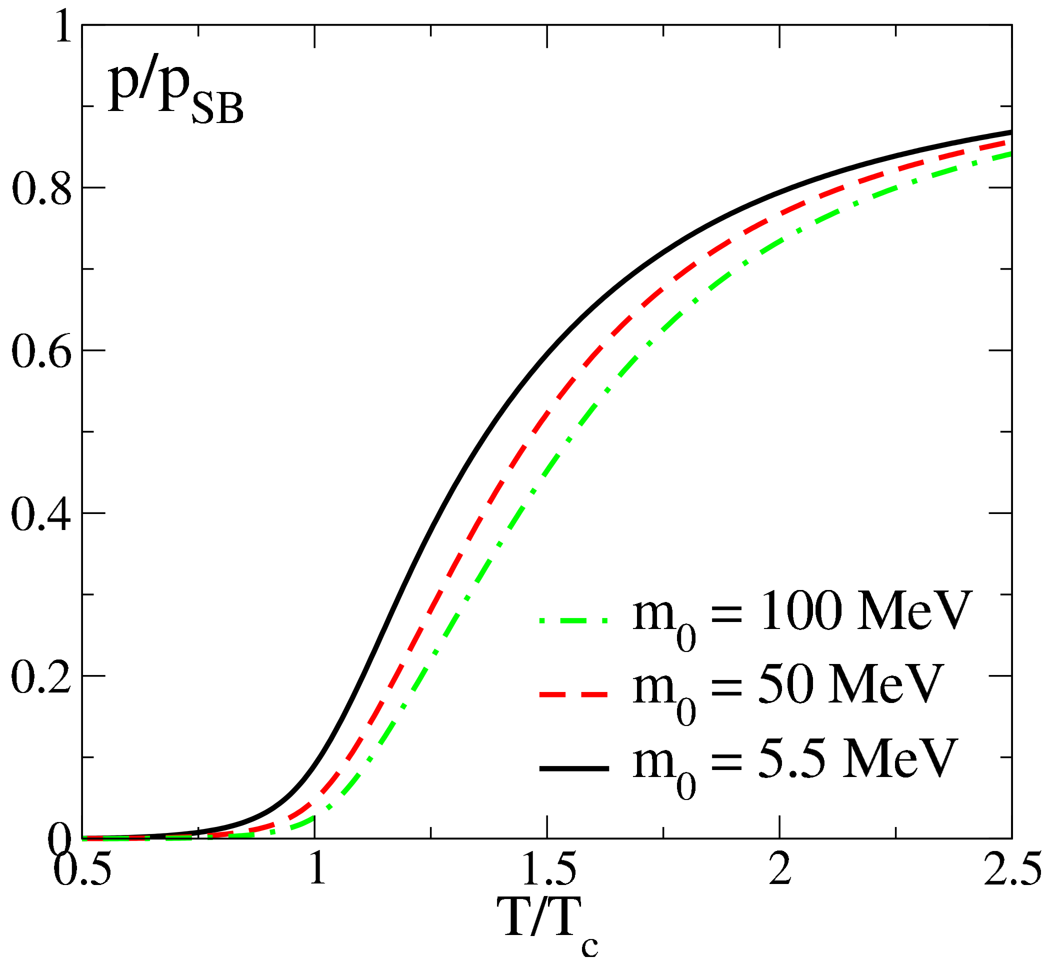
Finite μ results in the standard NJL model





- Comparison between PNJL and standard NJL model results at finite μ
- NJL model alone fails in reproducing lattice data
- Incorporation of confinement crucial to reproduce lattice results

Quark mass dependence



Parameter fixing

We have three free parameters in the model: $m_0,\ \Lambda,\ G$. They are fixed by:

- The pion decay constant f_π is evaluated in the NJL model through the following relation:

$$f_\pi^2 = 4m^2 I_\Lambda^{(1)}(m) \quad \text{where} \quad I_\Lambda^{(1)}(m) = -iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{\left(p^2 - m^2 + i\epsilon\right)^2}.$$

The empirical value is $f_\pi = 92.4$ MeV.

- The quark condensate becomes

$$\left\langle \bar{\psi}_u \psi_u \right\rangle = -4m I_\Lambda^{(0)}(m) \qquad \text{with} \qquad I_\Lambda^{(0)}(m) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{p^2 - m^2 + i\epsilon}.$$

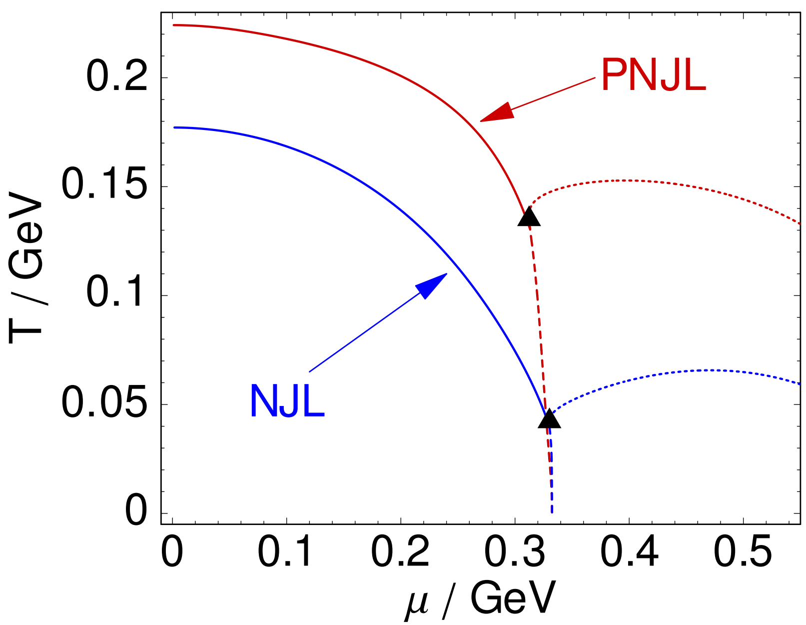
Its “empirical” value derived from QCD sum rules is

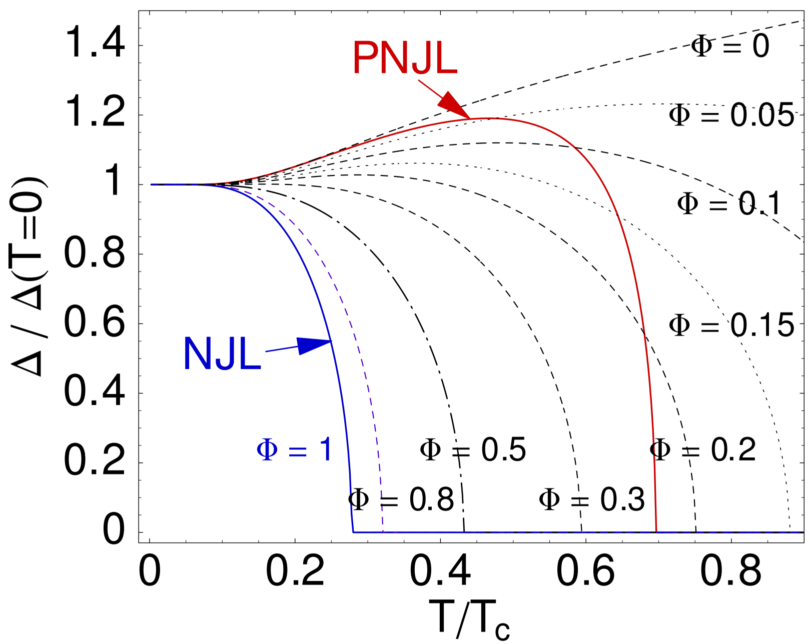
$$\langle \bar{\psi}_u \psi_u \rangle^{1/3} \simeq \langle \bar{\psi}_d \psi_d \rangle^{1/3} = -\left(240 \pm 20\right)\ \text{MeV}.$$

- The current quark mass m_0 is fixed from the Gell-Mann, Oakes, Renner (GMOR) relation:

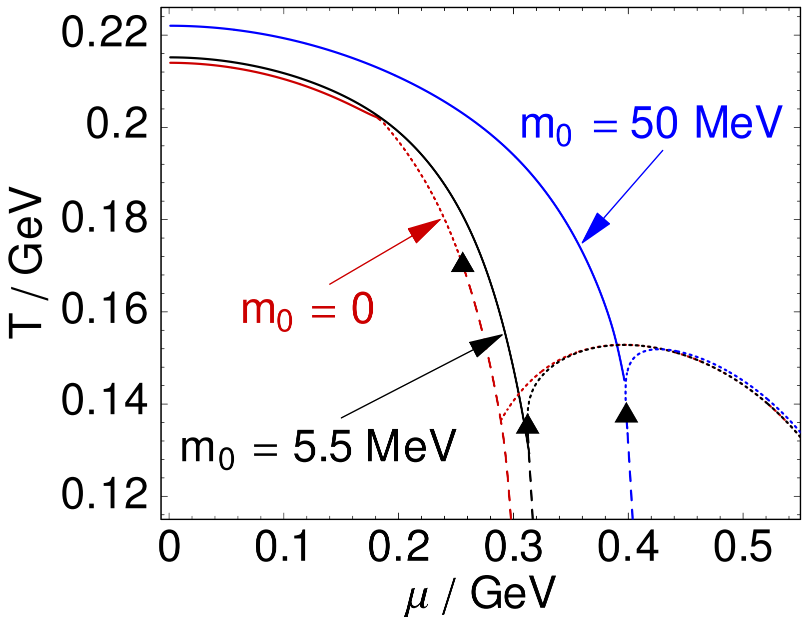
$$m_\pi^2 = \frac{-m_0 \left\langle \bar{\psi}\psi \right\rangle}{f_\pi^2}.$$

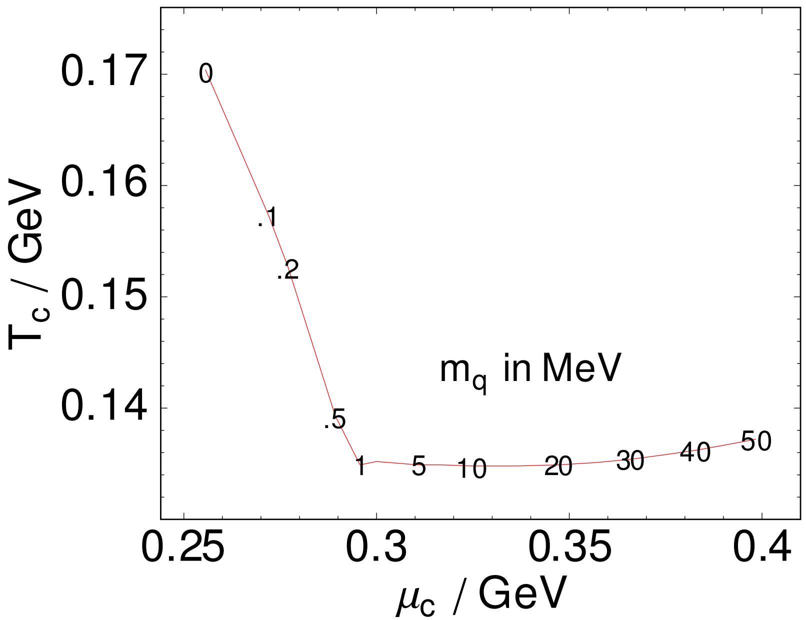
Phase diagram at high μ: NJL vs PNJL model





Quark mass dependence of phase diagram and critical endpoint





Thermodynamic potential: intermediate steps

The thermodynamic potential of the system is:

$$\Omega\left(T,\mu\right)=V\left(\Phi,T\right)-T\sum_n\int\frac{d^3p}{\left(2\pi\right)^3}\frac{1}{2}\,\mathrm{Tr}\,\ln\left(\frac{1}{T}\tilde{S}^{-1}\left(i\omega_n,\vec{p}\right)\right)+\frac{\sigma^2}{2G},$$

where $\omega_n=\left(2n+1\right)\pi\,T$ are the Matsubara frequencies for fermions and

$$\tilde{S}^{-1}\left(p^0,\vec{p}\right)=\begin{pmatrix}\not{p}-\hat{m}-\mu\gamma_0+gA^0\gamma_0 & 0\\ 0 & \not{p}-\hat{m}+\mu\gamma_0-gA^0\gamma_0\end{pmatrix}.$$

The constituent quark mass is defined as

$$m=m_0-\langle\sigma\rangle=m_0-G\langle\bar{\psi}\psi\rangle.$$

Final form of Ω:

$$\begin{aligned}\Omega\left(T,\mu\right)\quad&=\quad V\left(\Phi,T\right)+\frac{\sigma^2}{2G}-2N_f\,T\int\frac{\mathrm{d}^3p}{\left(2\pi\right)^3}\left\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-\left(E_p-\mu\right)/T}\right]\right.\\&+\quad\left.\mathrm{Tr}_c\ln\left[1+L^{\dagger}\,\mathrm{e}^{-\left(E_p+\mu\right)/T}\right]+\frac{E_p}{T}\theta\left(\Lambda^2-\vec{p}^{\;2}\right)\right\}\end{aligned}$$