Unconventional Color Superconductor: Beyond standard BCS theory

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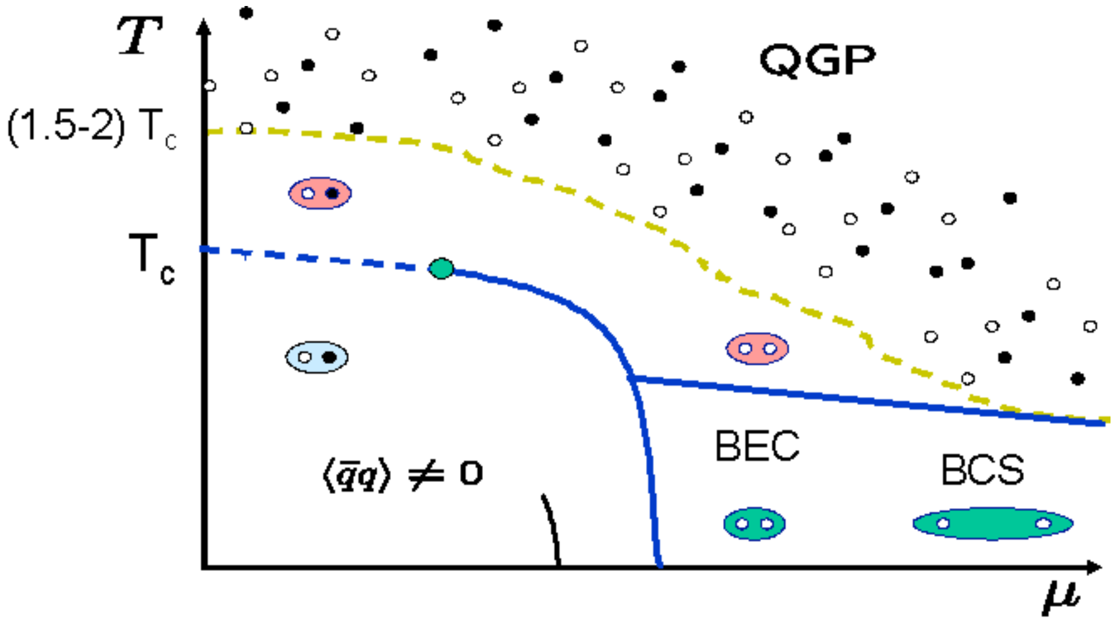
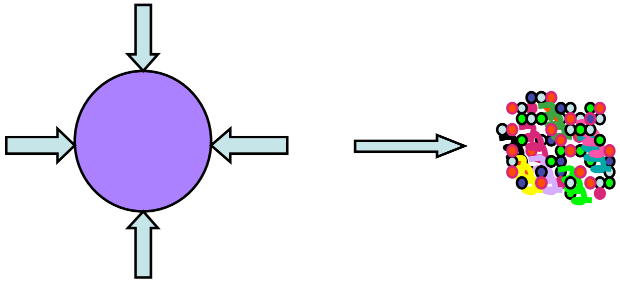
In recent 3 years, challenges for theorists ......

Hot quark matter: **Ideal gas** ⟶ **sQGP**

Cold quark matter: **Ideal BCS CSC** ⟶ **Unconventional CSC**



I. BCS pairing CSC



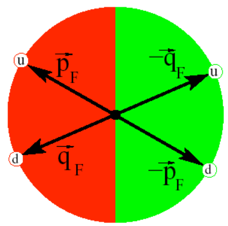
$$\langle qq\rangle_{\bar{3}_c}$$

Since 1977

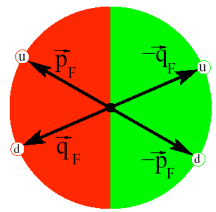
Barrois, NPB129 (1977),390;
……

Rapp, Schaefer, Shuryak,Velkovsky, PRL81, 53 (1998);
Alford, Rajagopal, Wilczek, PLB422, 247 (1998);
……

BCS pairing



Standard BCS pairing

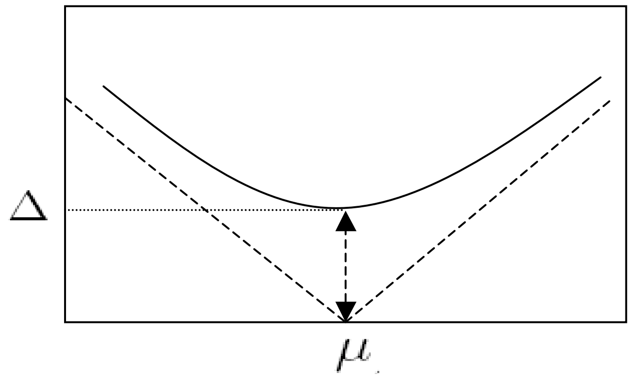


e.g. 2SC

SU(3)->SU(2)

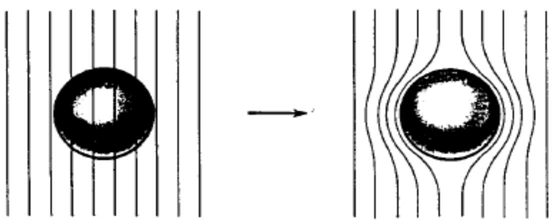
$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$

1) Quasiparticle excitation



$$E_\Delta^\pm = \pm\sqrt{(p-\mu)^2 + \Delta^2} \qquad E_b = \pm(p-\mu)$$

2) Meissner effect



Normal S/C

Anderson-Higgs mechanism

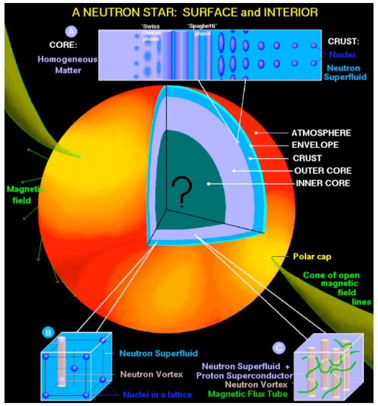
a=1,2,3 massless

a=4,5,6,7,8 massive

SU(3)->SU(2)

Since 2002

II. Pairing with mismatched Fermi surfaces



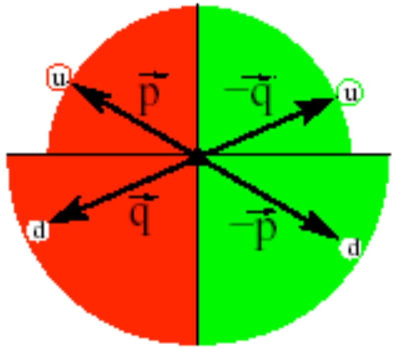
Lattimer & Prakash

beta-equilibrium, charge neutrality

2-flavor $n_d \approx 2n_u$

3-flavor $m_s > m_u$

$$\delta\mu, \delta m \rightarrow \delta p_F$$



Pair breaking?

Imbalanced cold atom system, Asymmetric nuclear matter, ...

II.1 Results and problems from BCS at Mean-field

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] + G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]$$

Beta-equilibrium:

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)_{\alpha\beta}$$

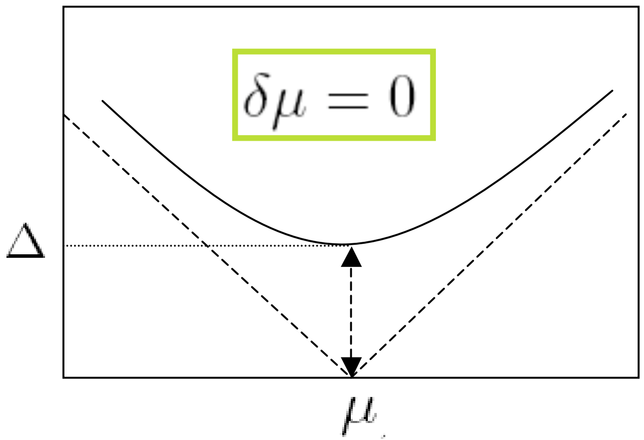
$$\bar{\mu} = \mu - \mu_e/6 + \mu_8/3$$
$$\delta\mu = \mu_e/2$$

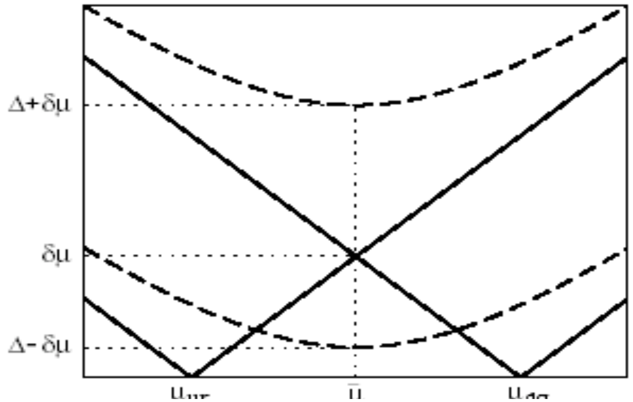
BCS at mean-field(MF)

$$\Delta = |\Delta|$$

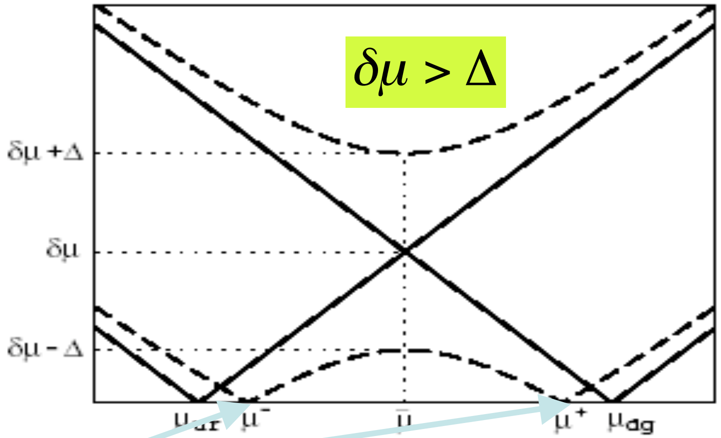
Gapless mode

$$E^{\pm}_{\Delta} = \pm\sqrt{(p-\mu)^2+\Delta^2} \quad \longrightarrow \quad E^{\pm}_{\Delta\pm} \;=\; E^{\pm}_{\Delta}(p) \pm \delta\mu.$$





$\delta\mu < \Delta$

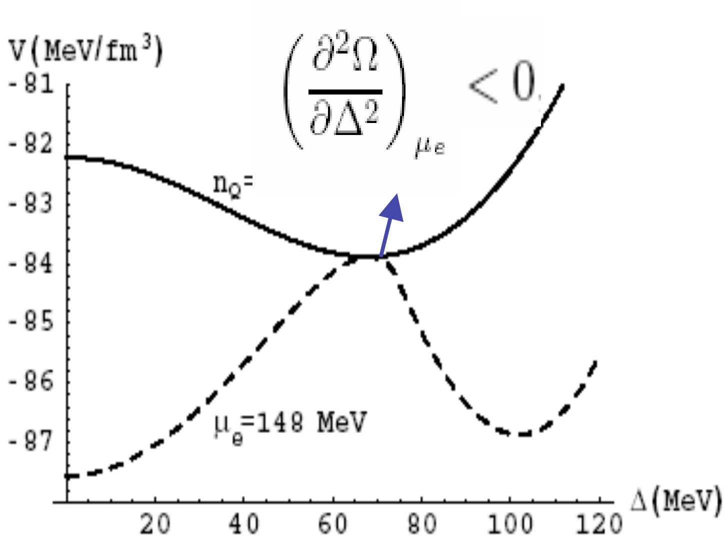


Energy gap is different from Pairing gap !!!

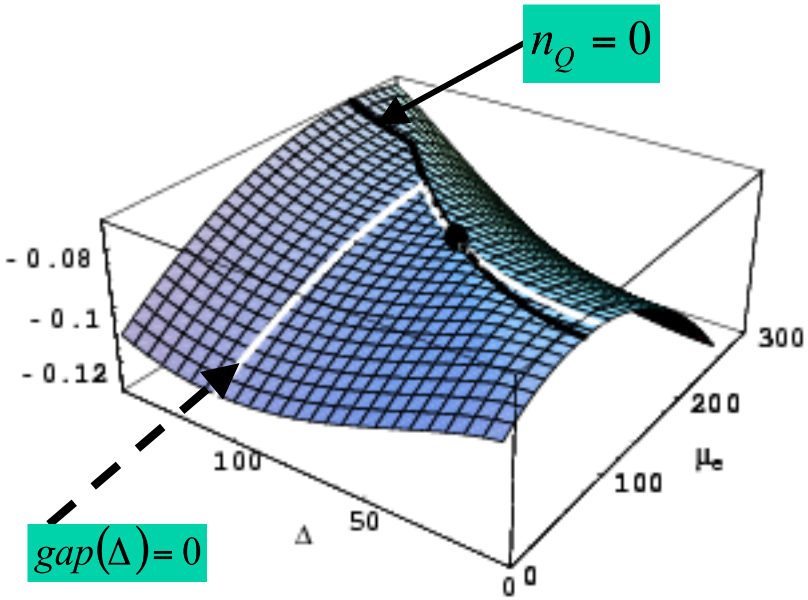
Gapless Mode !

I. Shovkovy, M.H, Phys.Lett.B564:205,2003

Sarma instability



$$E_{\text{Coulomb}} \sim n_Q^2 R^5$$

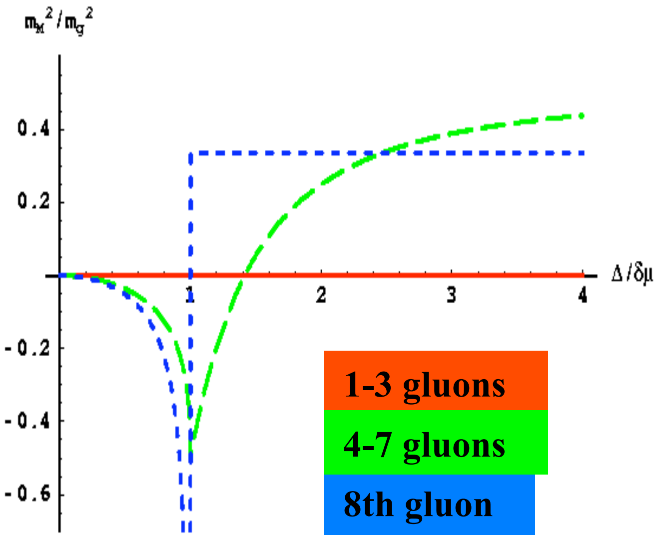


I. Shovkovy, M.H, Phys.Lett.B564:205,2003

G.Sarma, J.Phys.Chem.Solids 24, 1029 (1963)

BP (**gapless phase in cold atom system**) **Liu, Wilczek** 2003

Chromomagnetic instability driven by mismatch!



$$(m_M^a)^2 < 0, a = 4, 5, 6, 7, 8$$

MH, I.Shovkovy, PRD70:051501,2004; 094030,2004

(Chromo)Magnetic instability in other gapless phases

BP

Wu, Yip, PRA67: 053603, 2003

gCFL

Casalbuoni, et.al., PLB605:362-368,2005

Alford, Wang, J.Phys.G31:719-738,2005

K. Fukushima, Phys.Rev.D72:074002,2005

Possible ground states:

1. LOFF (Larkin Ovchinnikov Fulde Ferrell)

$\Delta(r) = |\Delta_q| e^{(iqr)}$ (FF)

P. Fulde and A. Ferrell Phys. Rev. 135, A550 (1964).

Anisotropic

also:

multi-plane wave

$\Delta(r) = |\Delta_q| \cos(qr)$ (LO)

A.I. Larkin and Yu.N. Ovchinnikov Zh. Eksp. Teor. Fiz. 47, 1136 (1964).

2. Phase separation

Inhomogeneous

M.Alford, K.Rajagopal, S.Reddy, F.Wilczek,PRD64(2001), 074017;
F. Neumann, M. Buballa, M. Oertel, NPA 714, 2003;
I. Shovkovy, M. Hanauske, M.H, PRD67:103004,2003;
S. Reddy and G. Rupak, nucl-th/0405054, ……

II.2 A new framework for mismatched systems

----- but at the very beginning

Based on the work:

M.H. PRD73:045007, 2006; Int.J.Mod.Phys.A21, 910, (2006)

I. Giannakis, D.F.Hou, M.H., H.C.Ren,

hep-ph/0606178; hep-ph/0609098

Full theory:

Group G → **Subspace: Group H** | **Coset space: M=G/H**

$$\Delta(\vec{r}) = (\Delta + H(\vec{r}))e^{i\varphi(\vec{r})}$$

$$\Omega = \Omega_M + \Omega_H + \Omega_{NG}$$

BCS theory at MF

$$\Delta = |\Delta| \qquad \Omega = \Omega_M$$

Full order parameter in the 2SC phase:

$$\begin{pmatrix} \Delta^1(\vec{r}) \\ \Delta^2(\vec{r}) \\ \Delta^3(\vec{r}) \end{pmatrix} = \exp\left[i \sum_{a=4}^{8} \varphi_a(\vec{r}) T_a \right] \begin{pmatrix} 0 \\ 0 \\ \Delta + H(\vec{r}) \end{pmatrix}$$

new quark field:

$$q = \mathcal{V}\chi \quad , \quad \bar{q} = \bar{\chi}\,\mathcal{V}^\dagger$$

$$\mathcal{L}_{nl} \equiv \bar{X}\,\mathcal{S}_{nl}^{-1}\,X - \frac{\Phi^+\Phi^-}{4G_D}$$

$$\mathcal{S}_{nl}^{-1} \equiv \begin{pmatrix} [G_{0,nl}^+]^{-1} & \Phi^- \\ \Phi^+ & [G_{0,nl}^-]^{-1} \end{pmatrix}$$

$$[G_{0,nl}^+]^{-1} = i\,\not{D} + \hat{\mu}\,\gamma_0 + \gamma_\mu V^\mu, \quad V^\mu \simeq -\sum_{a=4}^{8} (\partial^\mu \varphi_a)\, T_a$$

Shovkovy, Rischke, Phys.Rev.D66:054019,2002
M.H. PRD73:045007

A. Instability in the NG sector & anisotropy

$$\Omega_{NG} = \frac{1}{2}\int d^3\vec{r}\sum_{a=1}^{8} m_a^2(\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a)(\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a) + higher\, orders$$

Meissner mass

NG currents ---> (LO)FF-like state

$$(m_a)^2 < 0, \quad a = 4, 5, 6, 7 \qquad \sum_{a=4}^{7} < \vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a > \neq 0$$

Gluon phase, Gorbar, Hashimoto, Miransky, **hep-ph/0507303**

$$(m_8)^2 < 0 \qquad < \vec{\mathbf{A}}^8 - \frac{1}{g}\vec{\nabla}\varphi^8 > \neq 0.$$

U(1) (LO)FF-state, Giannakis, Ren, **hep-ph/0412015**
Cold atom system, **He, Jin, Zhuang, cond-mat/0601147**

B. Instability in Higgs sector & spatial inhomogeneity

I. Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0606178; hep-ph/0609098

Higgs sector

$$\Omega_M = -\frac{T}{2}\sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}\ln([\mathcal{S}_M(P)]^{-1}) + \frac{\Delta^2}{4G_D}$$

$$\Omega_H = \frac{T}{2}\sum_{k_0} \int \frac{d^3\vec{k}}{(2\pi)^3} H^*(\vec{k})\Pi_H(k)H(\vec{k}).$$

$\Pi_H =$

Inhomogeneous Higgs field induces inhomogeneous charge distribution

Coulomb energy

$$\delta\mathcal{F} = \frac{1}{2}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n \delta\Delta^2 + \frac{1}{2}\sum_{\vec{k}\neq 0}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^*_{\vec{k}}\partial\Delta_{\vec{k}}}\right)_n \delta\Delta^*_{\vec{k}}\delta\Delta_{\vec{k}} + E_{\mathrm{coul.}}$$

$$\delta\rho(\vec{k}) = \kappa(k)H(\vec{k})$$

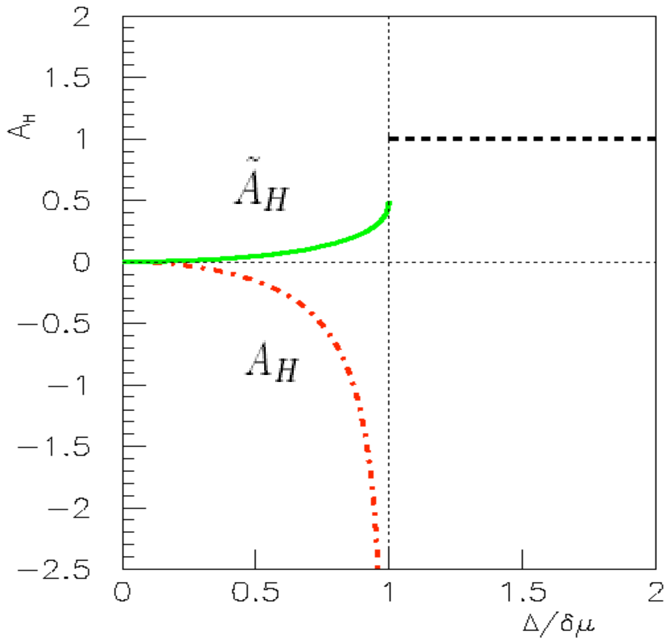
$$E_{\mathrm{coul.}} = \frac{1}{2V}\sum_{\vec{k}\neq 0}\frac{\delta\rho(\vec{k})^*\delta\rho(\vec{k})}{k^2+m_D^2(k)}$$

$$\tilde{\Pi}(k) \equiv \left(\frac{\partial^2\mathcal{F}}{\partial H^*(\vec{k})\partial H(\vec{k})}\right)_{n_Q} = \Pi(k) + \frac{\kappa^*(k)\kappa(k)}{k^2+m_D^2(k)}$$

Sarma Instability can be removed by Coulomb energy

for $k << \Delta$

$$\Pi(k) = A_H + B_H k^2 \qquad A_H = \left(\frac{\partial^2 \Omega}{\partial \Delta^2}\right)_{\mu} = \frac{4\bar{\mu}^2}{\pi^2}\left[1 - \frac{\delta\mu}{\sqrt{(\delta\mu)^2 - \Delta^2}}\right]$$

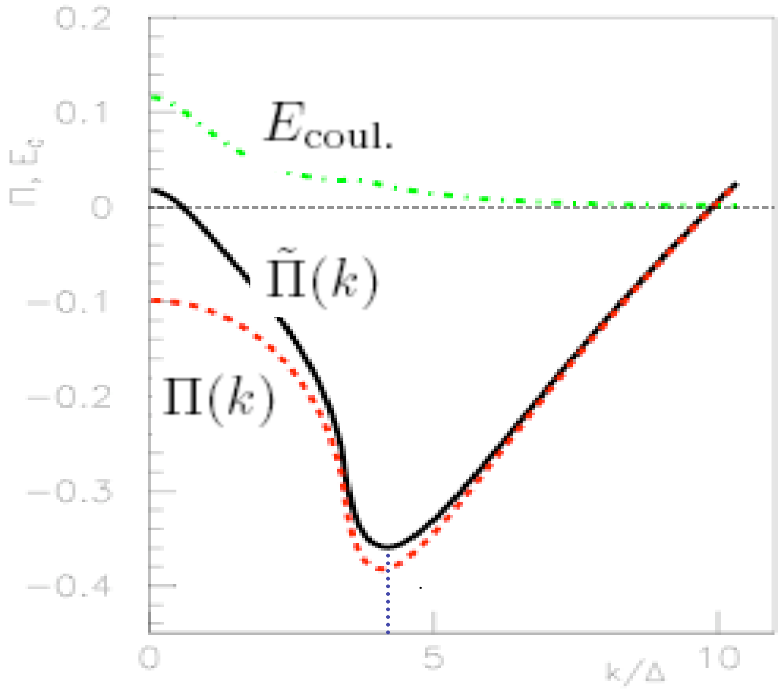


$$\left(\frac{\partial^2 \Omega}{\partial \Delta^2}\right)_{\nu} < 0$$

$$\left(\frac{\partial^2 \mathcal{F}}{\partial \Delta^2}\right)_{n} = \left(\frac{\partial^2 \Omega}{\partial \Delta^2}\right)_{\nu} + \frac{\left(\frac{\partial n}{\partial \Delta}\right)_{\nu}^2}{\left(\frac{\partial n}{\partial \nu}\right)_{\Delta}} > 0.$$

Electric Coulomb energy is not strong enough to compete the Higgs Instability in g2SC

numerical results in whole momentum space



$\Delta/\delta\mu = 1/2$ and $\alpha_e\bar{\mu}^2/\Delta^2 = 1$

coherence length ξ

$$\xi \simeq \Delta^{-1}$$

wavelength for the unstable mode

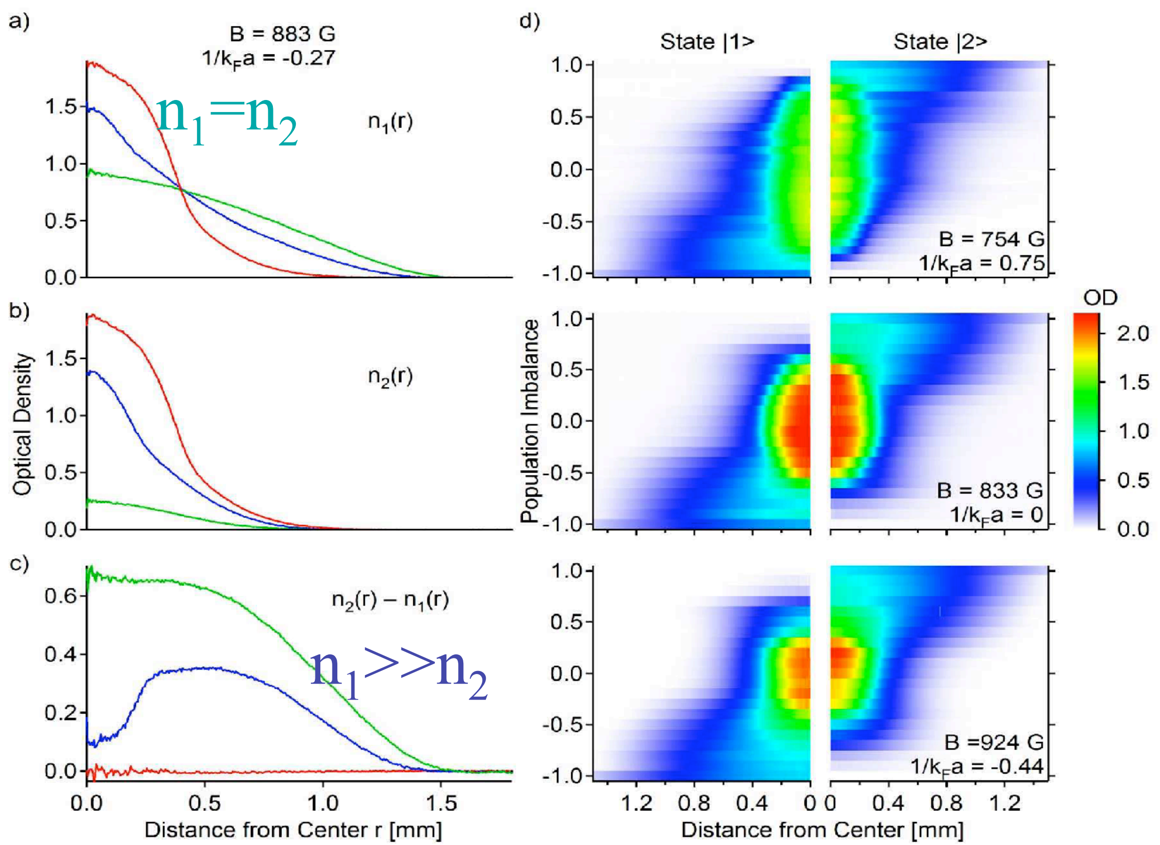
$$l \simeq k_{min}^{-1}$$

phase separation

$$l/\xi < 1$$

Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0609098

For gapless superfluid systems, Higgs instability remains, phase separation is favored



phase separation

Zwierlein, Schirotzek, Schunck, & Ketterle, Science 2005, cond-mat/0511197
Partridge, Li, Kamar, Liao, & Hulet, Science 2005, cond-mat/0511752.

Summary & Outlook

I. Two instabilities driven by mismatch:
Instability in NG sector --> anisotropy
Instability in Higgs sector --> inhomogeneity

II. Gapless superfluidity (BP) state, no other mechanism compete with Higgs instability, phase separation is more favored.

III. G2SC phase, electric Coulomb energy is not strong enough to compete with the Higgs instability

IV. gCFL phase, whether color Coulomb energy is strong enough to remove Higgs instability?