Elliptic flow fluctuations
in 200 GeV Au+Au collisions at RHIC

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for the PHOBOS collaboration

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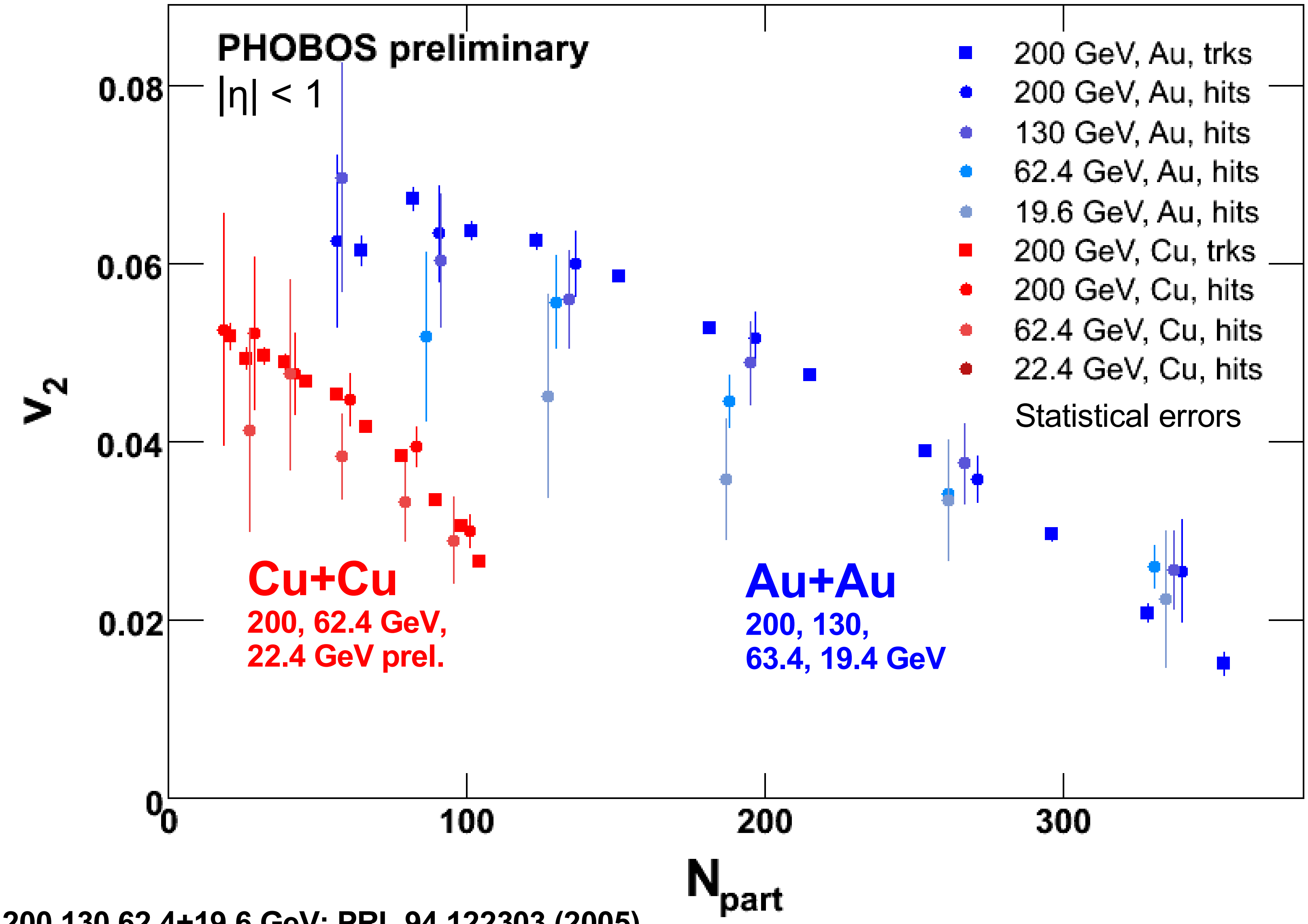
UNIVERSITY OF MARYLAND

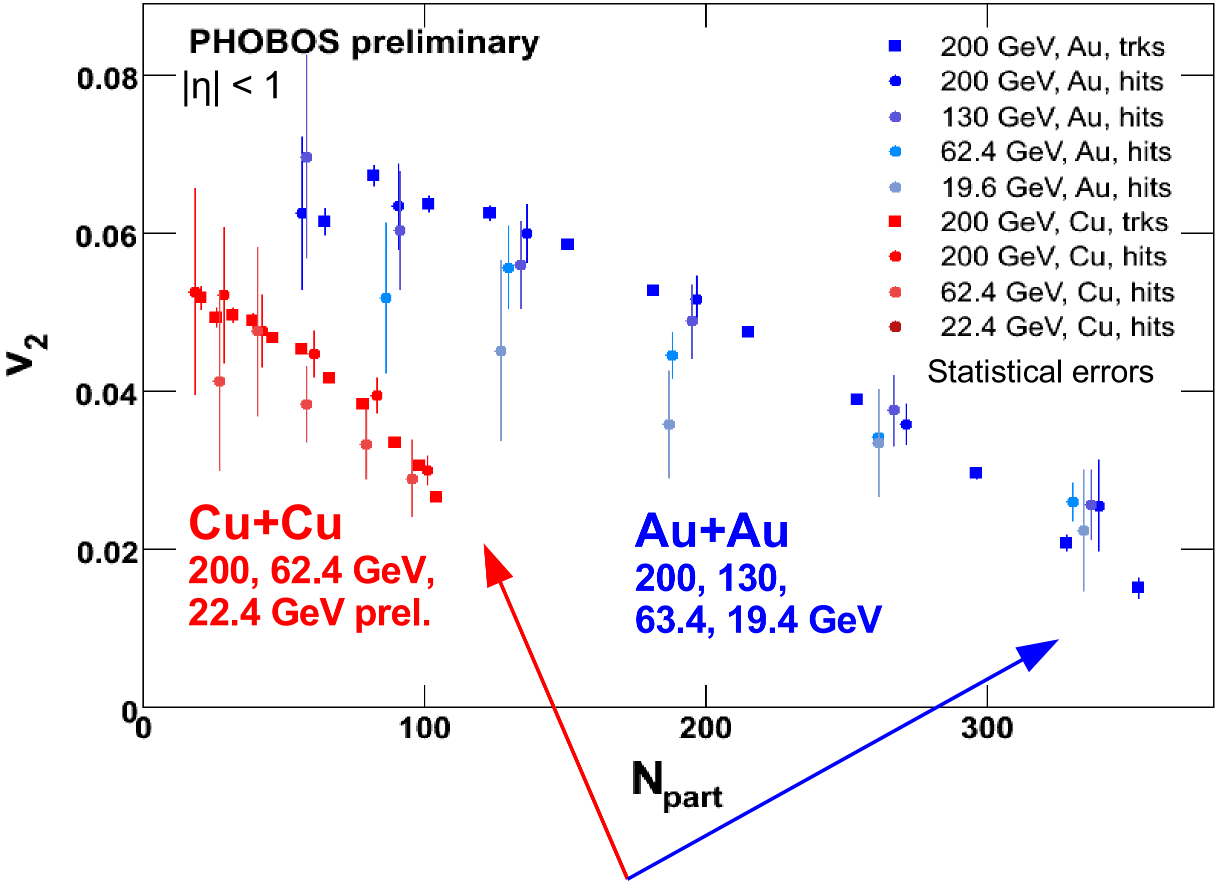
BROOKHAVEN NATIONAL LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
UNIVERSITY OF ILLINOIS AT CHICAGO
UNIVERSITY OF ROCHESTER

Elliptic flow for different species



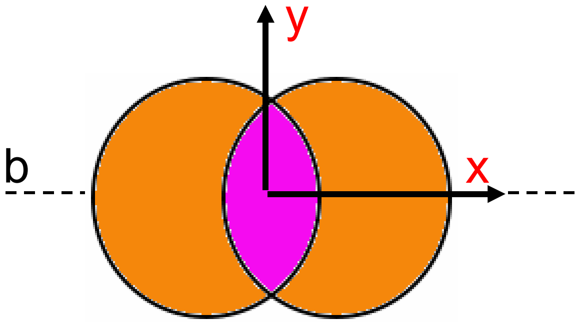
Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Elliptic flow and standard eccentricity

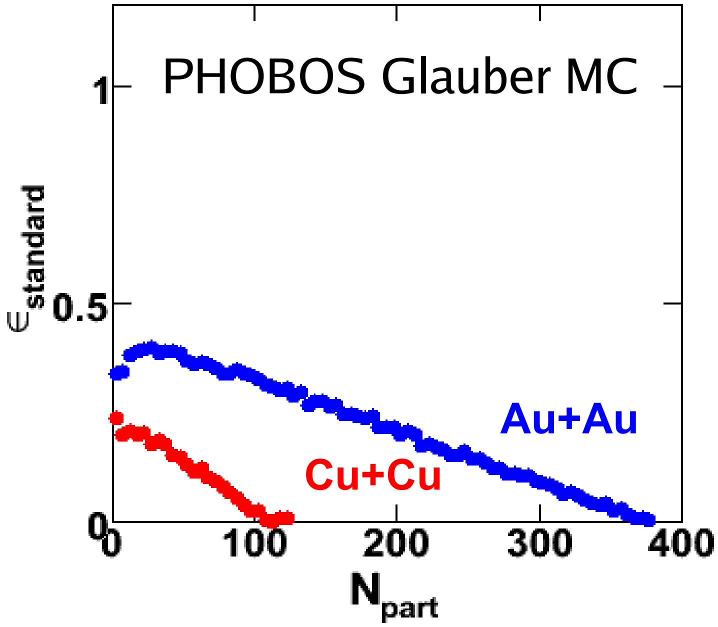


Large flow in central Cu+Cu compared to central Au+Au

Standard Eccentricity



$$\epsilon_{standard} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
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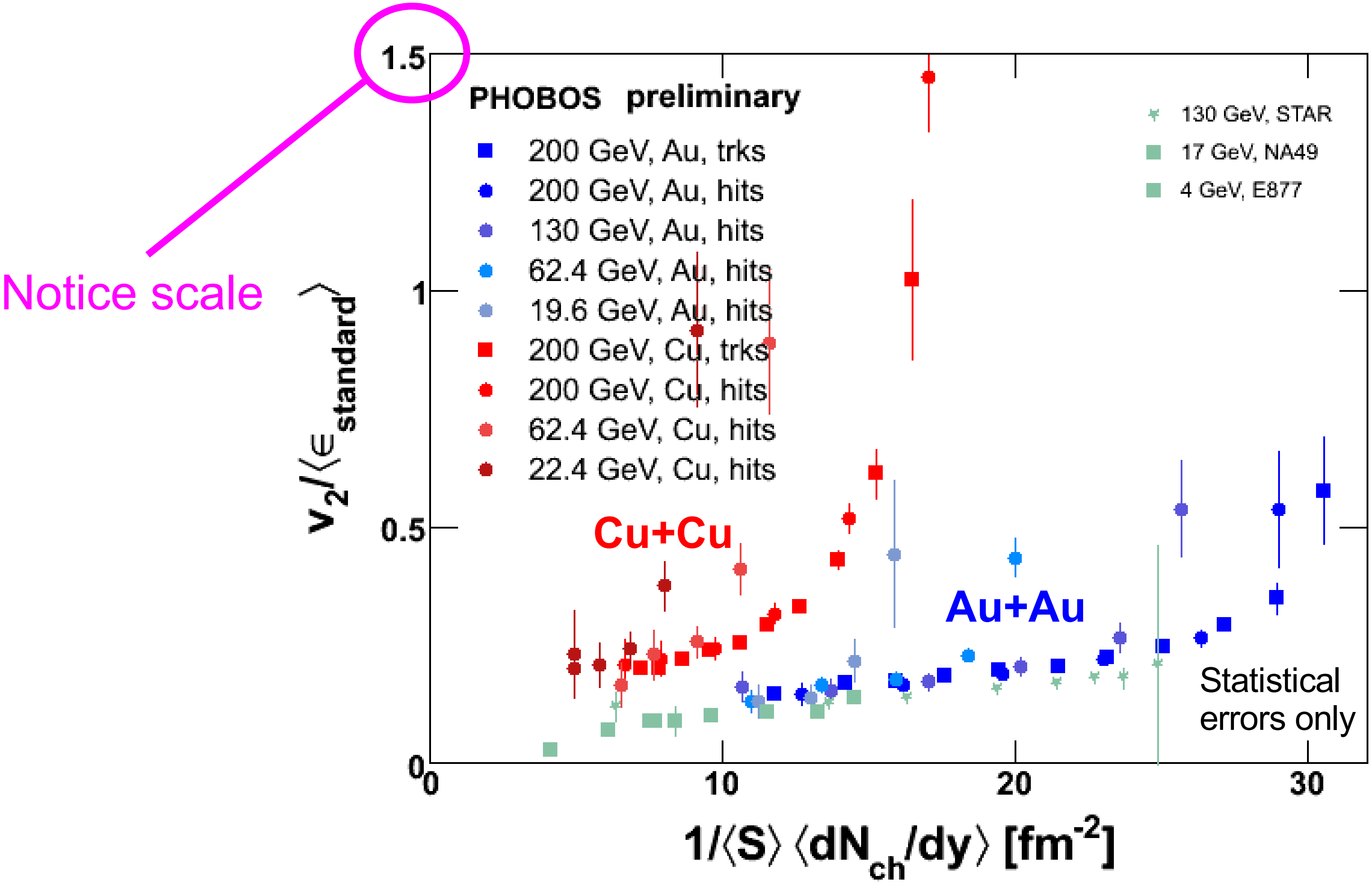
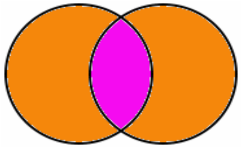
Elliptic flow scaled with $\varepsilon_{standard}$





Small print:

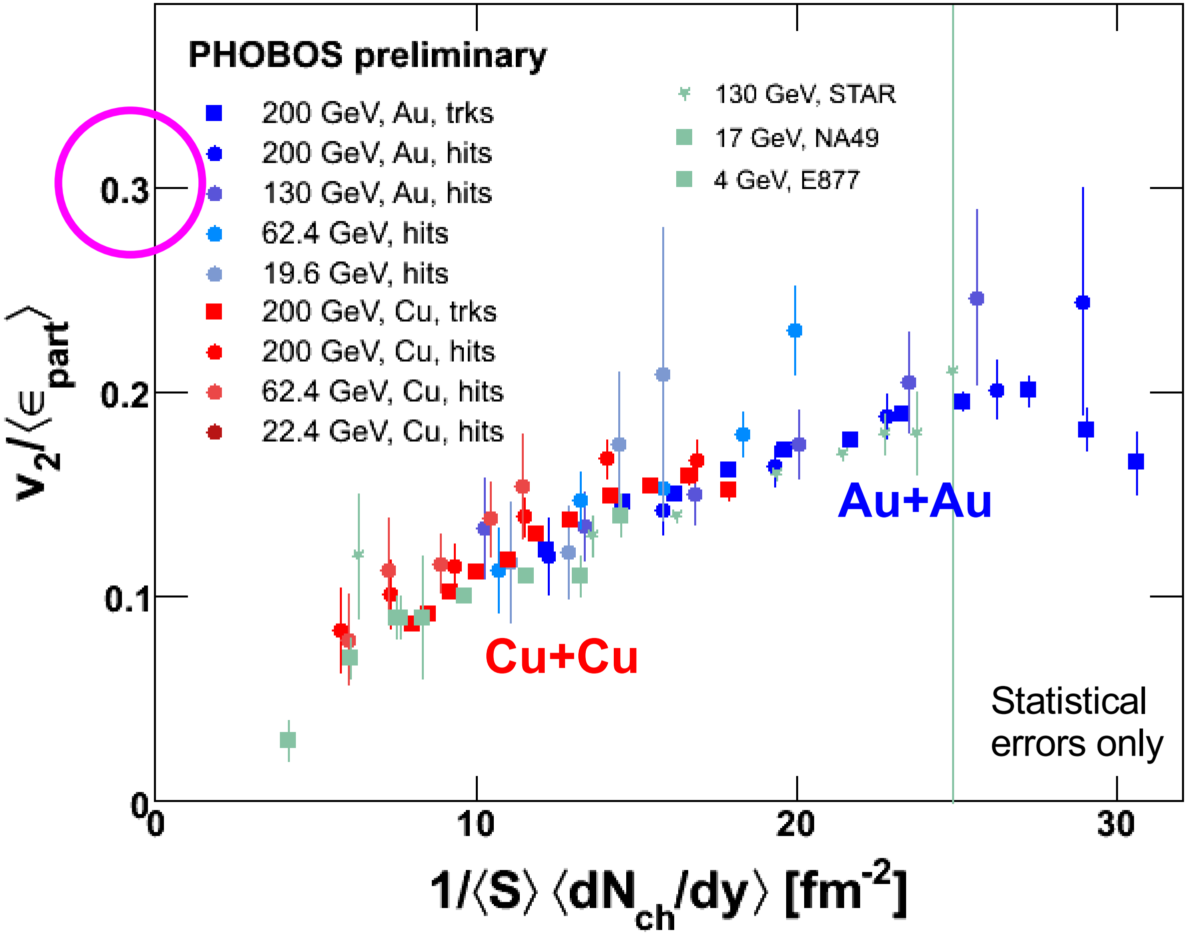
- Scale $v_2(\eta)$ to $v_2(y)$ (10% lower)
- Scale $dN/d\eta$ to dN/dy (15% higher)
- S is overlap area (MC Glauber)

No scaling between Cu+Cu and Au+Au

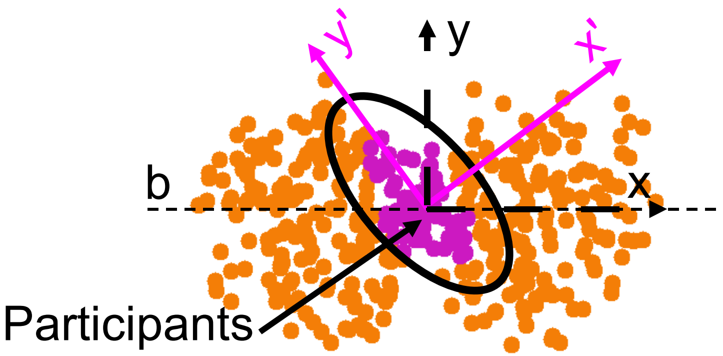
Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Heiselberg, Levy, PRC 59 2716, (1999)
Voloshin, Poskanzer, PLB 474 27 (2000)
STAR, PRC 66 034904 (2002)

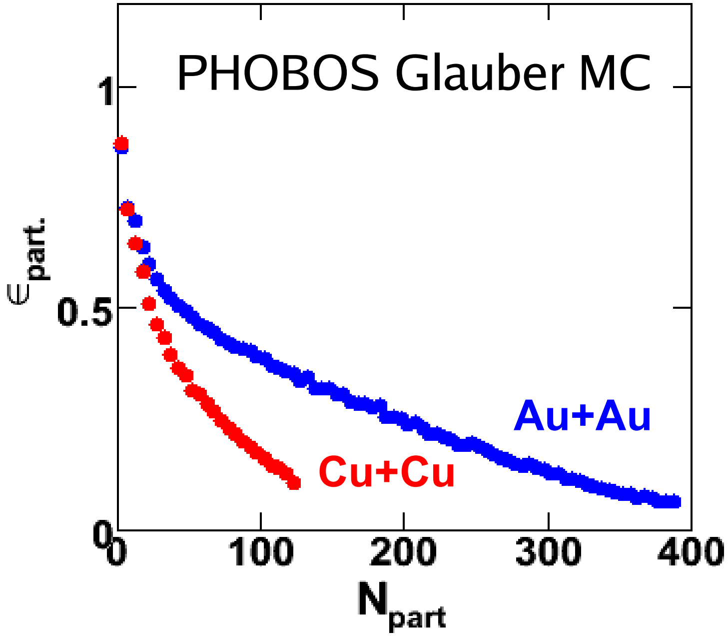
Elliptic flow and participant eccentricity



Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



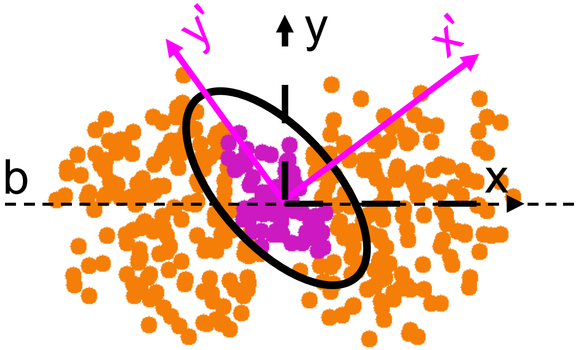
Approximate scaling between Cu+Cu and Au+Au

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Expected elliptic flow fluctuations

Elliptic flow seems to be developed event-by-event with respect to the overlap region

$$v_2 \sim \epsilon_{part}$$

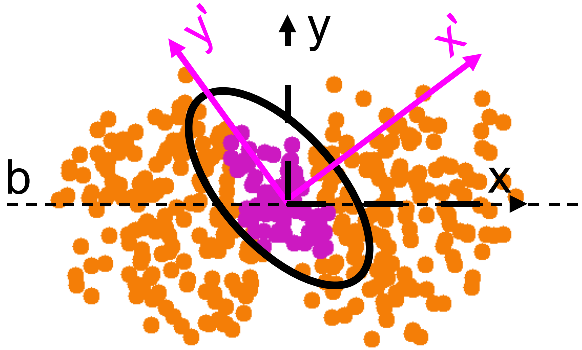


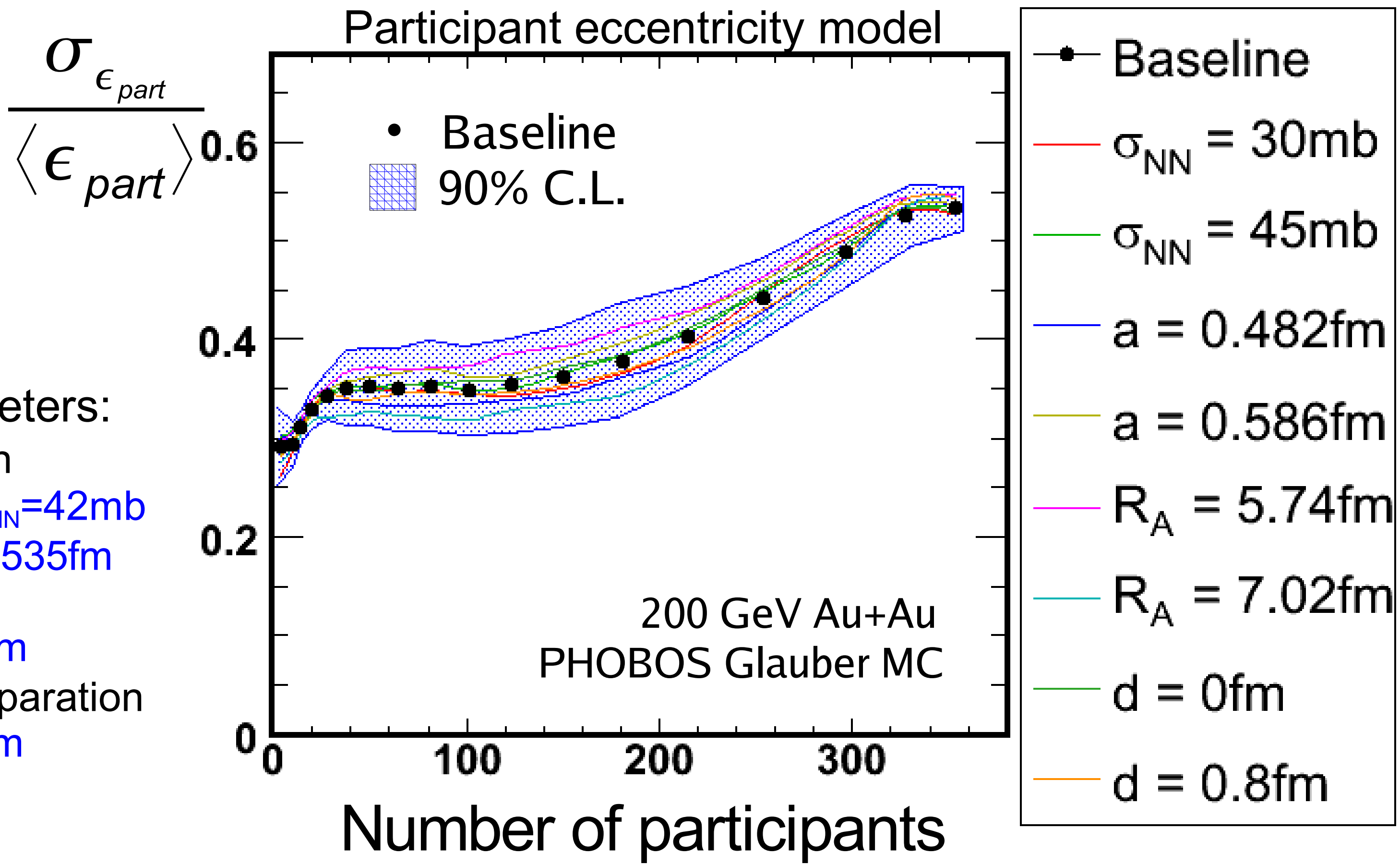
$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

Expected elliptic flow fluctuations

Elliptic flow is developed event-by-event with respect to the overlap region

$$v_2 \sim \epsilon_{part}$$



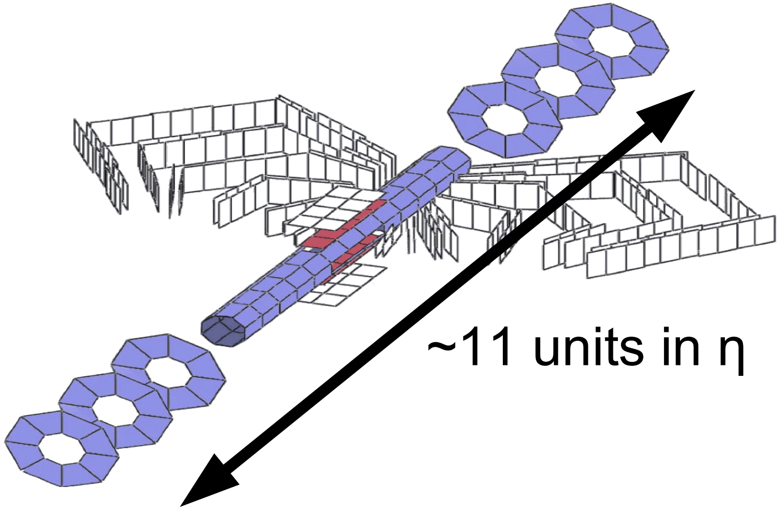


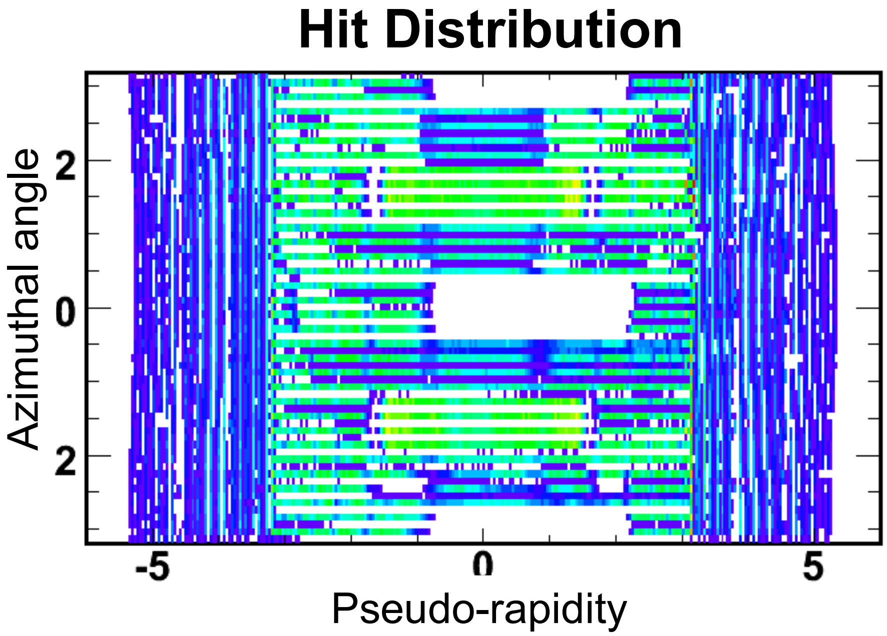
Baseline parameters:

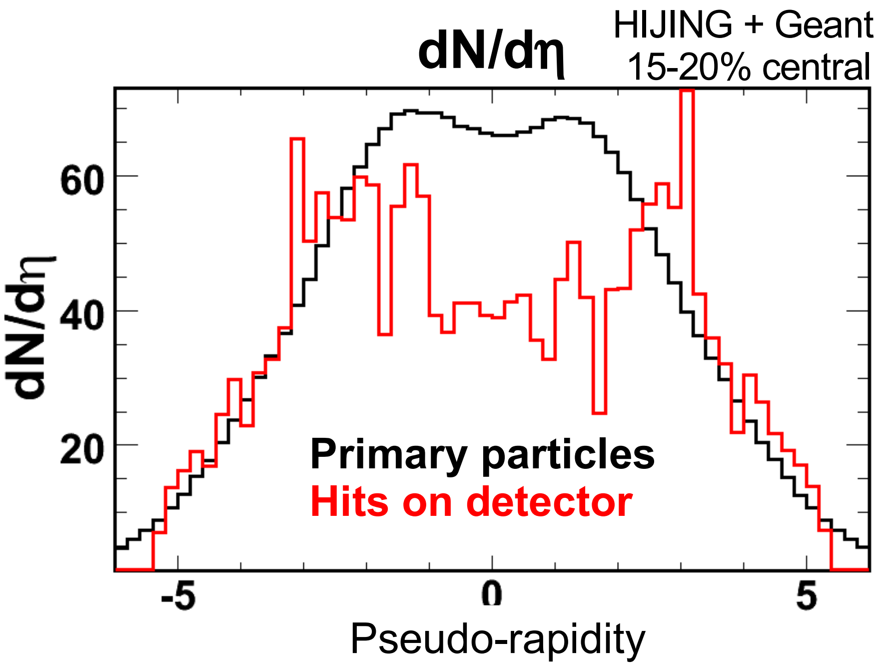
- Nucleon-nucleon cross section: σ_{NN}=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: R_A=6.38fm
- Inter-nucleon separation distance: d=0.4fm

Event-by-event measurement of v_2^{obs}

- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information in event to determine event-by-event a single value for v_2^{obs}

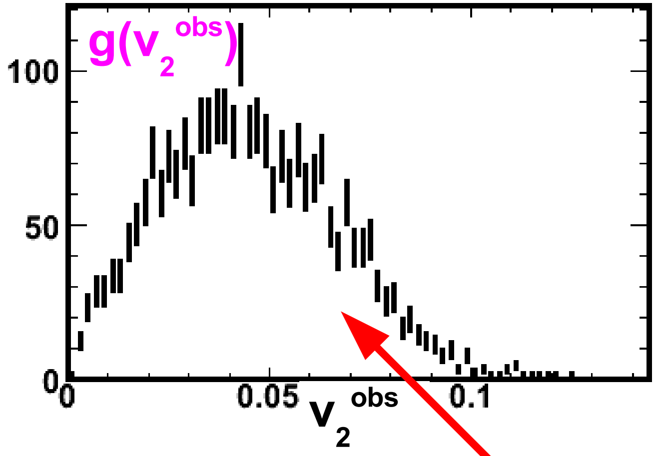




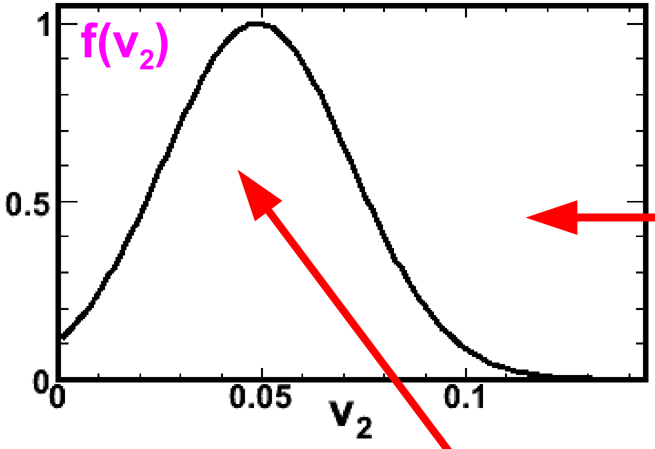


Measuring elliptic flow fluctuations

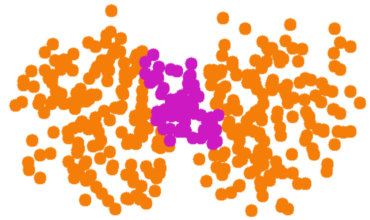
Observed v_2 distribution



True v_2 distribution

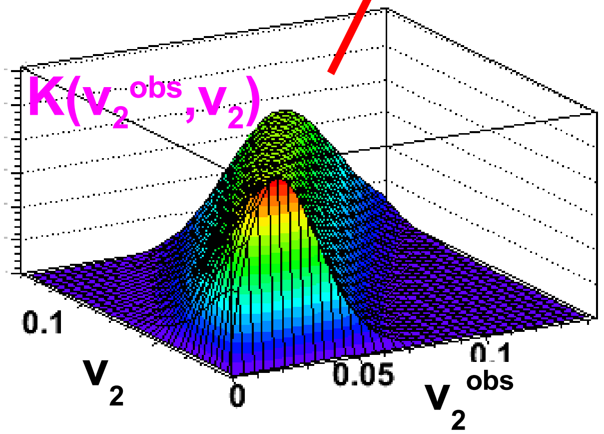


Source of v_2 fluctuation

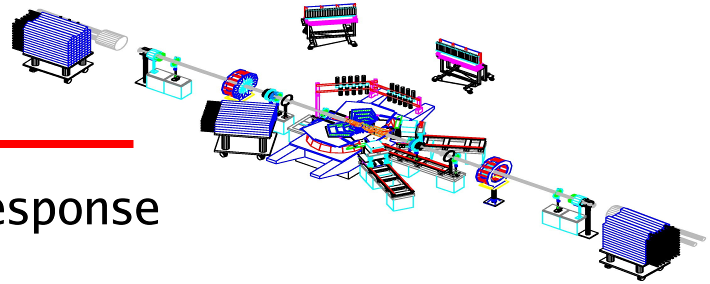


$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Kernel



Detector response

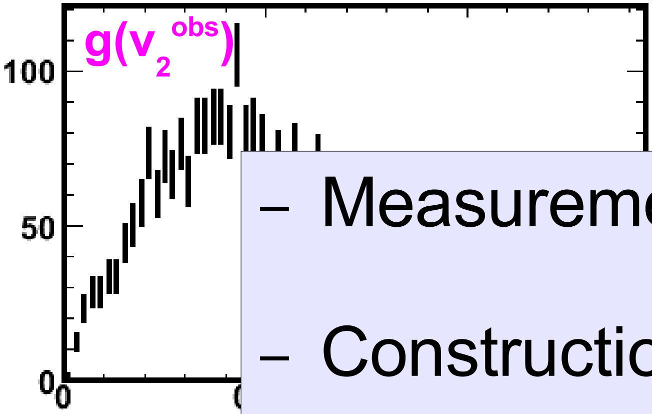


Kernel

- Detector and acceptance effects
- Finite-number fluctuations
- Multiplicity fluctuations

Measuring elliptic flow fluctuations

Observed v_2 distribution



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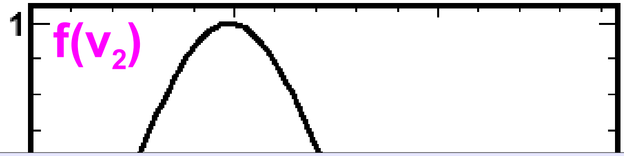
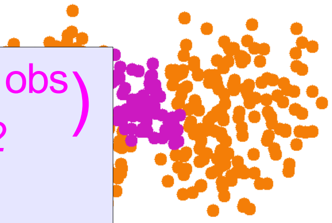


Source of v_2 fluctuation



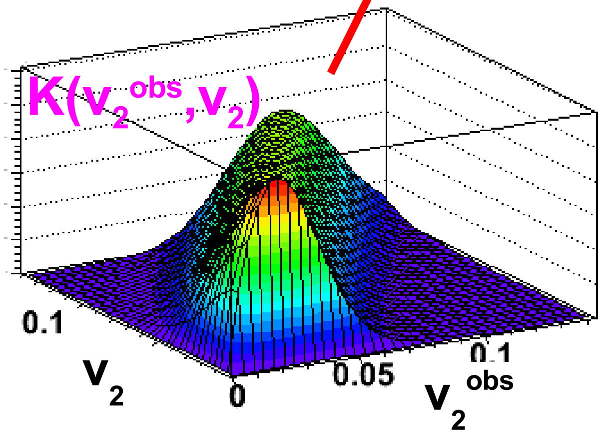
- Measurement of v_2^{obs} event-by-event: $g(v_2^{obs})$
- Construction of the kernel: $K(v_2^{obs},v_2)$
- Extraction of dynamical fluctuations: $f(v_2)$

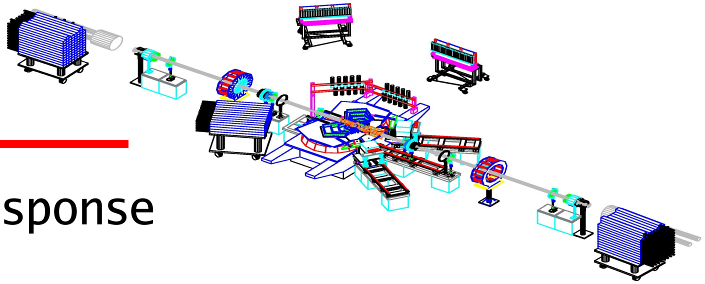
Kernel

- Detector and acceptance effects
- Finite-number fluctuations
- Multiplicity fluctuations

$$g(v_2^{obs})=\int_0^1 K(v_2^{obs},v_2)\,f(v_2)\,dv_2$$

Kernel



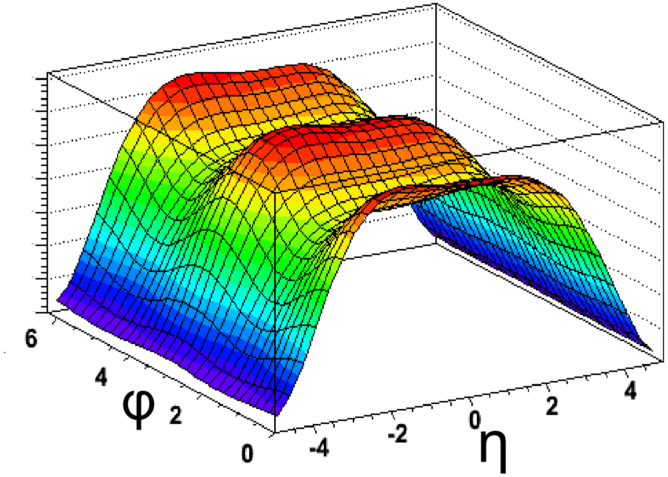


Detector response

Event-by-event measurement of v_2^{obs}

- Event-by-event measurement of v_2^{obs}
  - Deal with acceptance effects
  - Use all available hit information

Probability distribution function



- Probability distribution function for hit positions:

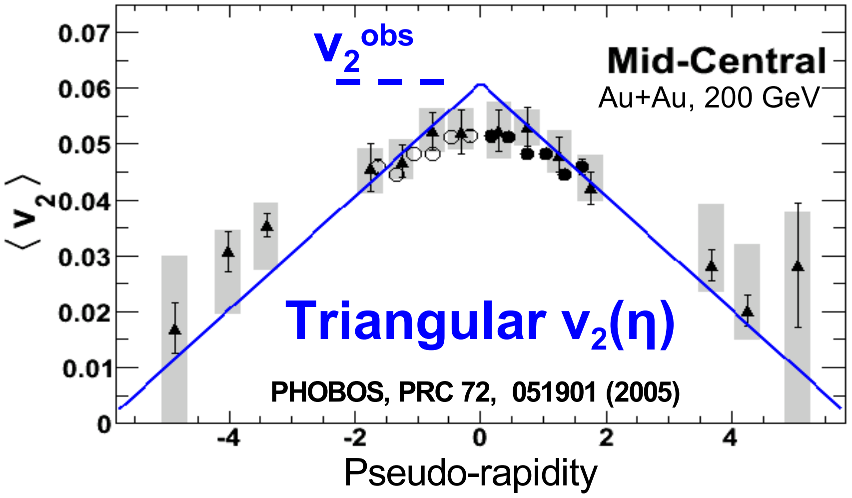
$$P(\eta,\phi;\ v_2^{obs},\phi_0)=\underbrace{p(\eta)}_{\text{Normalization incl. acceptance}}\underbrace{[1+2v_2(\eta)\cos(2\phi-2\phi_0)]}_{\text{Probability of hit in }(\phi,\eta)}$$

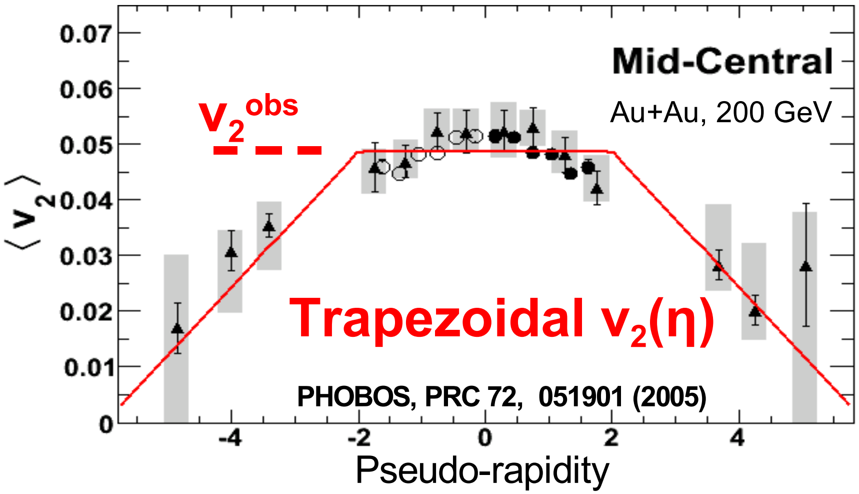
  - Maximize the likelihood function to obtain v_2^{obs} and φ^0 (event plane angle)

$$L(v_2^{obs},\ \phi_0)=\prod_{i=1}^{n}P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

See Burak Alver's Poster 16, QM2006
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Event-by-event measurement of v_2^{obs}





$$P(\eta,\phi;\, v_2^{obs},\phi_0) = p(\eta)[1+2\underline{v_2(\eta)}\cos(2\phi-2\phi_0)]$$

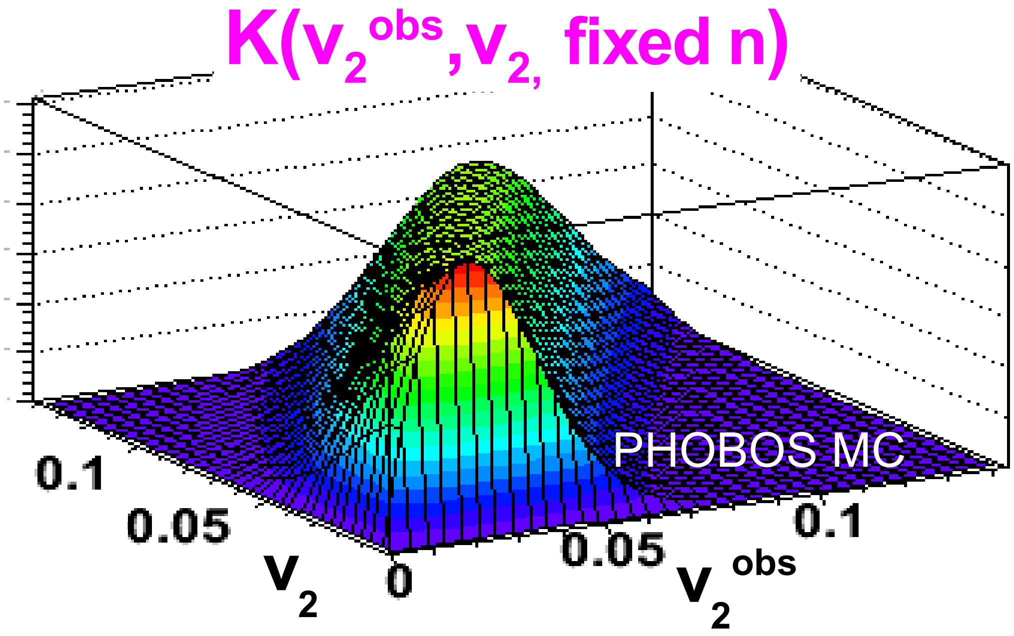
Use known, measured shape

Analysis is run completely independent on triangular and trapezoidal shape. Results are averaged at the end.

See Burak Alver's Poster 16, QM2006
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Determining the kernel

- “Measure” and record the v_2^{obs} distribution in bins of v_2 and multiplicity (n) from large MC samples
  - 1.5·10^6 HIJING events
  - Modified φ to include triangular or trapezoidal flow



- Fit response function (ideal case)

$$K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs}+v_2^2}{2\sigma^2}\right)} I_0\left(\frac{-v_2^{obs} v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

- Changed to account for detector effects

$$v_2 \rightarrow (An+B) v_2 \qquad \sigma = \frac{C}{\sqrt{n}} + D$$

(suppression) (finite resolution)

See Burak Alver's Poster 16, QM2006
B.Alver et.al. (PHOBOS), nucl-ex/0608025

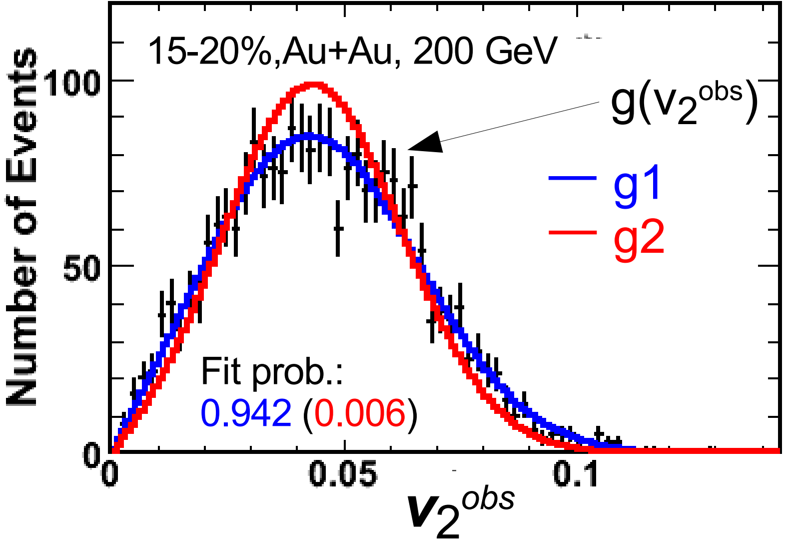
Extracting dynamical fluctuations

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs}, v_2)}_{\text{Constructed from MC}} \underline{f(v_2)} dv_2$$

Gaussian Ansatz:

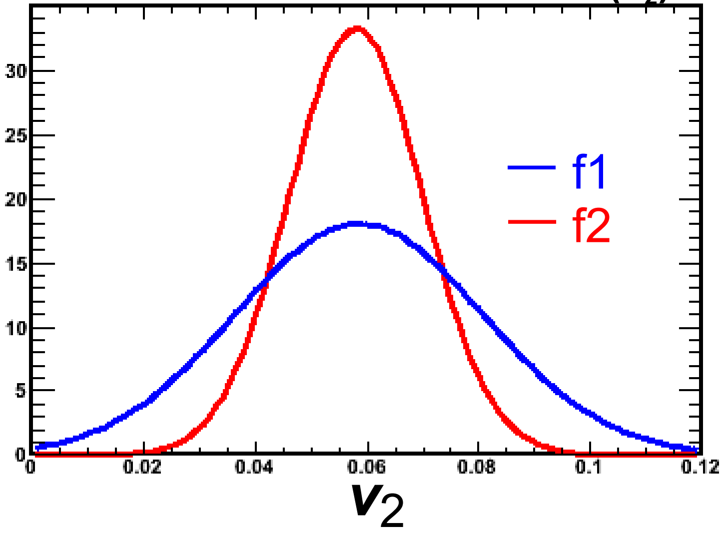
$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

Comparison with data $g(v_2^{obs})$



Use kernel + integrate

Different trials for Ansatz $f(v_2)$



Compare expected $g(v_2^{obs})$ for trials with data:

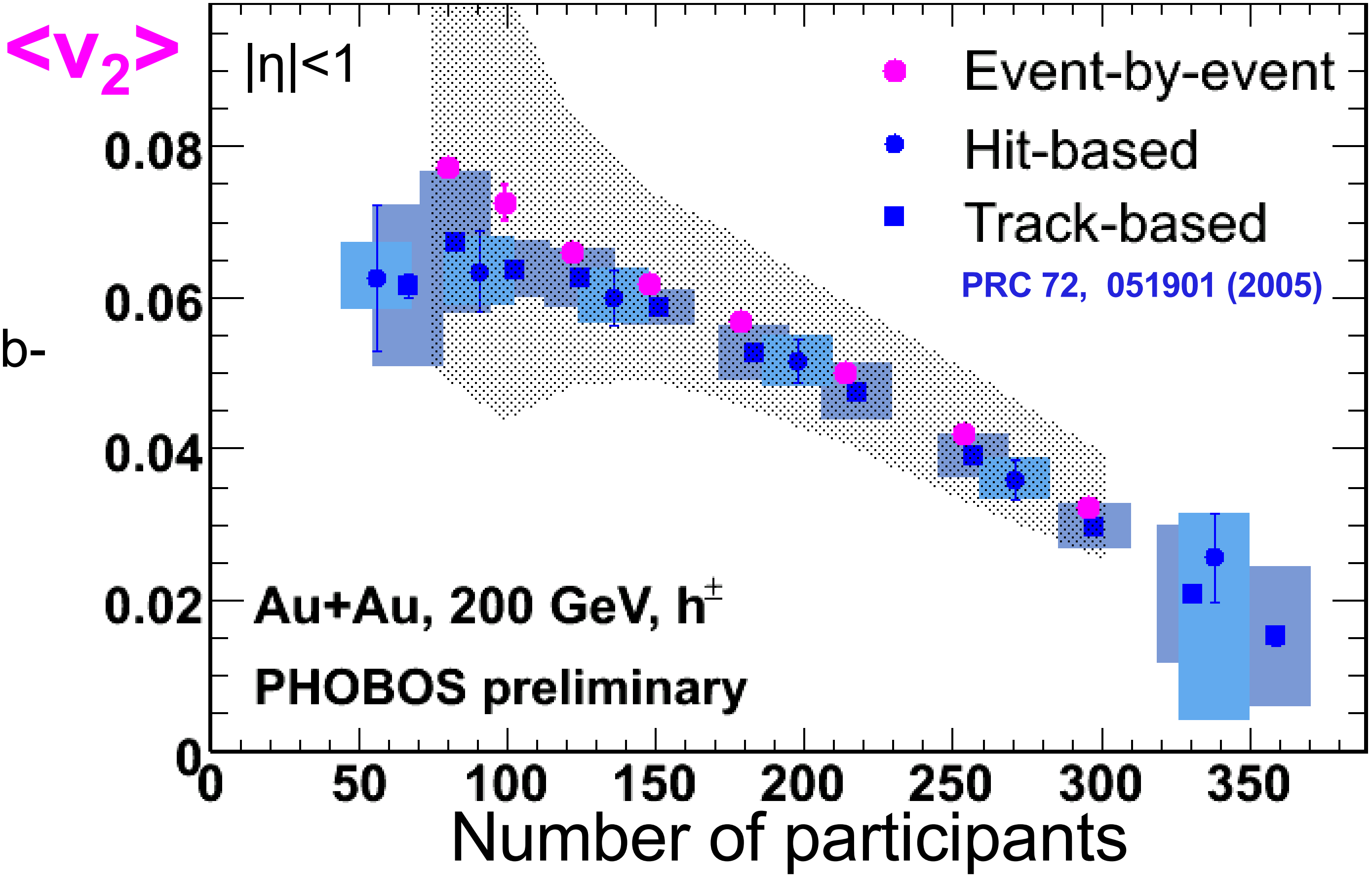
Maximum-Likelihood fit → $<v_2>$ and σ_{v2}

See Burak Alver's Poster 16, QM2006
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Event-by-event mean v_2 vs published results

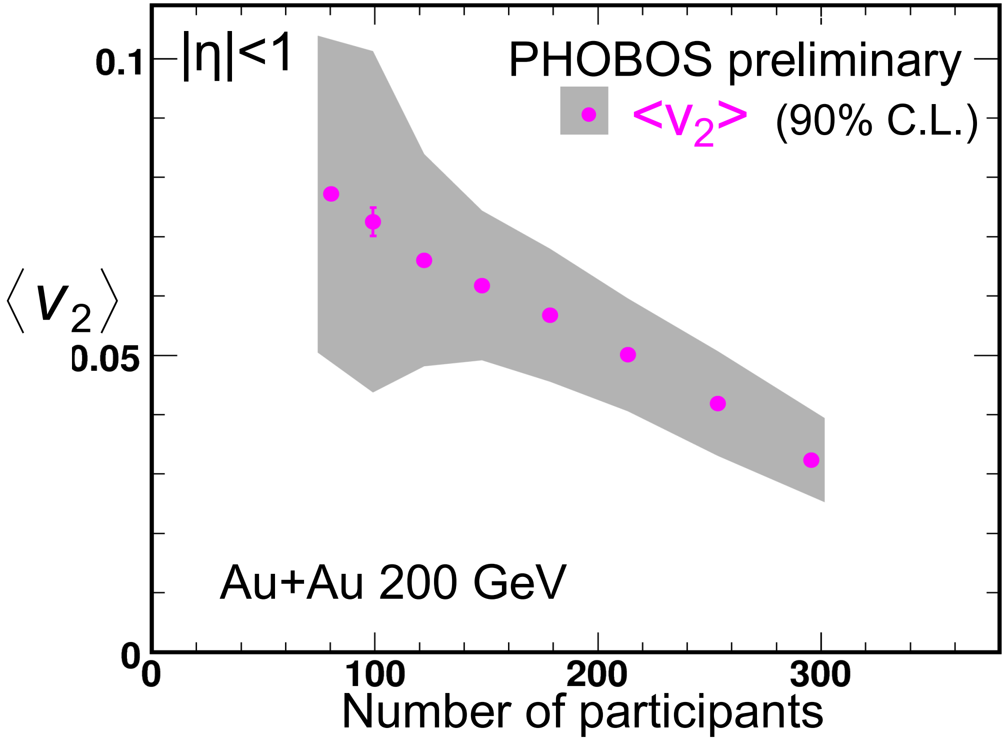
- Standard methods
  - Averaged over events to measure the mean
  - Hit- and track-based
  - Use reaction plane sub-event technique
- Event-by-event:
  - PR04 Au+Au data
    - No magnetic field
    - 500.000 events
    - 10 vertex bins ($-10cm<z_{vertex}<10cm$)
  - **Relate v_2^{obs} to $\langle v_2 \rangle$:**

$$\langle v_2 \rangle(|\eta|<1) = 0.5 \times (11/12 \langle v_2^{triangular} \rangle + \langle v_2^{trapezodial} \rangle)$$

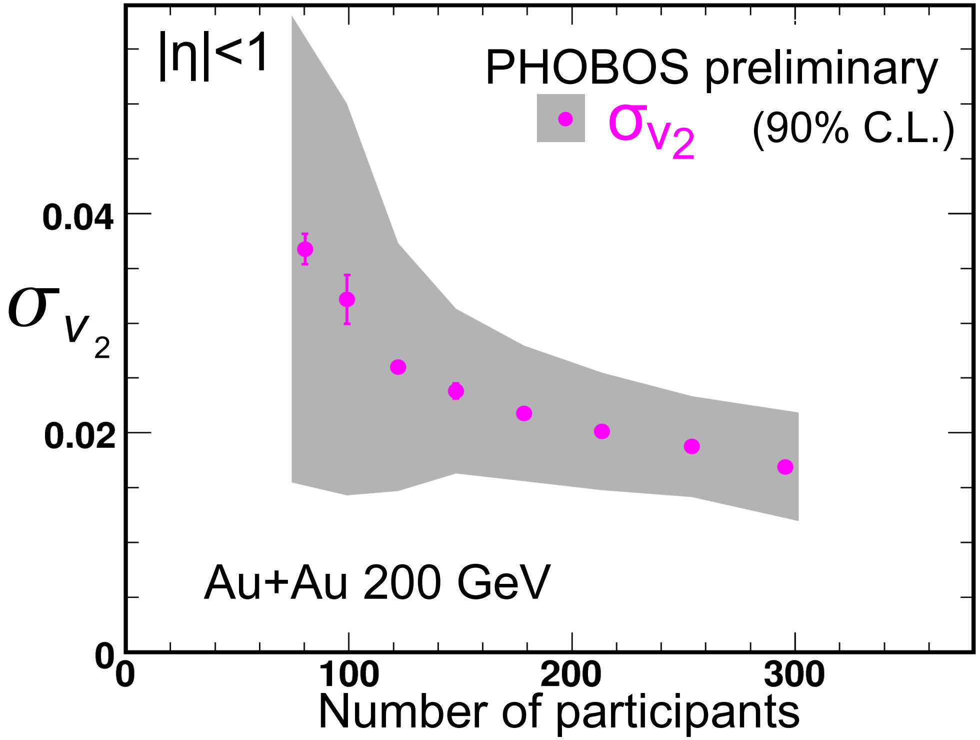


Very good agreement of the event-by-event measured mean v_2 with the hit- and tracked-based, event averaged, published results

Elliptic flow fluctuations: $\langle v_2 \rangle$ and σ_{v_2}



Mean elliptic flow



Dynamical flow fluctuations

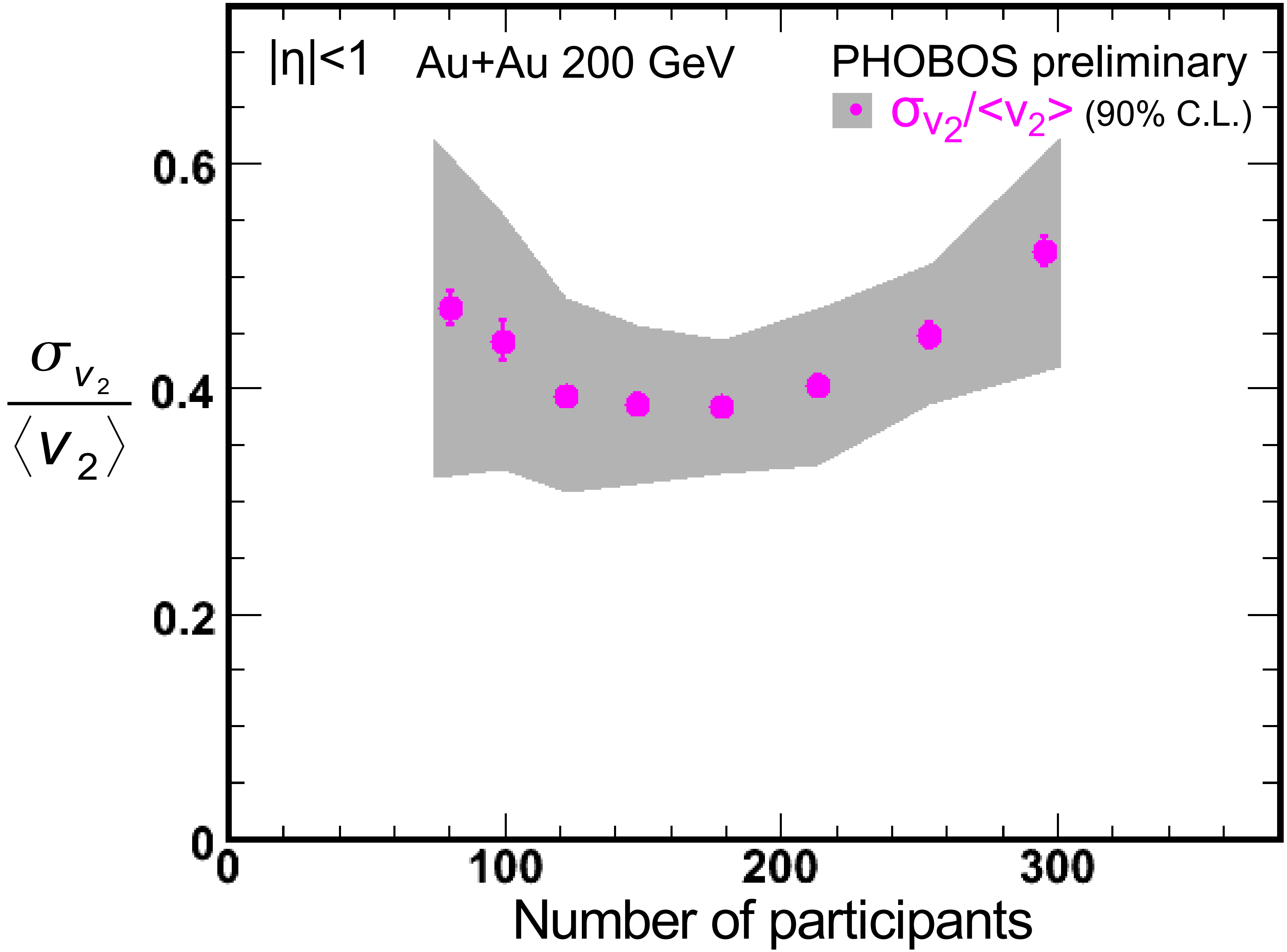
<u>Systematic errors:</u>

- Variation in η-shape
- Variation of $f(v_2)$
- MC response
- Vertex binning
- Φ_0 binning

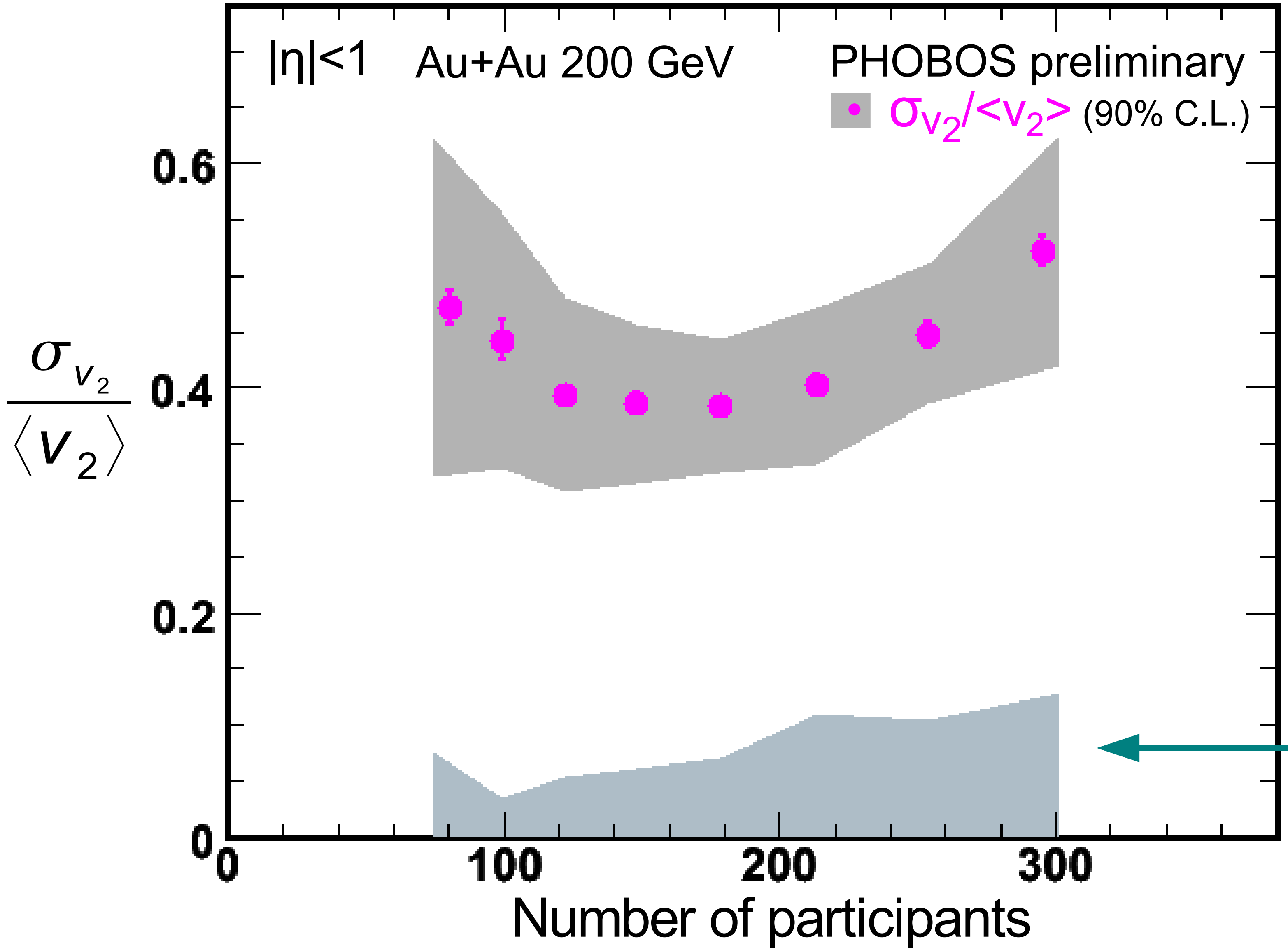
"Scaling" errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/\langle v_2 \rangle$

see next slide →

Elliptic flow fluctuations: $\sigma_{v_2}/\langle v_2\rangle$

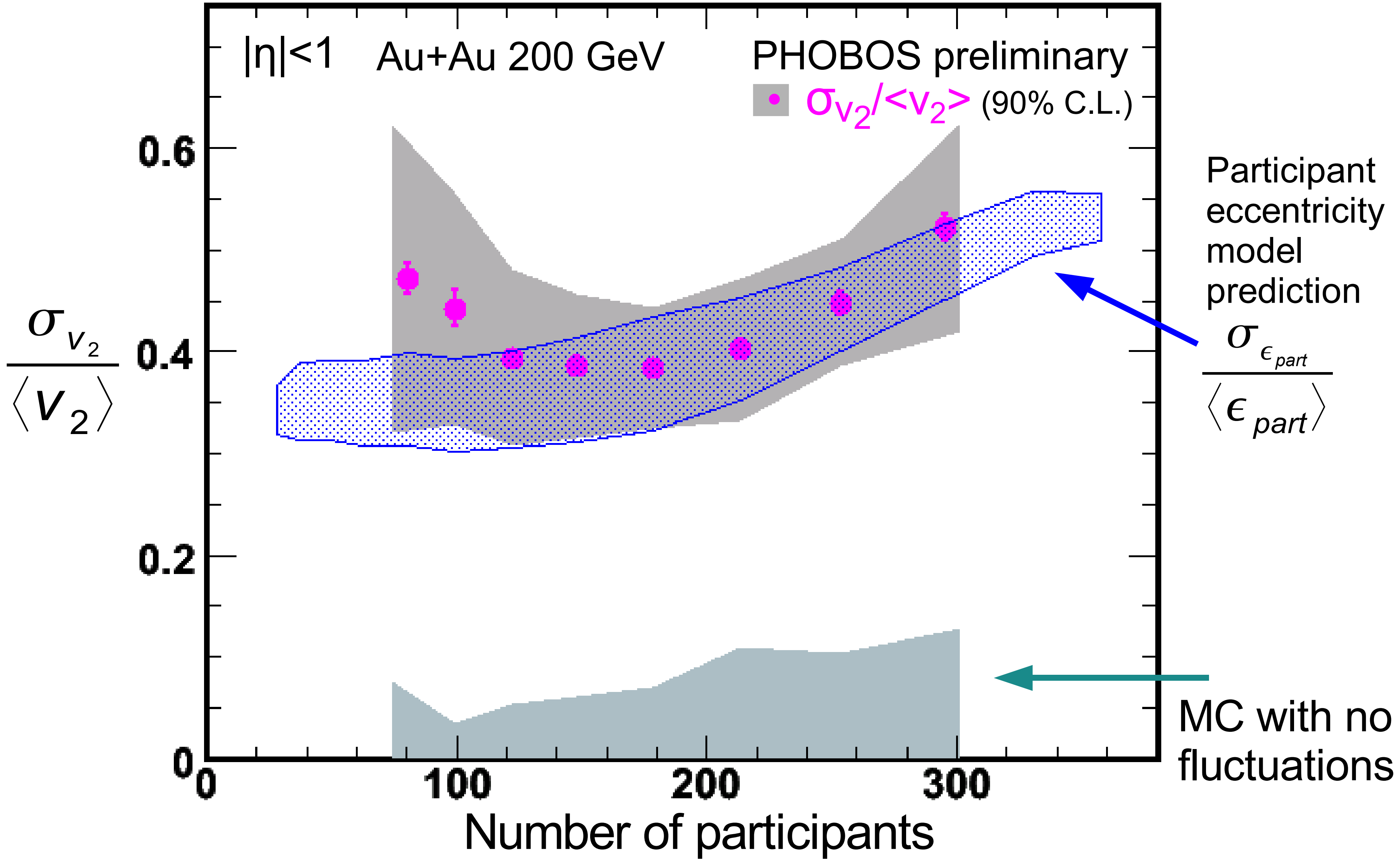


Elliptic flow fluctuations: $\sigma_{v_2}/\langle v_2\rangle$



MC with no fluctuations

Participant eccentricity compared to data



Participant eccentricity model prediction $\frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part}\rangle}$

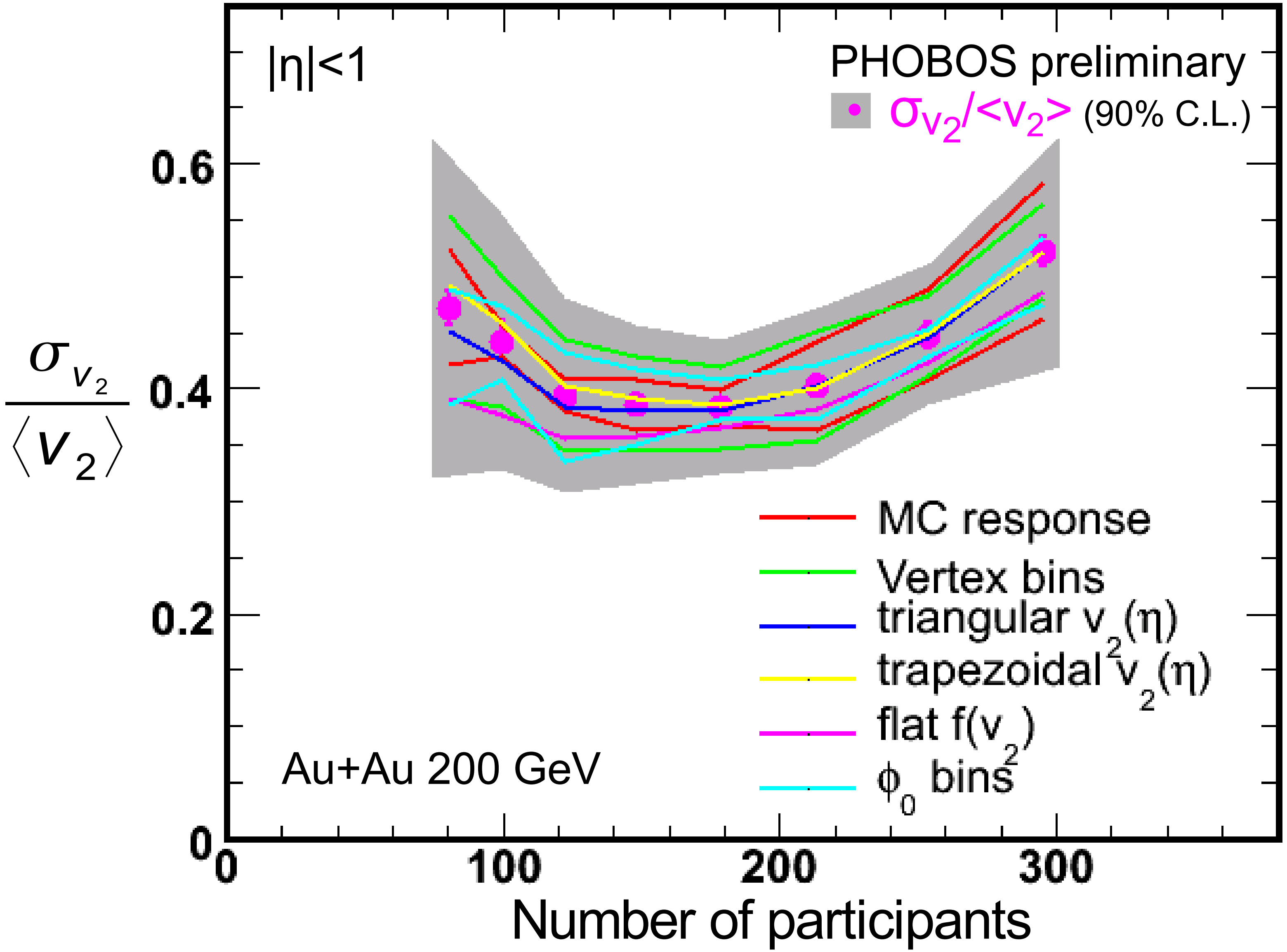
MC with no fluctuations

Summary

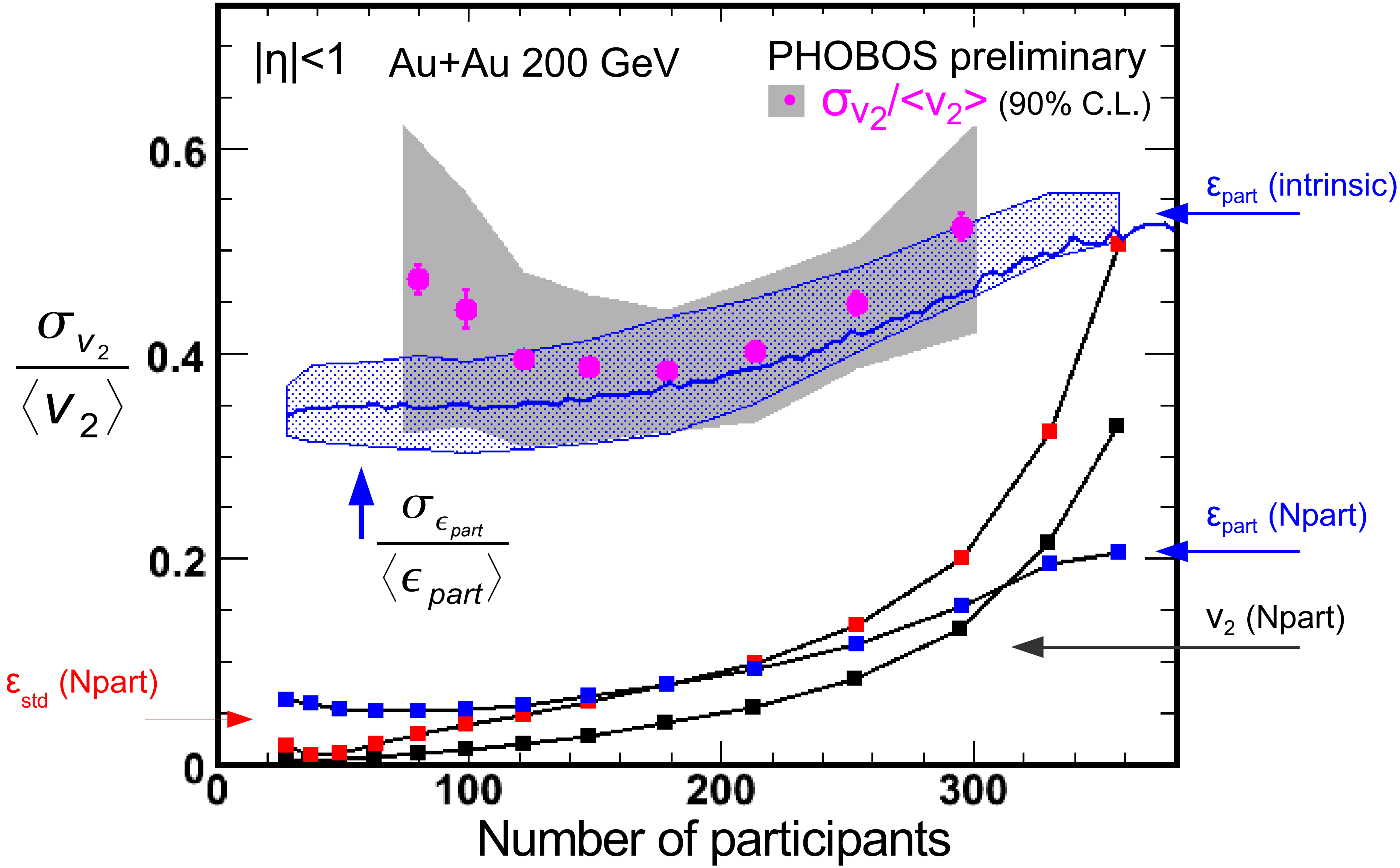
- PHOBOS has measured elliptic flow fluctuations in peripheral to semi-central Au+Au collisions at 200 GeV
  - Absolute fluctuations (σ_{v2}) are about 0.02
  - Relative fluctuations ($\sigma_{v2}/\langle v_2 \rangle$) are about 40%
  - The participant eccentricity predictions for the magnitude of the relative fluctuations are in striking agreement with the measurement
- Modeling of interaction points with MC Glauber interpreted event-by-event, the participant eccentricity model, appears to be able to explain both
  - The magnitude of the mean elliptic flow in Cu+Cu wrt Au+Au
  - The magnitude of the elliptic flow fluctuations in Au+Au

Backup slides

Systematic error sources

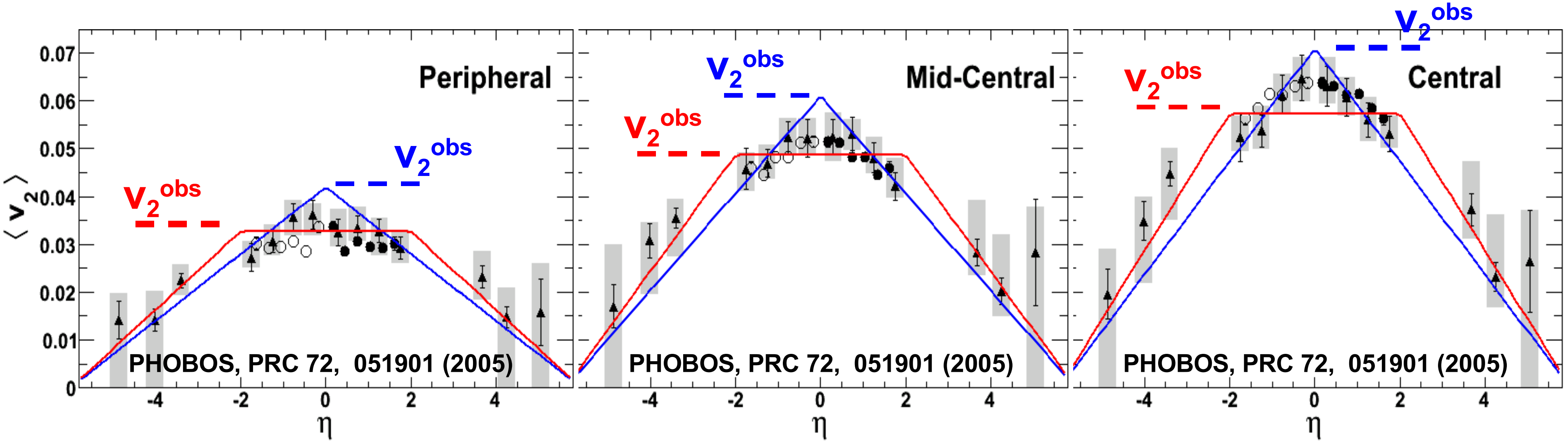


Contributions from Npart fluctuations



Fluctuations in Npart are calculated by folding f(Npart) with a Gaussian with mean and sigma as obtained from the centrality selection used in PHOBOS

Event-by-event measurement of v_2^{obs}



$$P(\eta,\phi;\, v_2^{obs},\phi_0)=p(\eta)[1+2\underline{v_2(\eta)}\cos(2\phi-2\phi_0)]$$

↑

Use known, measured shape

See Burak Alver's Poster 44, QM2006
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Fluctuations estimates: nucl-th/0208052

- Estimation of δv_2 in e-by-e analysis using the RP method
  - Statistical noise(*) $\delta_{v_2}^{stat} = 1/\langle R \rangle / \sqrt{2\langle N \rangle} \approx 0.01 - 0.07$
  - Impact para (Npart) (**) $\delta_{v_2}^{b} = f(b)\, \delta_{Np} \quad [0.024\ for\, b = 5\mathrm{fm}]$
  - Multiplicity fluctuation(*) $\delta_{v_2}^{N} = 0.4 \langle v_2 \rangle / \sqrt{\langle N \rangle} = 0.56 \langle v_2 \rangle \langle R \rangle \delta_{v_2}^{stat}$
  - Cluster formation $\delta_{v_2}^{cl} = 0.4 \langle v_2 \rangle / \sqrt{\langle N \rangle} (\sqrt{1 - f} + \sqrt{kf}) \approx 2\, \delta_{v_2}^{N}$
  - Fluctuations due to filamentation instability
    - Random process occurring dominantly for b->0
    - No prediction about the distribution: $0 < v_2 < 0.46$
- Participant eccentricity $\delta_{v_2}^{epart} = 1/\langle v_2 \rangle \delta_\epsilon / \langle \epsilon \rangle \approx 0.02$

(*) taken out by kernel, (**) estimated to be small

S.Mrowczynski, E.Shuryak, nucl-th/0208052

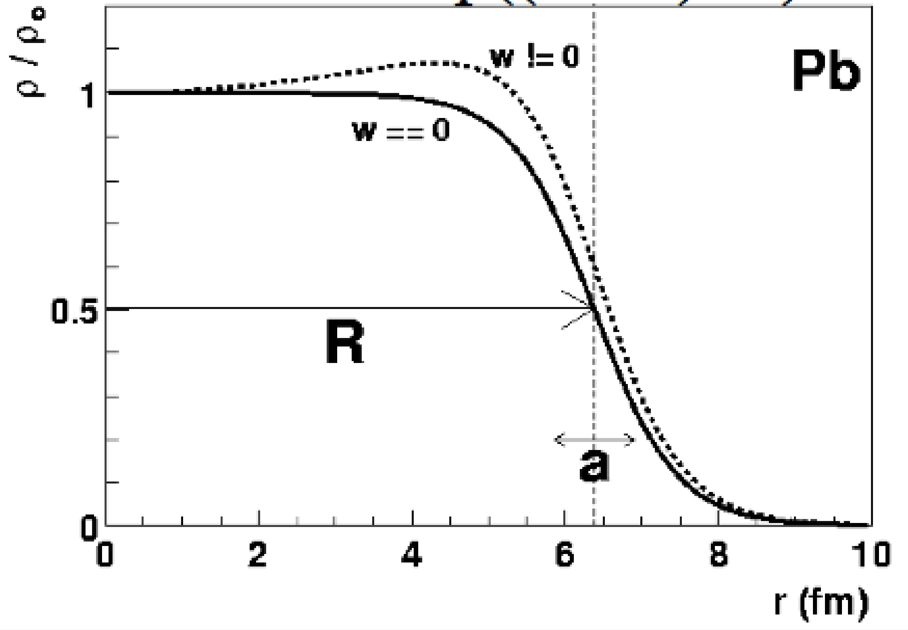
Non-flow contributions to flow fluctuations

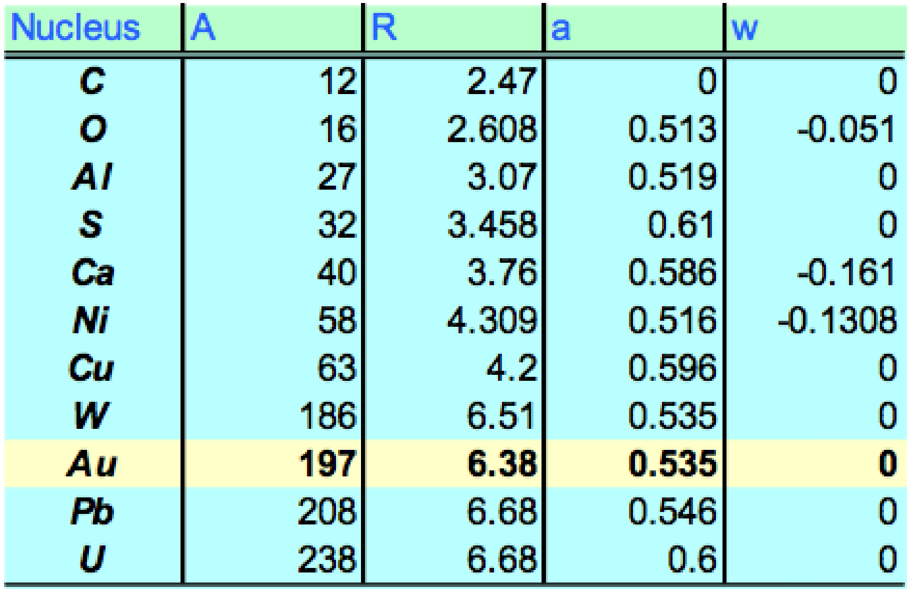
- Non-flow correlations mimic dynamical fluctuations and will contribute to the width of the v_2 distribution
  - The resolution of our method depends on the kernel
    - Modified Hijing: particle multiplicity defines the resolution
    - Data (AMPT): clusters flow and therefore the cluster multiplicity determines the resolution
  - The fluctuations we measure are real (present at particle level) but might not be the ones we are after
- Kernel could compensate for non-flow effects if they are correctly described by the MC used to construct it
  - Construct and tune MC on data
  - Two-particle correlation measurements can be used as input to disentangle the different contributions

Glauber parameters

Systematic Source	Standard	How Much We Vary
Nucleon-nucleon cross-section	42 mb (for 200GeV)	30 mb (<20GeV) 45 mb (>200GeV)
Nuclear skin depth	0.535fm(Au)0.596fm(Cu)	±10%
Nuclear radius	6.38fm (Au)4.2fm (Cu)	±10%
Minimum nucleon separation (center-to-center)	0.4fm (like HIJING)	0fm 0.8fm

$$\rho(r) = \frac{\rho_0\left(1 + wr^2 / R^2\right)}{1 + \exp((r - R)/a)}$$





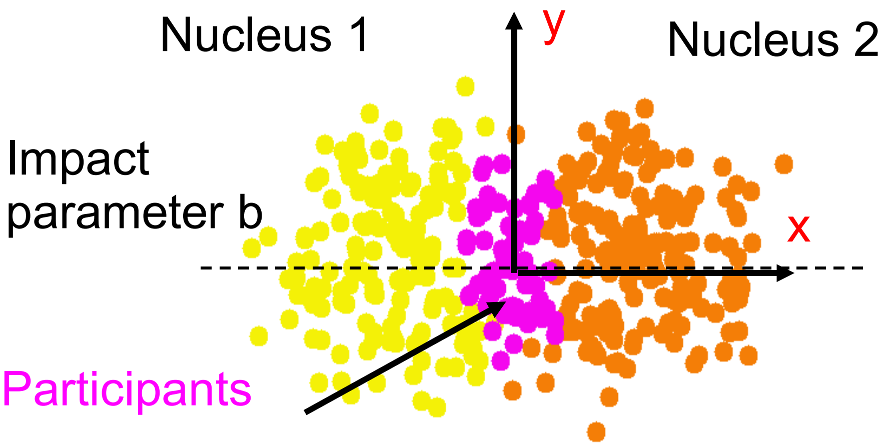
Nucleus	A	R	a	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

H. DeVries, C.W. De Jager, C. DeVries, 1987

Glauber MC

- Glauber Monte Carlo
  - Radial distribution of nucleons (in nucleus) drawn from Wood-Saxon distribution
  - Isotropic angular distribution
  - Separate by impact parameter
  - Nucleons travel on straight-line paths and interact inelastically when

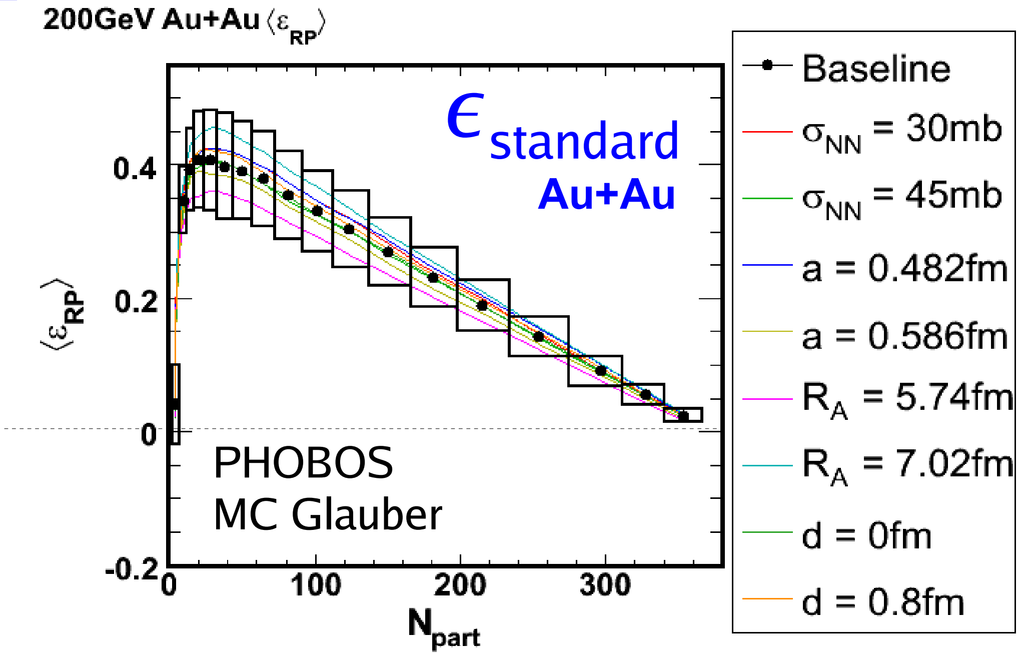
$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}<\sqrt{\sigma_{NN}/\pi}$$

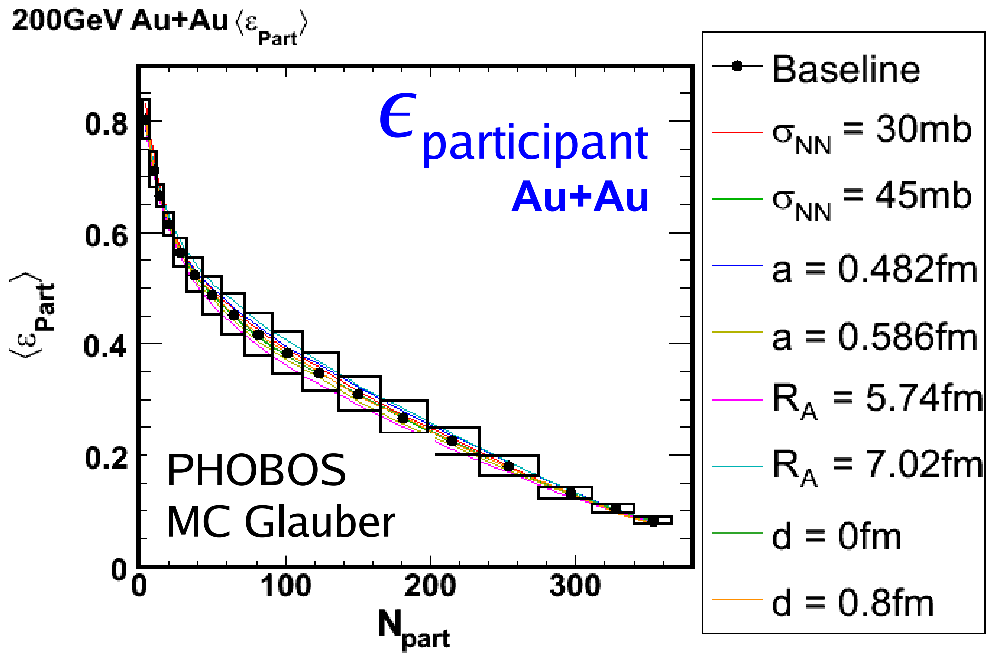


- Centrality of collision
  - #Participants
    - Nucleons that interact at least once
  - Related to cross section and impact parameter range
- Eccentricity of collision zone
  - Given by participants position distributions

Eccentricity: $\epsilon_{std}=\frac{\sigma_y^2-\sigma_x^2}{\sigma_y^2+\sigma_x^2}$

Robustness with geometry variables





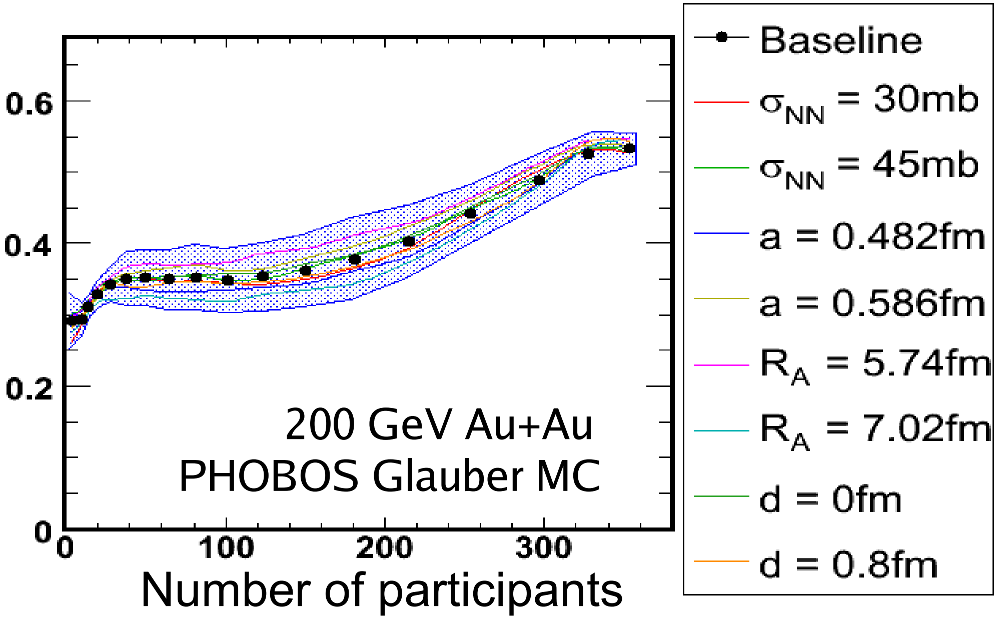
- Variation of
  - Nucleon-nucleon cross section (30-45mb)
  - Nuclear radius (±10% from the nominal value)
  - Skin depth (0.482-0.586fm)
  - Minimum separation distance between nucleons (d=0-0.8fm)

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R)/a)}$$

$\epsilon_{\mathrm{participant}}$ even slightly more robust than $\epsilon_{\mathrm{standard}}$

Expected elliptic flow fluctuations

$$\frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$



$$\frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$