Even the "minimal" viscosity matters at RHIC

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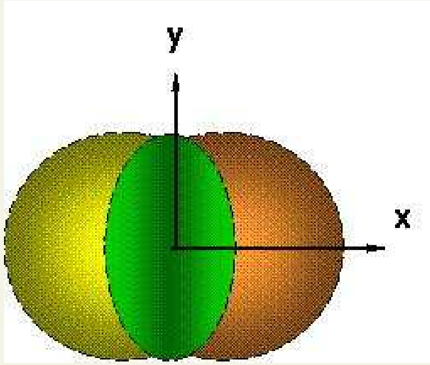
XIX$^{\underline{th}}$ Int. Conf. on Ultra-Relativistic Nucleus-Nucleus Collisions

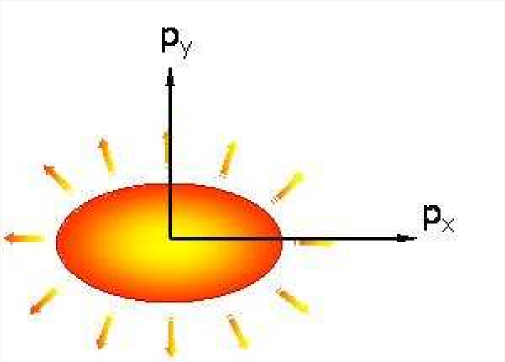
Nov 14-20, 2006, Shanghai Science Hall, Shanghai, China

- **remarkable collectivity at RHIC - “elliptic flow” (v_2)**

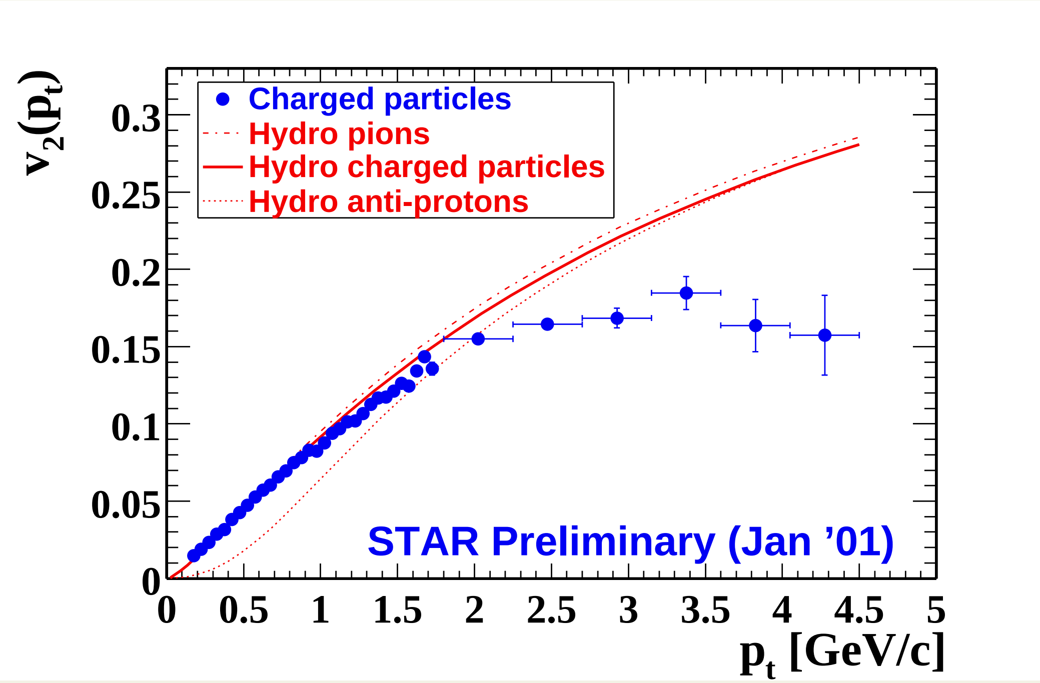
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



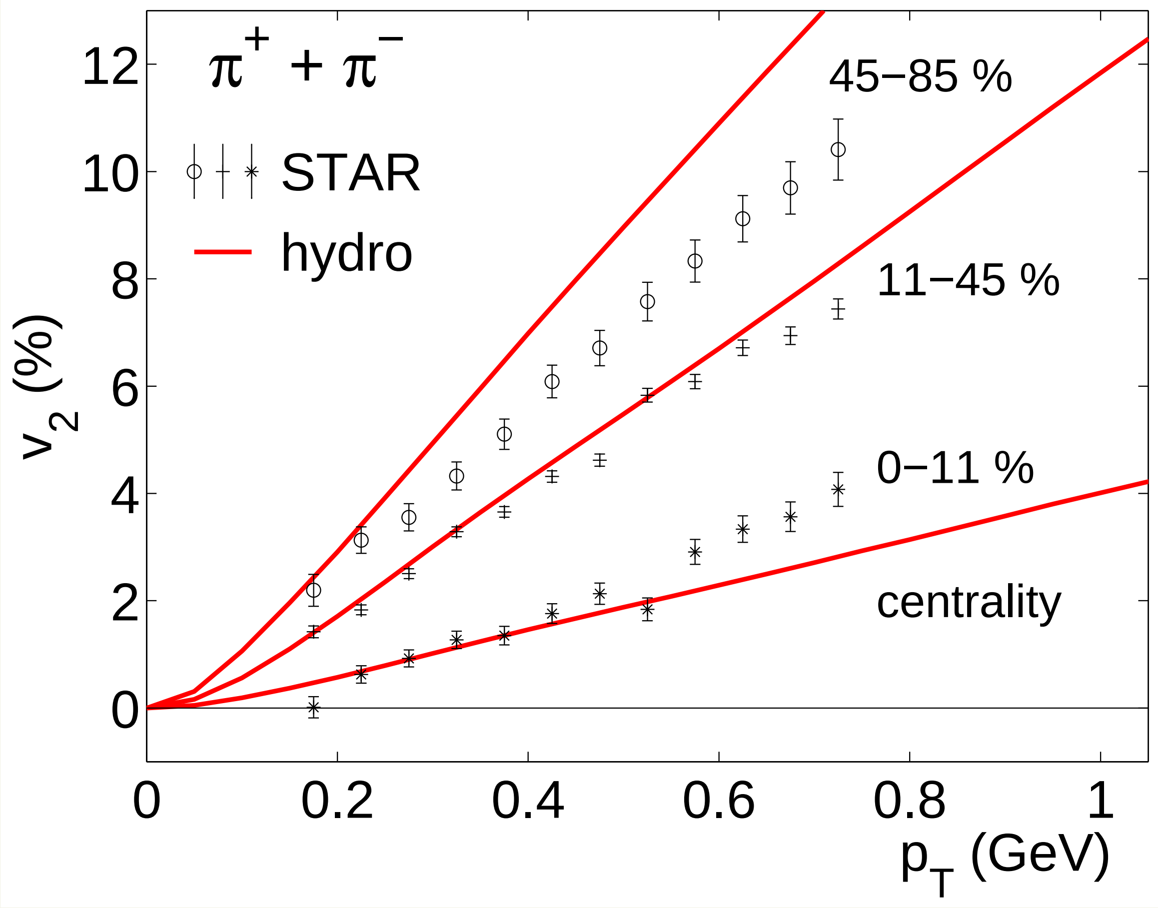
$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$



ideal hydro vs minbias Au+Au - good agreement for $p_T < 1.5$ GeV



a better microscope reveals $\sim 10 - 20\%$ deviations already at $p_T = 0.7$ GeV



but: ideal hydro ignores dissipation in both plasma and hadron stage

quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

$$\eta \approx 4/5 \cdot T/\sigma_{tr}$$

$$s \approx 4n$$

$\Rightarrow$ **minimal viscosity:** $\eta/s = \frac{\lambda_{tr} T}{5} \geq 1/15$

$\mathcal{N} = 4$ **SYM + gauge-gravity duality:** $\eta/s \geq 1/4\pi$

Policastro, Son, Starinets, PRL87 ('02) Kovtun, Son, Starinets, PRL94 ('05)

no general proof - but this might be a universal lower bound

$\Rightarrow$ **no ideal fluids - only "perfect" ones at best (those with minimal viscosity)**

[re-branding - '50s, '60s: perfect $\equiv$ ideal]

PHYSICAL REVIEW D VOLUME 31, NUMBER 1 1 JANUARY 1985

Dissipative phenomena in quark-gluon plasmas

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Transport coefficients of small-chemical-potential quark-gluon plasmas are estimated and dissipative corrections to the scaling hydrodynamic equations for ultrarelativistic nuclear collisions are studied. The absence of heat-conduction phenomena is clarified. Lower and upper bounds on the shear-viscosity coefficient are derived. QCD phenomenology is used to estimate effects of color-electric and -magnetic shielding, and nonperturbative antiscreening. Bulk viscosity associated with the plasma-to-hadron transition is estimated within the relaxation-time approximation. Finally, effects of dissipative phenomena on the relation between initial energy density and final rapidity density are estimated.

I. INTRODUCTION

Ultrarelativistic collisions of heavy nuclei offer the possibility of creating a new state of matter—the quark-gluon plasma. Current estimates[1] indicate that this plasma state could be reached by increasing the energy density of hadronic matter by an order of magnitude over that found in nuclei ($\epsilon_{\rm nuc} \sim 0.15$ GeV/fm^3). Unfortunately, such high energy densities are expected to be reached only for short times, $\Delta\tau \sim$ few fm/c, due to the rapid longitudinal expansion of the plasma.[1] However, if the expansion proceeds hydrodynamically,[2–5] then information about the interesting early stages of the collisions can be extracted from the *final* rapidity density dN/dy of hadrons. Specifically, the initial energy density ϵ_0 can be related to dN/dy via[6]

$$\epsilon_0 \propto (dN/dy)^{1+c_0^2}$$

where c_0^2 is the speed of sound. The above relation is ob-

κ for small-μ_B/T plasmas. A possible source of bulk viscosity ξ is considered due to the finite relaxation time of the plasma-to-hadron phase transition. Finally, we solve the Navier-Stokes equation with the scaling boundary condition to estimate the magnitude of entropy production due to dissipative process in ultrarelativistic nuclear collisions. We find that dissipative effects could reduce the estimated initial energy density by a few GeV/fm^3 relative to ideal hydrodynamics estimates.

II. HEAT CONDUCTION

In this section we derive an expression for heat conduction κ including both quark and gluon degrees of freedom. The gluon contribution to κ has been derived in Ref. (11). However, we show that the quark contribution is singular in the limit of zero baryon density. The singularity indicates that the Landau-Lifshitz definition of hydrodynamic frame is preferable over any other, for small-baryon-number problems. With the Landau-Lifshitz choice of

Viscous hydro

Navier, Stokes, ..., Landau, ..., Baym, Gyulassy, Danielewicz, ...

expansion in gradients $\rightarrow$ limited to small gradients and viscosities

zero baryon density limit: $\partial_\mu T^{\mu\nu} = 0$ **with**

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - p g^{\mu\nu} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta \Delta^{\mu\nu}\nabla_\alpha u^\alpha,$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu$$

η, ζ **- shear and bulk viscosities**

features:
- **entropy production:** $\partial_\mu S^\mu_{NS} \geq 0$
- **sound attenuation:** $\omega(k) = c_s k - \frac{i}{2} k^2 \Gamma_s$, $\Gamma_s \equiv \frac{4\eta/3+\zeta}{\varepsilon+p}$
- **slower cooling (1D Hubble):** $\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon+p}{\tau}(1 - \frac{\Gamma_s}{\tau})$ Gyulassy & Danielewicz '85

back in '80s, estimate for QGP was: $\eta/s \sim 1$

with recent "perfect liquid" assumption: $\eta/s = 1/4\pi$

DM @ RHIC/AGS Users' Mtg (June 2005):

entropy production relevant at RHIC already for "minimal" viscosity:

for 1D Bjorken expansion, ideal gas $\epsilon = 3p \propto T^4$**,** $\partial_i u_j \neq 0$ **(**$i, j \in \{0, z\}$**)**

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2} \approx \frac{4}{3}\kappa\frac{\epsilon + p}{T\tau^2}$$

$$\Rightarrow \frac{T(\tau)}{T_0} = \left(\frac{\tau_0}{\tau}\right)^{1/3}\left[1 + \frac{2\kappa}{3\tau_0 T_0}\left(1 - \left(\frac{\tau_0}{\tau}\right)^{2/3}\right)\right]$$

$$\frac{\tau s}{\tau_0 s_0} = \frac{\tau T^3}{\tau_0 T_0^3} \approx 1.1 - 1.2$$

($\tau_0 = 0.6$ fm/c, $T_0 \sim 300$ MeV, $\tau_{QGP} \sim 3 - 5$ fm/c, $\kappa = 1/4\pi$)

even larger in realistic case (not uniform 1D, also mixed phase)

Causal viscous hydro

Mueller ('67), Israel & Stewart ('79), ..., Muronga, Teaney, Chaudhuri, Heinz, Romatschke, ...

naive Navier-Stokes violates causality (parabolic modes)

causal formulation i) has new dynamical variables

heat flow q_α, bulk pressure Π, shear tensor π_{ij}

ii) needs further input: relaxation times $\tau_i(p, n_B)$

e.g, for 1D Bjorken flow:

instead of

$$\pi_{zz} = -\frac{4\eta}{3\tau} \equiv \pi_{zz}^{naive}$$

we have

$$\tau_\pi \dot{\pi}_{zz} + \pi_{zz} \left[1 + \frac{\tau_\pi}{2\tau} + \frac{\eta T}{2} \dot{\left(\frac{\tau_\pi}{\eta T} \right)} \right] = \pi_{zz}^{naive}$$

$\longrightarrow$ **sufficiently close to equilibrium, causality is restored**

Key point

- with (viscous) hydro we can study the hydrodynamic limit of the "real" theory (which we otherwise cannot solve)

- several different theories have the same(!) (viscous) hydrodynamic limit

⇒ we can get our (viscous) hydro solutions via solving any one of the equivalent theories

this way, we can study viscous hydro today, while viscous hydro algorithms being developed

Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

Covariant, nonequilibrium approach - formulated in terms of local rates.

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma \frac{n^2(x)}{2}$$

transport eqn.: $f_i(\vec{x}, \vec{p}, t)$ - **phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x},\vec{p},t) = \overbrace{S_i(\vec{x},\vec{p},t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x},\vec{p},t)}^{2\to 2\ (\text{ZPC, GCP, ...})} + \overbrace{C_i^{inel.}[f](\vec{x},\vec{p},t)}^{2\leftrightarrow 3\ (\text{MPC, Xu-Greiner})} + \ldots$$

algorithms: OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR

HERE: utilize MPC algorithm DM, NPA 697 ('02)

Hydrodynamic limit of transport

- **energy-momentum tensor:** $T^{\mu\nu}(x) = \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(x, \vec{p})$

  charge current: $N_c^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu c_i f_i(x, \vec{p})$

  entropy current: $S^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu f_i(x, \vec{p}) \left\{1 - \ln[f_i(x, \vec{p}) h^3]\right\}$

  conservation laws: $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N_c^\mu = 0$

  dissipation: $\partial_\mu S^\mu \geq 0 \quad \Rightarrow$ **entropy production, in general**

- **Ideal fluid limit** $\lambda \to 0$: **local equilibrium** $f = (2\pi)^{-3} \exp[p_\mu u^\mu(x)/T(x)]$

  $T_{id}^{\mu\nu} = (e + p) u^\mu u^\nu - p\, g^{\mu\nu}$

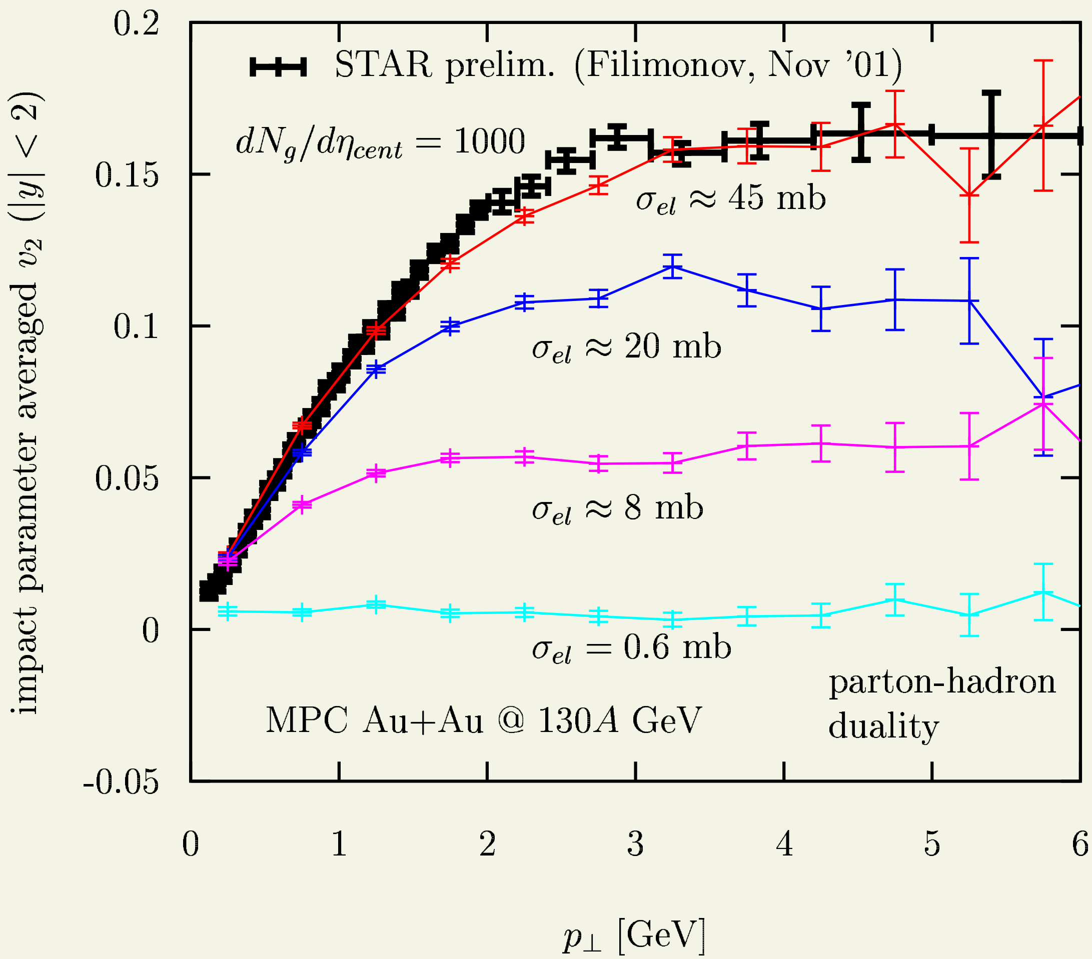
  $\partial_\mu S^\mu = 0 \quad \Rightarrow$ **entropy conserved**

- **Viscous hydro** $\lambda \ll$ *length & time scales*: **near equilibrium - slowly varying**

$$\text{shear viscosity } \eta \approx 0.8 \frac{T}{\sigma_{tr}}, \qquad \text{relaxation time } \tau_\pi \approx 1.2 \lambda_{tr}$$

Israel, Stewart ('79) ...

DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$



parton transport model MPC

$2 \to 2$ only

Au+Au @ 130 GeV, $b = 8$ fm

- minijet initconds
- 1 parton $\longrightarrow$ 1 π hadronization

elliptic flow sensitive to dissipation (finite rates)

need $\sigma_{gg} \approx 45$ mb $\to$ $15\times$ perturbative $2 \to 2$ rates

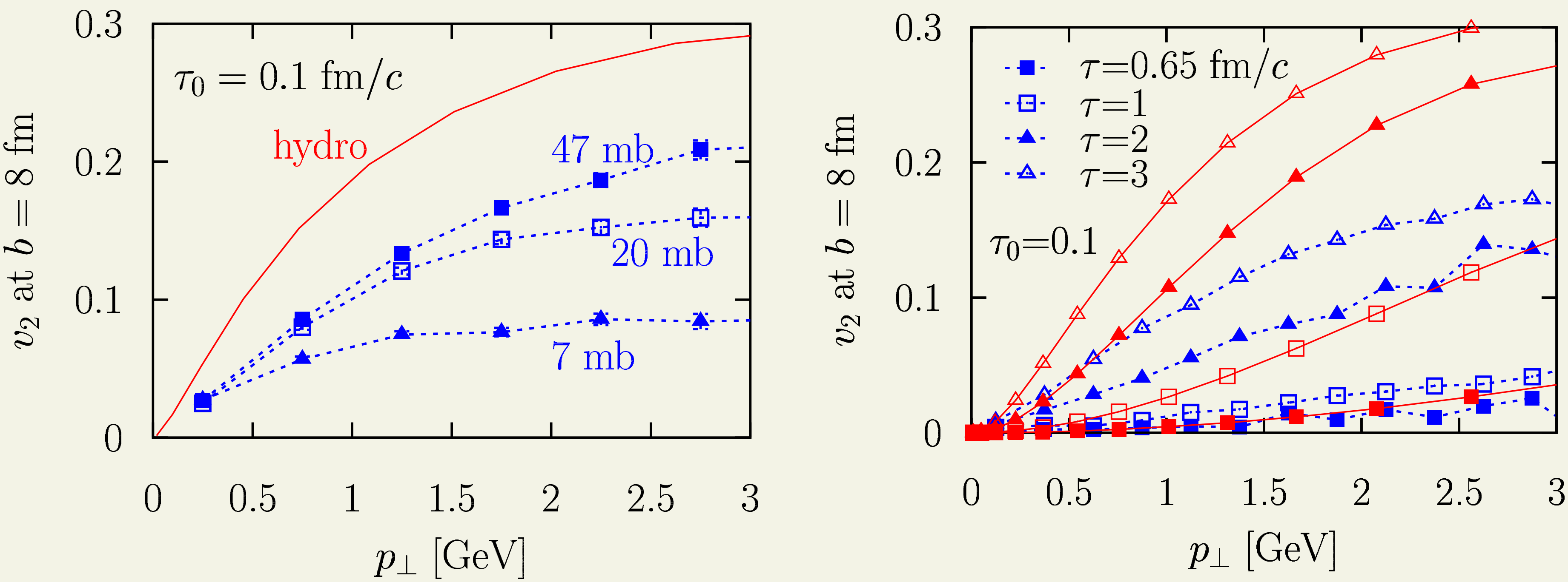
inelastic $3 \leftrightarrow 2$ helps by factor $\sim 2 - 4$ (enhances σ_{tr}), still not enough Xu, Greiner

No, still not ideal fluid

ideal hydro vs transport comparison for ultra-relativistic $\varepsilon = 3p$ with $2 \to 2$

from identical RHIC Au+Au initconds, $b = 8$ fm

DM & Huovinen, PRL94 ('05): **final $v_2(p_T)$** $\qquad$ $v_2(\tau, p_T)$ **for $\sigma_{gg} = 47$ mb**



even a tiny viscosity matters, if gradients are large

- **need** $|\Delta T^{\mu\nu}| \sim \eta\, |\partial^{<\mu} u^{\nu>}| \ll p$, **i.e.,** $\eta/s \ll 3/16 \cdot T\tau \approx\ 0.1 - 0.2$

η/s for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ fm at $\tau_0 = 0.1$ fm

estimate from average density: $\lambda_{tr} = \frac{1}{\langle n \rangle \sigma_{tr}}$

for $b = 8$ fm @ RHIC, transport with 47 mb gives

$$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2} \text{ fm}$$

estimate from transport opacity χ: assuming 1D Bjorken expansion

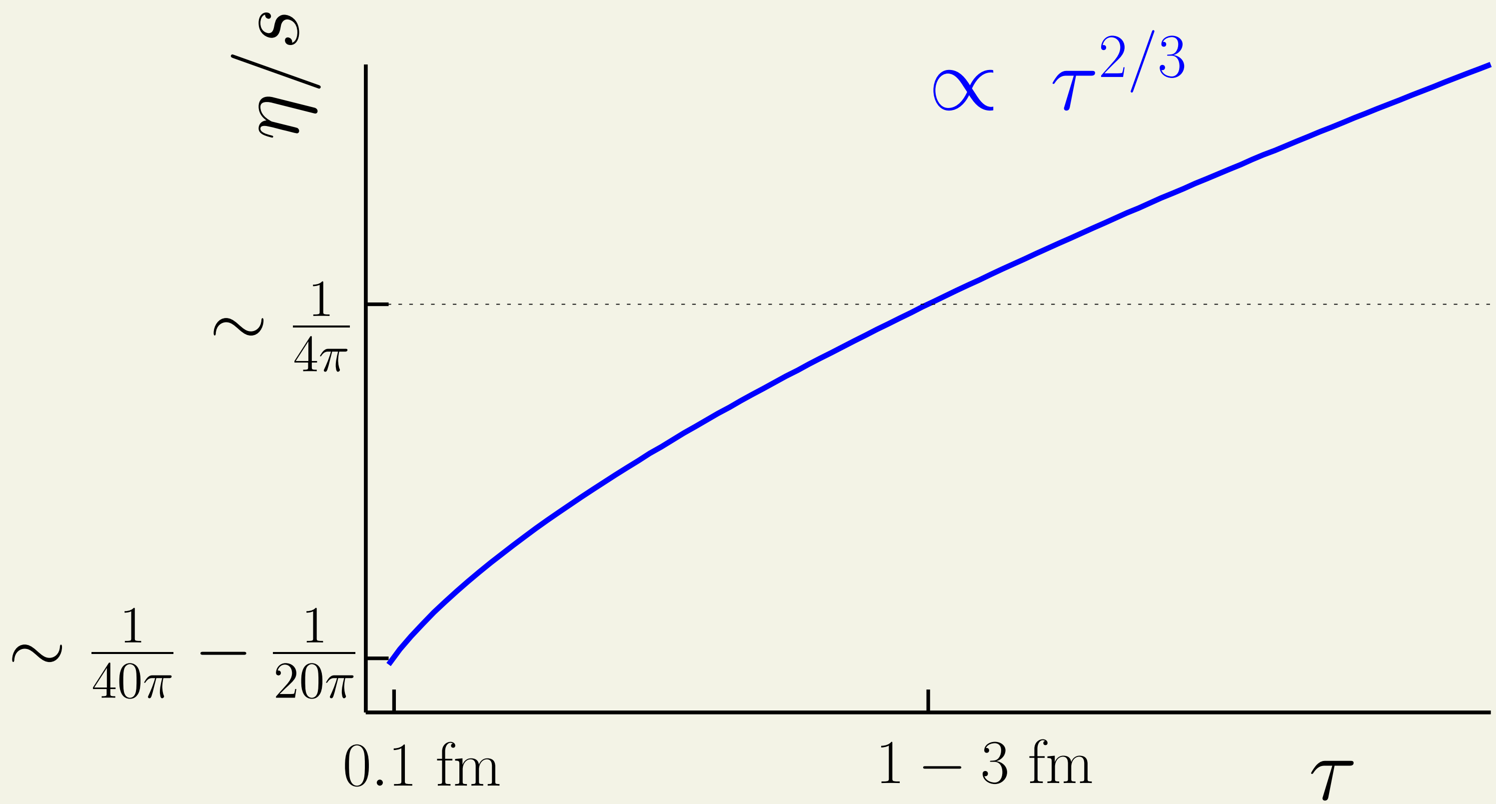
$$\chi = \int dz\, \rho(z)\sigma_{tr} \sim \int d\tau \rho_0 \frac{\tau_0}{\tau} \sigma_{tr} = \frac{\tau_0}{\lambda_{tr}(\tau_0)} \ln \frac{L}{\tau_0}$$

for $b = 8$ fm @ RHIC, transport with 47 mb gives $\chi \approx 20$

$\rightarrow \lambda_{tr}(\tau_0) \sim 1.5 - 2 \times 10^{-2}$ **fm (!)**

$\Rightarrow \sigma_{gg} \approx 50$ mb is already better than best-case scenario

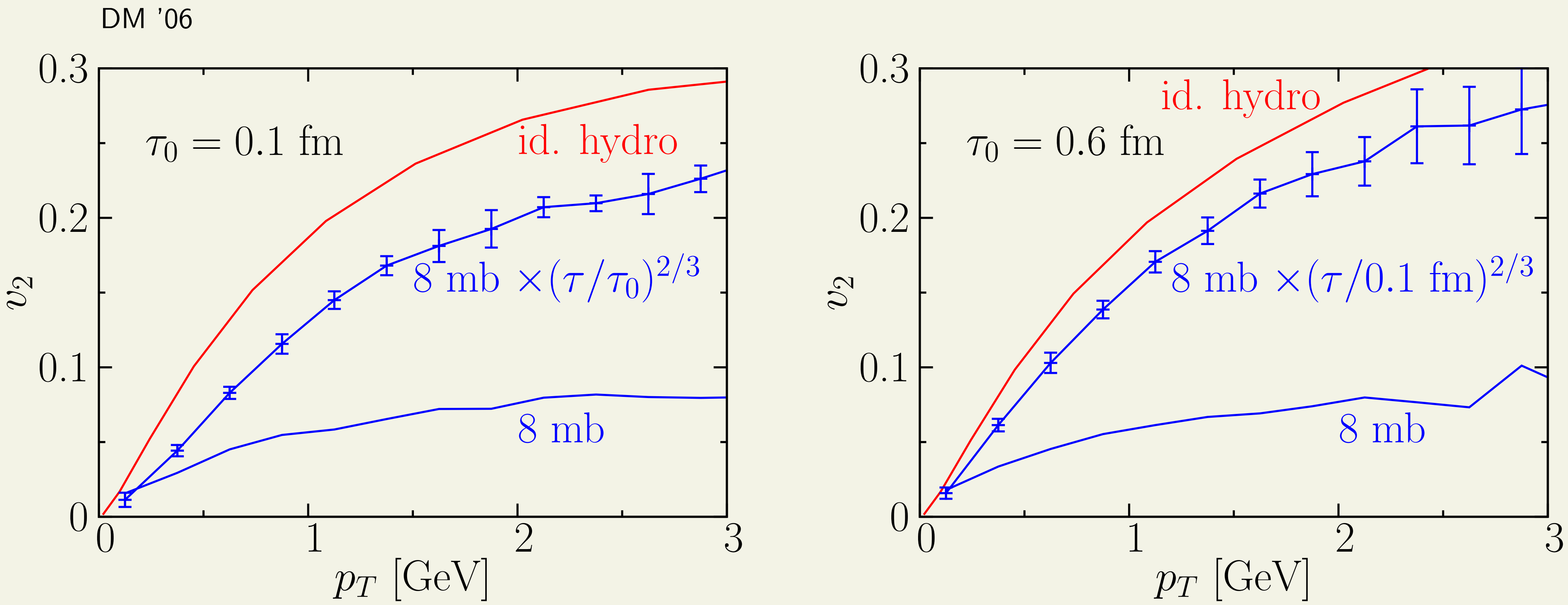
$\sigma \approx 47$ **mb dynamics corresponds to** $\eta/s \sim 1/(\sigma T^2)$



initially "better than perfect", after $\tau \sim 1-3$ fm "less than perfect"

$\eta/s = const$, **needs growing** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

redo comparison with "minimal viscosity" - $\sigma_{gg}(\tau = 0.1 \text{ fm}) \sim 4 - 9$ **mb**



$\Rightarrow$ **still** $20 - 30$**% reduction, even at low** p_T

main effect of viscosity - momentum anisotropy: $p_{longit}/p_{transv} > 1$

$$p_x = p_y \equiv p - \frac{\pi_{zz}}{2} \quad > \quad p_z \equiv p + \pi_{zz}$$

causal viscous hydro for 1D Bjorken:

DM & Huovinen ('06):

$$\begin{aligned} \dot{p} + \frac{4p}{3\tau} &= -\frac{\pi_z z}{3\tau} \\ \dot{\pi}_{zz} + \frac{\pi_{zz}}{\tau}\left(\frac{5\tau}{6\lambda_{tr}(\tau)} + \frac{4}{3}\right) &= -\frac{8p}{9\tau}\ . \end{aligned}$$

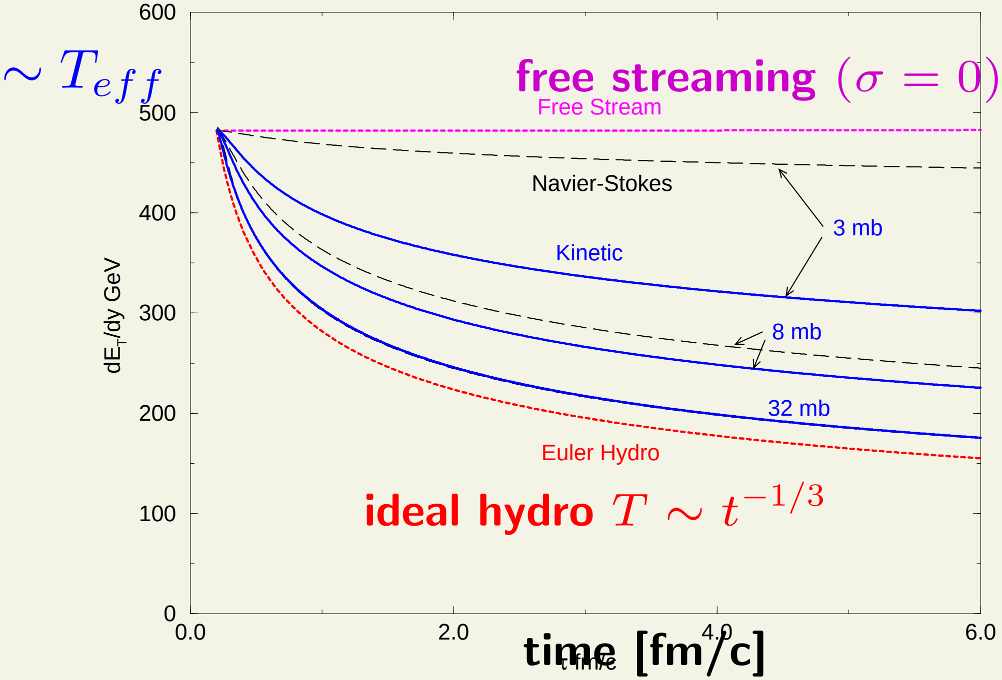
i) constant cross section: $\lambda_{tr}(\tau) = \lambda_{tr}(\tau_0)\tau/\tau_0$

ii) constant η/s: $\lambda_{tr}(\tau) = \lambda_{tr}(\tau_0)\frac{T^2\tau}{T_0^2\tau_0} = \lambda_{tr}(\tau_0)\frac{p^2(\tau)\tau^3}{p^2(\tau_0)\tau_0^3}$

these two cases are quite different because after $\tau >\sim few\ \tau_0$:

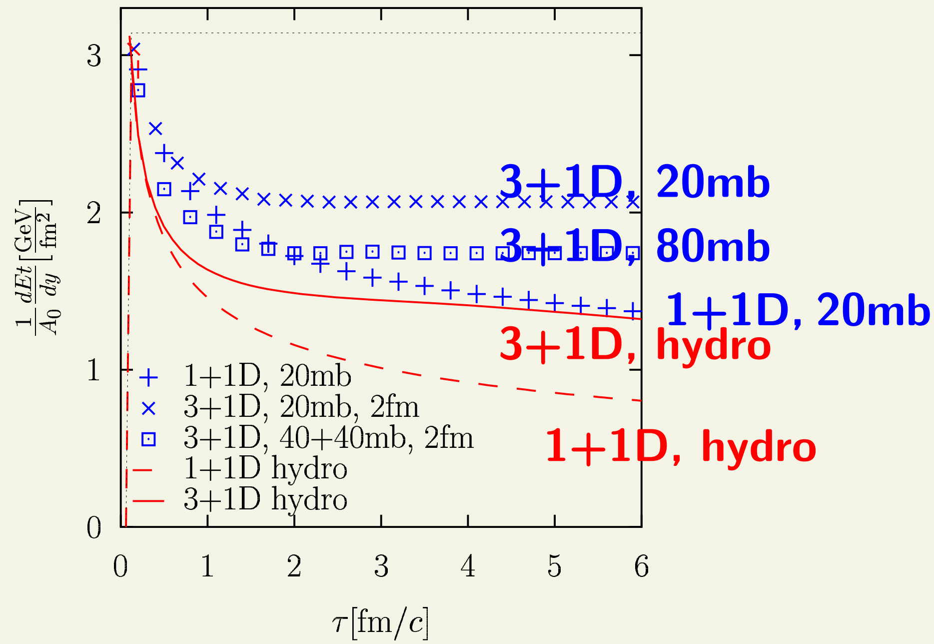
- **i) gives** $p_z/p_x = const < 1$
- **ii) gives** $p_z/p_x \to 1$ **slowly from below**

Gyulassy, Pang, Zhang ('97): 1+1D kinetic

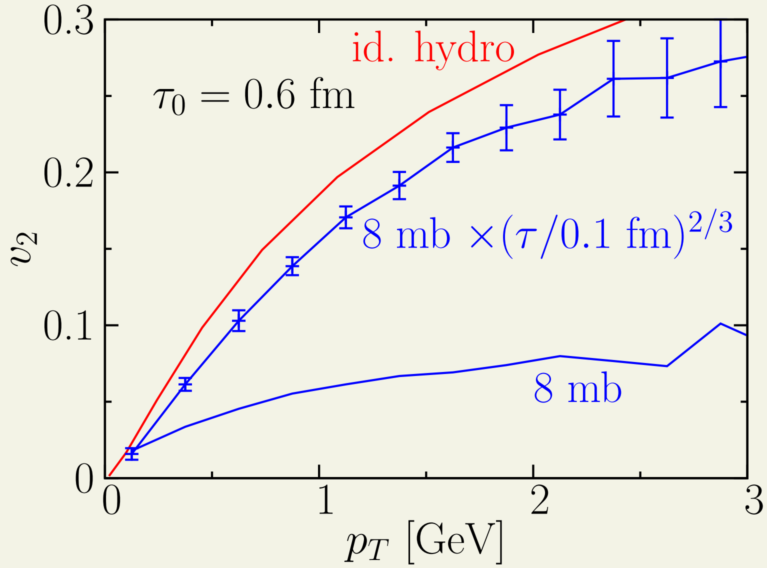


DM & Gyulassy ('00): 3+1D kinetic theory

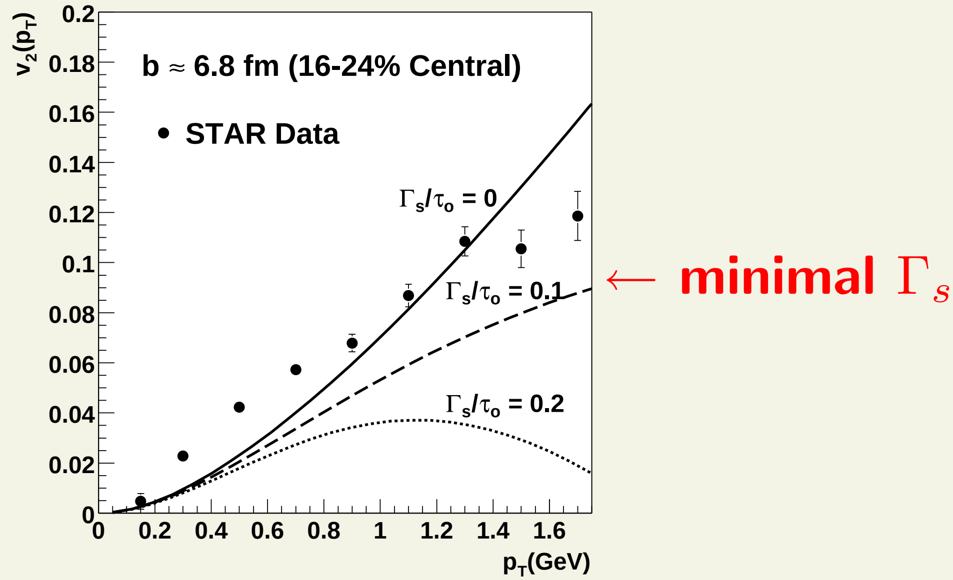
MPC vs hydro (1+1D and 3+1D)



DM ('06): $\eta/s \sim 1/(4\pi)$



Teaney ('04): **estimate at FO from id.hydro** $\Gamma_s \equiv \frac{4\eta}{3sT}$

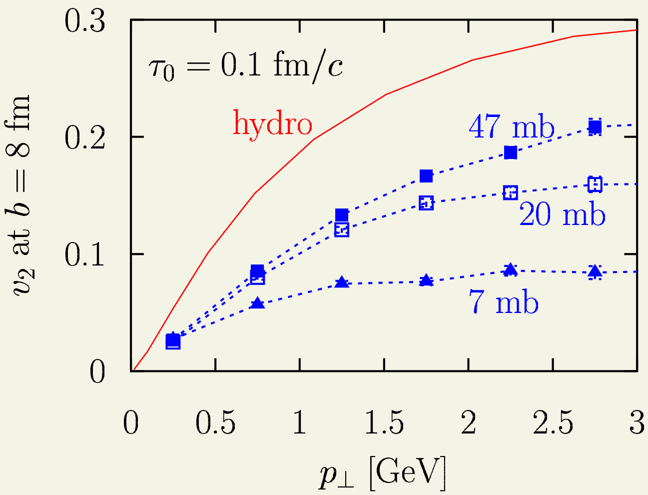


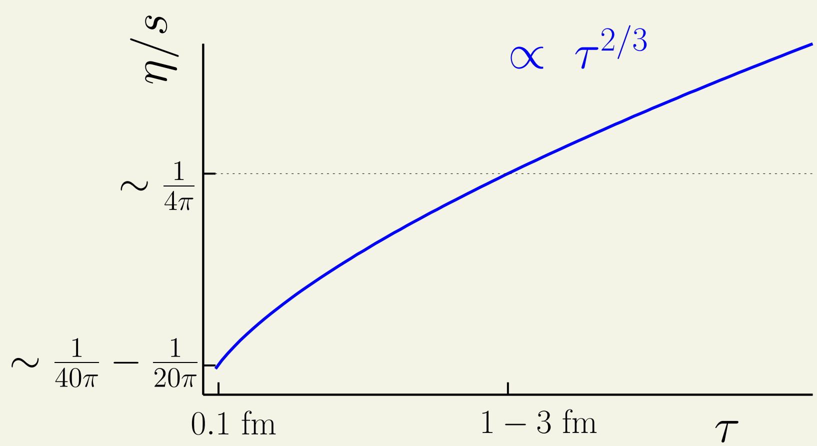
Conclusions

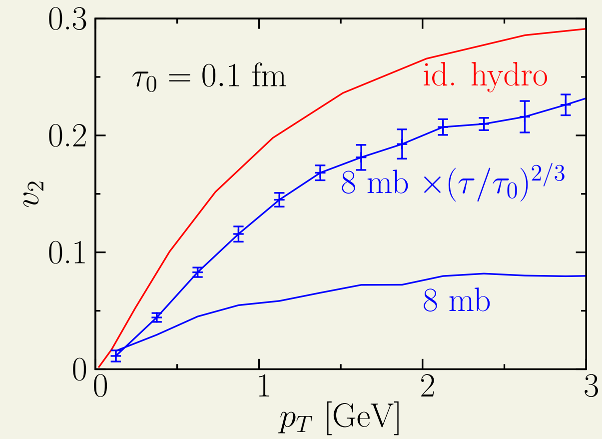
can study viscous hydro with dense transport - no acausal behavior

e.g., shear viscosity $\eta \approx 4T/5\sigma_{tr}$

$\sigma_{gg} = 50$ **mb gives strong dissipation at RHIC, however early evolution is inconsistent with (conjectured) lower viscosity bound - "better than perfect"**







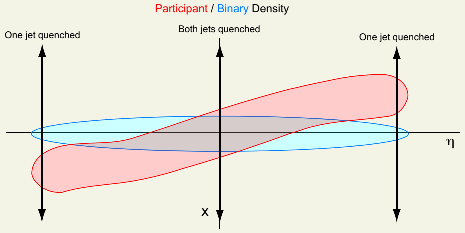
results for ultra-relativistic system ($\varepsilon = 3p$**) indicate that even for "minimal"** $\eta/s = 1/(4\pi)$**, elliptic flow is** $20-30\%$ **reduced by dissipation**

List of assumptions

initial conditions

- **geometry** - **assumed Glauber-like** $dN/dy \times f(\vec{x}_T)$

  other possibilities - **“octupole twist”** Adil & Gyulassy, PRC72 ('05)



  color glass condensate Adil, Gyulassy, Hirano, Kovchegov, Heinz, Dumitru, Nara, ... ('05 - '06)

- **question of initial radial flow**

- **fluctuations**

dynamics

- **limited equation of state** - **no phase transitions**

  but: most of elliptic flow builds up in the plasma phase

- **shear viscosity and shear relaxation time related, determined by same transition probability**

  but: much faster relaxation violates causality

- $2 \to 2$ **conserves particle number**

  but: this mainly affects temperature, not energy-momentum balance

  also: in local equilibrium, number is conserved - $s = 4n$

- **universal** $\eta/s \geq 1/(4\pi)$ - **does it apply to QCD?**

it is nontrivial whether causal viscous hydro is a consistent limit

naive Navier-Stokes: comes from separation of scales (long time and length scales)
- keeps only **linear terms** in gradients, e.g., $\partial_\mu u_\nu$

causal formulation: keeps second derivatives but ignores the square of first derivatives
- mixes short and long scales

e.g., kinetic theory: difference between **Chapman-Enskog expansion** and **Grad's 14-moment approximation**

Example: Molnar's Parton Cascade

Elementary processes: elastic $2 \to 2$ **processes** + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x, \vec{p})$**:** $i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$$
\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x, \vec{p}_1) = & \frac{\pi^4}{2} \sum_{jkl} \int_2\!\!\int_3\!\!\int_4 \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j\right) \left|\bar{\mathcal{M}}_{12\to34}^{i+j\to k+l}\right|^2 \delta^4(12-34) \quad \swarrow 2 \to 2 \\
& + \frac{\pi^4}{12} \int_2\!\!\int_3\!\!\int_4\!\!\int_5 \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i\right) \left|\bar{\mathcal{M}}_{12\to345}^{i+i\to i+i+i}\right|^2 \delta^4(12-345) \quad \swarrow 2 \leftrightarrow 3 \\
& + \frac{\pi^4}{8} \int_2\!\!\int_3\!\!\int_4\!\!\int_5 \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i}\right) \left|\bar{\mathcal{M}}_{45\to123}^{i+i\to i+i+i}\right|^2 \delta^4(123-45) \quad \swarrow 3 \leftrightarrow 2 \\
& + \tilde{S}^i(x, \vec{p}_1) \quad \leftarrow \textbf{initial conditions}
\end{aligned}
$$

with shorthands:

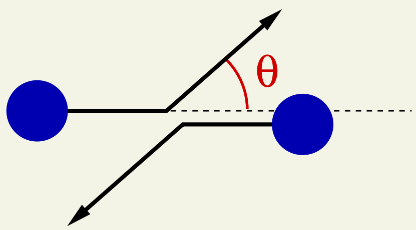
$$
\tilde{f}_i^q \equiv (2\pi)^3 f_q(x, \vec{p}_i), \quad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1+p_2-p_3-p_4) \equiv \delta^4(12-34)
$$

mean free path: characterizes local conditions

$$\lambda(x) \equiv \frac{1}{\textbf{cross section} \times \textbf{density}(\mathbf{x})} \qquad \begin{cases} \lambda = 0 & \text{– ideal hydrodynamics} \\ \lambda = \infty & \text{– free streaming} \end{cases}$$

transport opacity: time-integrated, spatially averaged [DM & Gyulassy NPA 697 ('02)]

$$\chi \equiv \langle n_{coll} \rangle \langle \sin^2 \theta_{CM} \rangle \quad \sim \# \text{ of collisions per parton} \times \text{mom. transfer efficiency}$$

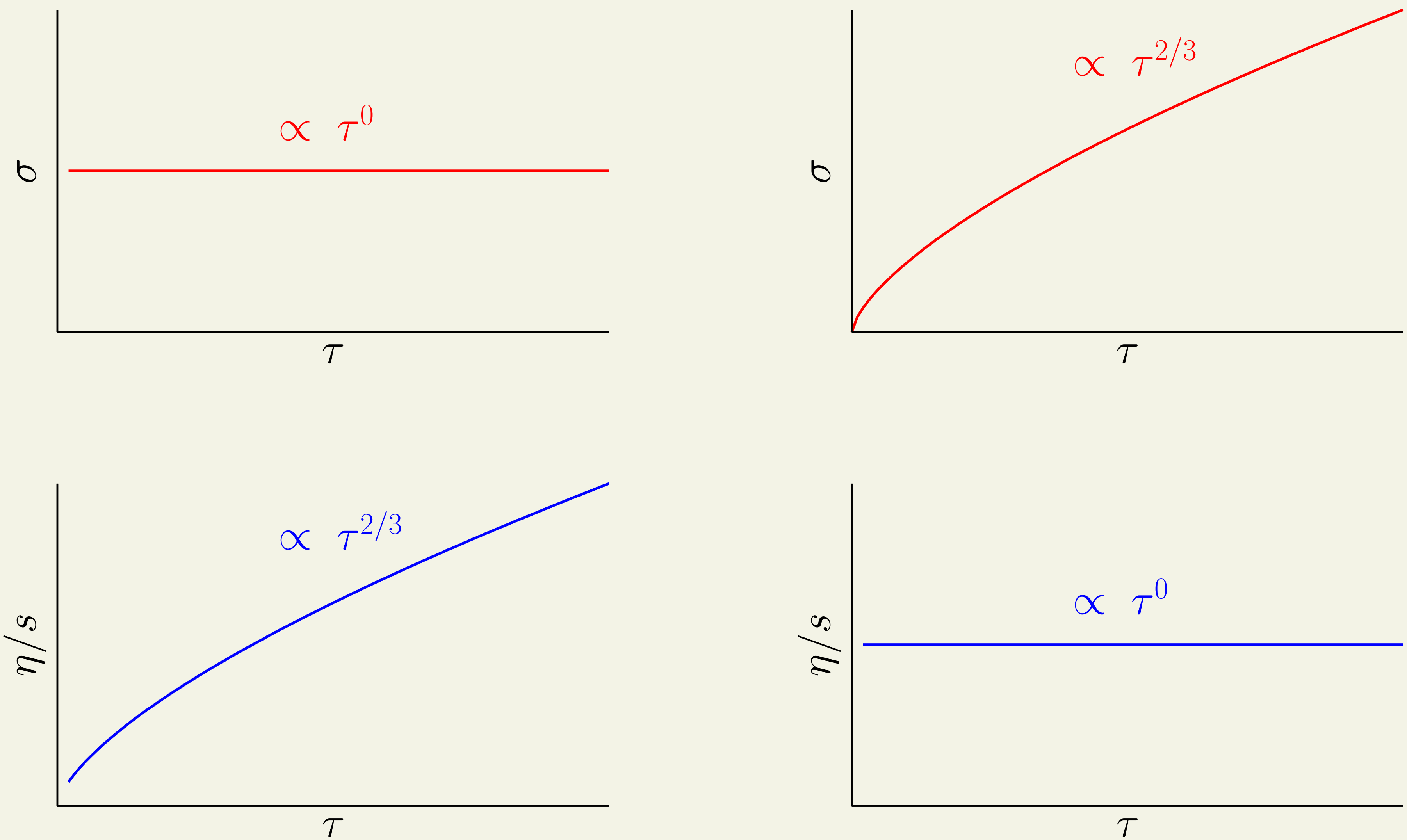


$$\chi = \int dz\, \rho(z)\, \sigma_{transp} = \int dz\, \frac{1}{\lambda_{tr}(z)}$$

near equilibrium: related to **transport coefficients** (viscosity, diffusion constants)

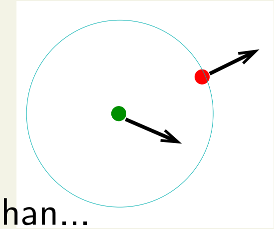
e.g., shear viscosity $\eta \approx \frac{4}{5} \frac{T}{\sigma_{tr}}$

$\sigma = const$ vs $\sigma \propto \frac{1}{T^2} \sim \tau^{2/3}$ (1D Bjorken)



since $\eta/s \propto 1/(\sigma T^2)$

Lorentz covariance violation

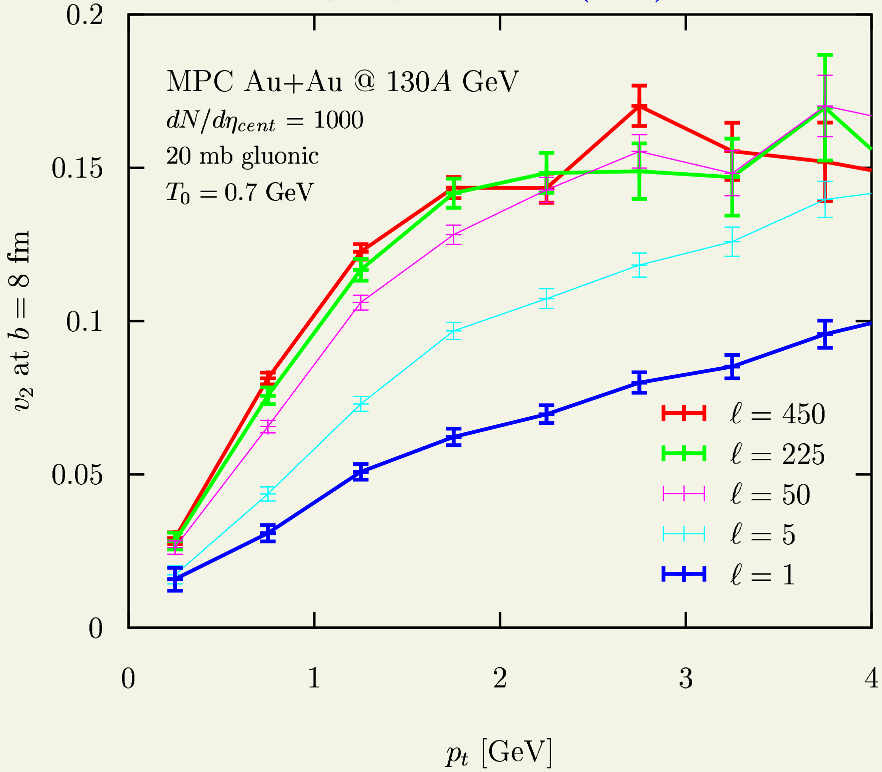


Naive $2 \to 2$ cascade nonlocal - action at distance $d < \sqrt{\frac{\sigma}{\pi}}$

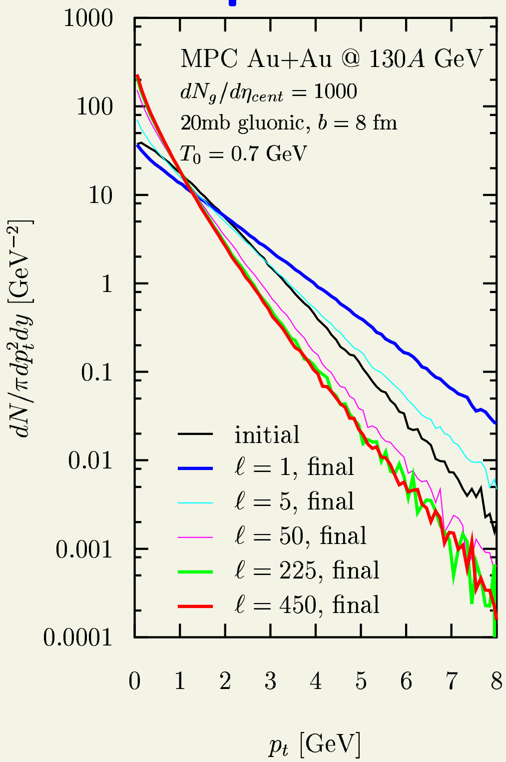
recall - NO-GO theorems in relativistic Hamilton dynamics Currie, Jordan, Sudarshan...

subdivision: rescale $f \to f \cdot \ell$, $\sigma \to \sigma/\ell$ $\Rightarrow$ $d \propto \ell^{-1/2}$ local as $\ell \to \infty$

DM & Gyulassy ('02): $v_2(p_T)$



spectra



RHIC (b=8 fm): need subdivision $\ell \sim 200$ to eliminate $2 \to 2$ artifacts