Radius studies of $^8\text{Li}$ and $^8\text{B}$ using the optical-limit Glauber Model in conjunction with relativistic mean-field theory

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Abstract We study the reaction cross sections ($\sigma_R$) and root-mean-square (RMS) radii of $^8\text{Li}$ and $^8\text{B}$, the halo-like nuclei, with stable target $^{12}\text{C}$, $^{27}\text{Al}$ and $^9\text{Be}$ within the standard optical-limit Glauber Model, using densities obtained from relativistic mean-field (RMF) formalisms and other types of distributions. It is found that the experimental $\sigma_R$ can be reproduced well at high energy. The RMS radius and $\Delta r$ extracted by RMF-theory and harmonic oscillator distribution are compared. We find that the RMS radius and $\Delta r$ of $^8\text{B}$ are larger than those of $^8\text{Li}$. Besides, we analyze in detail the relationship between $\sigma_R$ and density distribution.

Key words Glauber-model, root-mean-square radius, relativistic-mean-field theory, reaction cross sections

PACS 21.10.Gv, 24.10.Jv, 25.60.-t

1 Introduction

The nuclear size and density distribution are important bulk properties of nuclei which determine the nuclear potential, single-particle orbitals, wave functions, and so on. So it is very important to get the precise values for the nuclear size and density distribution.

Since the 1980th, as a great progress with accelerator technology has been made, radioactive ion beams (RIBs) have been used widely for studying structures and properties of isotopes near and far away from $\beta-$Stability. The quantities measured in the studies include various inclusive cross sections, for instance, reaction or interaction cross sections, nucleon-removal cross sections, coulomb breakup cross sections and momentum distributions of a fragment, and so on. Among those quantities, the reaction cross section (Interaction cross section at high energy\textsuperscript{[1]}) is one of the most fundamental quantities characterizing the nuclear reactions and to probe for nuclear structure details. In 1985, I. Tanihata et al, Ref\textsuperscript{[2]}, found the neutron halo nucleus $^{11}\text{Li}$ by measuring the interaction cross section in the relativistic energy region. Later on many experiments confirmed this result. But for $^8\text{B}$ multipole moment measurements, indicating a halo structure due to the weakly bound ($E_s = 0.137 \text{MeV}$) p-wave valence proton\textsuperscript{[3]}, triggered a set of experimental studies devoted to the $^8\text{B}$ structure investigations. All of those experiments found a large reaction cross section, a relatively narrow momentum distribution of the $^7\text{Be}$ fragments and a large one-proton removal cross section, thus confirming the halo-like structure of $^8\text{B}$. However, the matter radius has not been found to be anomalously large compared to the neighboring nuclei, and it is important to note that these experimental results seem not to be consistent. The values of the RMS radius from different experiments vary from 2.39 to 2.74 fm, which is a difference almost 15%\textsuperscript{[4-6]}. The same problem occurs with...
its mirror nucleus, \(^{8}\)Li, which has a valence neutron with a separation energy of 2 MeV. The thickness of the neutron skin obtained by different methods ranges from 0.18 to 0.56 fm\(^7\)\(^-\)\(^8\).

Although we have no straightforward model-independent method for determining the matter-density distribution of unstable nuclei now, the Glauber model has been used extensively. It is based on the independent individual nucleon-nucleon collisions in the overlap zone of the colliding nuclei, accounting for a significant part of breakup effects that play an important role in the reaction of a weakly bound nucleus, and successfully explains the observed nuclear reaction cross section for various systems at high energies. Although the model is simple, it shows reasonable results in many cases. This model requires the structure information, namely the density distribution, of the nuclei involved (projectile and target) and N-N cross section. The structure information has to be provided by nuclear structure models, like the Harmonic-oscillator-type (HO-type) distribution, the Gaussian distribution, and so on. In this work, beside these two distributions, we also use the RMF-theory as involved nuclear structure calculations to reproduce the \(\sigma_R\). The free N-N cross sections are inappropriate, however, because the effective values may differ from the free-nucleon values due to nuclear-matter effects and Fermi-motion\(^9\)\(^-\)\(^10\). To determine the effective N-N cross sections, we use a HO-type distribution of \(^{12}\)C and take \(\sigma_R\) of \(^{12}\)C\(^-\)\(^{13}\)C as a calibration. The reason why we use a HO-type distribution and \(\sigma_R\) of \(^{12}\)C will be explained below. The obtained value is 71% of the free N-N cross sections.

The main objective of the present work is to study the RMS radii and \(\Delta r\) \(\left(R_{p,n} - R_{p,n}\right)\) for \(^8\)Li and \(^8\)B and to extract the density distribution of \(^{12}\)C, \(^{9}\)Be and \(^{27}\)Al, which in July 2009 were used as experimental targets for \(^8\)Li in HIMAC. This article is presented as follows. In Sec. 2, we discuss in brief the formalism used in the present work. In Sec. 3, we extract the effective N-N cross section and the density distributions of the stable targets \(^{12}\)C, \(^{27}\)Al and \(^{9}\)Be. In Sec. 4, we discuss the RMS radii of \(^8\)Li and \(^8\)B using RMF-theory in conjunction with the Glauber model. The summary and conclusion are given in Sec. 5.

2 formalism

2.1 A. Glauber Model

The standard Glauber model form for the reaction cross section, given by R. J. Glauber\(^10\), is deduced from the eikonal approximation is given by:

\[
\sigma_R = 2\pi \int_0^\infty b [1 - T(b)] db, \tag{1}
\]

where \(T(b)\) is the transmission as a function of the impact parameter \(b\). A straightforward calculation of \(T(b)\) is complicated. One of the simplest approximation to calculate \(T(b)\) is the optical limit. In this approximation we get:

\[
T(b) = \exp \left\{ -\sum_{i,j} \sigma_{ij} \int \rho^p_i \rho^p_j (|b-z|) ds \right\}, \tag{2}
\]

Here \(\rho^p_K(s)\) is an over the z-direction integrated nucleon-density distribution,

\[
\rho^p_K(s) = \int_{-\infty}^{\infty} \rho_K(\sqrt{s^2+z^2}) dz, \tag{3}
\]

where the index \(k = P\) (projectile) or \(T\) (target) means projectile or target nucleon-density distribution and \(\sigma_{ij}\) are the N-N cross sections, used to the replace profile function in the zero range limits, in which the indices \(i, j\) are used to distinguish protons and neutrons.

This model is successful in reproducing reaction cross sections for various systems at high energy\(^11\). To explain its success at high energy more detailed, we perform an analysis by comparing the \(\sigma_R\) derived with a Gaussian distribution and a HO-type distribution of \(^{12}\)C extracted in this work. Fig. 1 and Fig. 2 show us that the reaction cross sections at high energy, for example the \((b)\) in Fig.1 and Fig.2, are mainly depend on the core-nucleons and the reaction cross sections at low energy, for example the \((d)\) in Fig.1 and Fig.2, are sensitive to outer-nucleons. If peripheral distribution changes by 2%, the reaction cross sections show a 6.5% difference at low energy. At the same time, the RMS radii differ by 1%. So we can use it to discuss the RMS radii and \(\Delta r\) of \(^8\)Li and \(^8\)B at high energy.

2.2 B. nuclear structure models

To get the structure information, we employ three types of density distributions: HO-type distribution, Gaussian distribution and RMF-theory which recently has been effectively used for this purpose\(^12\).

The HO-type and the Gaussian distributions each have one size parameter. We get these parameters through experimental data: reaction cross sections and charge-RMS radius. For \(^8\)B, we get the RMS radius by using a HO-type distribution based on the Gaussian calculation. The RMF theory has several
sets of parameters to calculate the density distribution which we will use in this paper, for details please see Ref.[13].

The HO-type density distribution as an useful model is successful in explaining stable nuclei even such as \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\)[14]. On the other hand, for light stable nuclei (A \(\leq 20\)), the HO-type density distribution is sufficient to describe the experimental data. The HO-type density distributions including the contributions up to the sd-shell are given by the following equations:

For \(2 < Z < 8, 2 < N < 8\):

\[
\rho_n(r) = 2\pi^{-\frac{3}{2}}\lambda^{-3}(1 - \frac{1}{A})^{-\frac{3}{2}}e^{-r^2}(1 + \frac{N + 2}{3x^2}).
\]

\[
\rho_p(r) = 2\pi^{-\frac{3}{2}}\lambda^{-3}(1 - \frac{1}{A})^{-\frac{3}{2}}e^{-r^2}(1 + \frac{Z + 2}{3x^2}).
\]

For \(2 > Z > 8, 2 > N > 8\):

\[
\rho_n(r) = 4\pi^{-\frac{3}{2}}\lambda^{-3}(1 - \frac{1}{A})^{-\frac{3}{2}}N e^{-\frac{r^2}{8} } \times (1 + 2X^2\frac{N - 8}{15x^4}).
\]

\[
\rho_p(r) = 4\pi^{-\frac{3}{2}}\lambda^{-3}(1 - \frac{1}{A})^{-\frac{3}{2}}Z e^{-\frac{r^2}{8} } \times (1 + 2X^2\frac{Z - 8}{15x^4}).
\]

Here A, N and Z are the mass, neutron, and proton numbers, \(x = (\frac{r}{\bar{r}})^2\) and \(\lambda\) denotes the size parameter.

The Gaussian calculation uses:

\[
\rho_k(r) = x_k\exp\left\{\left[-\left(\frac{r}{b}\right)^2\right]\right\}.
\]

The index \(k\) denotes a proton or a neutron, \(x_k\) and \(b\) are the amplitude and width respectively.

## 3 Density distribution of \(^{12}\text{C}\), \(^{27}\text{Al}\) and \(^{9}\text{Be}\), The RMS radii of \(^8\text{Li}\) and \(^8\text{B}\)

As a calibration, \(^{12}\text{C}\) was measured by many experiments. The collected data are listed in the Table 1. The last one of \(2.46\pm0.009\text{fm}\) is the normalized result from these data by the equation: \(\sum A_i^2 / \sum A_i\), here \(x_i^2\) and \(A_i\) are experimental radii and errors. We use this value as \(^{12}\text{C}\) charge-RMS to get the effective N-N cross sections. In Table 2 we show the results calculated with the Gaussian and HO-type distribution. In Table 3 we show the results obtained from the RMF-theory. Reaction cross sections calculated with the Glauber model in conjunction with RMF-theory are listed in Table 4.

<table>
<thead>
<tr>
<th>(R_{c}(^{12}\text{C})/\text{fm})</th>
<th>Ref.</th>
<th>(R_{c}(^{27}\text{Al})/\text{fm})</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.46\pm0.025</td>
<td>[15]</td>
<td>3.06\pm0.09</td>
<td>[24]</td>
</tr>
<tr>
<td>2.453\pm0.008</td>
<td>[16]</td>
<td>3.05\pm0.05</td>
<td>[18]</td>
</tr>
<tr>
<td>2.468\pm0.016</td>
<td>[17]</td>
<td>3.035\pm0.040</td>
<td>[25]</td>
</tr>
<tr>
<td>2.462\pm0.022</td>
<td>[18]</td>
<td>3.057\pm0.013</td>
<td></td>
</tr>
<tr>
<td>2.471\pm0.0055</td>
<td>[19]</td>
<td>3.0554\pm0.0004</td>
<td></td>
</tr>
<tr>
<td>2.40\pm0.56</td>
<td>[20]</td>
<td>3.058\pm0.05</td>
<td></td>
</tr>
<tr>
<td>2.32\pm0.16</td>
<td>[21]</td>
<td>3.0554\pm0.0004</td>
<td></td>
</tr>
<tr>
<td>2.49\pm0.05</td>
<td>[22]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.472\pm0.016</td>
<td>[23]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.43\pm0.02</td>
<td>[2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.464\pm0.009</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Comparison of RMS radii obtained from a Gaussian distribution and a HO-type calculation with normalized result (See the first paragraph of Sec.3.). \( R_m \) and \( R_n \) mean matter-RMS radius and Neutron-RMS radius respectively. The parameters of the Gaussian distribution extracted here are: \(^{12}\)C(1.99, 1.0), \(^{27}\)Al(2.451, 1.0) and \(^9\)Be(2.06, 1.0), which have been normalized first. The parameters of the HO-type distribution extracted here are: \(^{12}\)C(1.7485, 1.7485), \(^{27}\)Al(1.9505, 1.98) and \(^9\)Be(1.889, 1.878), the first value is for protons. The RMS radii of \(^9\)Be and \(^8\)Li are taken from Ref.[27] and Ref.[28].

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>Exp. ( R_m/\text{fm} )</th>
<th>Gaussian ( R_m/\text{fm} )</th>
<th>HO-\text{Oscillator} ( R_m/\text{fm} )</th>
<th>( \Delta r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12})C</td>
<td>2.464 ± 0.009</td>
<td>2.437 ± 0.019</td>
<td>2.464 ± 0.019</td>
<td>0.232</td>
</tr>
<tr>
<td>(^{27})Al</td>
<td>3.055 ± 0.0004</td>
<td>3.006 ± 0.010</td>
<td>3.093 ± 0.009</td>
<td>0.390</td>
</tr>
<tr>
<td>(^9)Be</td>
<td>2.519 ± 0.012</td>
<td>2.523 ± 0.022</td>
<td>2.545 ± 0.020</td>
<td>0.570</td>
</tr>
<tr>
<td>(^8)Li</td>
<td>2.299 ± 0.032</td>
<td>2.486 ± 0.037</td>
<td>2.493 ± 0.023</td>
<td>0.570</td>
</tr>
<tr>
<td>(^8)B</td>
<td>2.572 ± 0.058</td>
<td>2.588 ± 0.029</td>
<td>2.725 ± 0.029</td>
<td>2.343</td>
</tr>
</tbody>
</table>

The \( R_m \) is obtained by calculations with a Gaussian distribution and with a HO-type distribution differ by about 1%. There are two major reasons for this: (I) there is a 2% difference of the outer-nucleon distribution in these two kinds of calculation, (II) the total cross sections for proton-proton, neutron-neutron, and proton-neutron scattering are not the same in the two systems \(^{27}\)Al and \(^9\)Be\(^9\). The \( R_m \) and \( \Delta r \) of \(^8\)B are larger than those of \(^8\)Li by about 3.8% ± 1.5% and 25.7% ± 15.2% respectively.

4 The RMS radii of \(^8\)Li and \(^8\)B using RMF-theory in combination with the Glauber model

The RMF-theory which naturally includes the strong three-body repulsion, responsible for the saturation, and a strong spin-orbit force which is responsible for the magic numbers, has gained considerable success in describing various facets of nuclear structure properties. For instance it leads to a quantitative description of the binding energy and equilibrium density of nuclear matter. With a very limited number of parameters it is also able to give a quantitative description of the ground-state properties at and off the stability line\(^{29, 30}\). Because of the success of both theories, Glauber model and RMF-theory, we combine them in order to study \( \sigma_R \) and the RMS-radii of \(^8\)Li and \(^8\)B. For TM1, \( R_m \) and \( \Delta r \) of \(^8\)B is larger than \(^8\)Li about 2.1% and 143.5% respectively. For TM2, these values are 1.9% and 146.1% respectively. The results are listed in Table. 3. and Table. 4. We also show results in Fig. 3 and Fig. 4. About the parameter TM1 and TM2 please see Ref.[31] and [32].

Table 3. Comparison of RMS radii of \(^8\)Li and \(^8\)B obtained from RMF-theory with different parameters. The experimental data of the binding energy are taken from Ref.[33].

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>( R_m/\text{fm} )</th>
<th>( R_n/\text{fm} )</th>
<th>( \Delta r )</th>
<th>Binding \text{- Energy}/\text{MeV}</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^8)Li((NL-\text{SH}))</td>
<td>2.507</td>
<td>2.277</td>
<td>2.508</td>
<td>0.231</td>
</tr>
<tr>
<td>(^8)Li(NL1)</td>
<td>2.660</td>
<td>2.404</td>
<td>2.683</td>
<td>0.279</td>
</tr>
<tr>
<td>(^8)Li(NLz)</td>
<td>2.654</td>
<td>2.404</td>
<td>2.672</td>
<td>0.268</td>
</tr>
<tr>
<td>(^8)Li(TM1)</td>
<td>2.556</td>
<td>2.313</td>
<td>2.566</td>
<td>0.253</td>
</tr>
<tr>
<td>(^8)Li(TM2)</td>
<td>2.522</td>
<td>2.284</td>
<td>2.527</td>
<td>0.243</td>
</tr>
<tr>
<td>(^8)B((NL-\text{SH}))</td>
<td>2.559</td>
<td>2.716</td>
<td>2.121</td>
<td>0.595</td>
</tr>
<tr>
<td>(^8)B(NL1)</td>
<td>2.720</td>
<td>2.893</td>
<td>2.263</td>
<td>0.630</td>
</tr>
<tr>
<td>(^8)B(NLz)</td>
<td>2.711</td>
<td>2.880</td>
<td>2.263</td>
<td>0.617</td>
</tr>
<tr>
<td>(^8)B(TM1)</td>
<td>2.610</td>
<td>2.776</td>
<td>2.160</td>
<td>0.616</td>
</tr>
<tr>
<td>(^8)B(TM2)</td>
<td>2.569</td>
<td>2.728</td>
<td>2.130</td>
<td>0.598</td>
</tr>
</tbody>
</table>
Table 4. Reaction cross sections calculated with the Glauber model in combination with the RMF-theory. The experimental data are taken from Ref.[8,34]. The energies are: 790MeV/u for $^{12}\text{C}$ and $^9\text{Be}$, 285MeV/u for $^{27}\text{Al}$.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$\sigma_R(^{12}\text{C})$/mb</th>
<th>$\sigma_R(^{27}\text{Al})$/mb</th>
<th>$\sigma_R(^{9}\text{Be})$/mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8\text{Li}(NL-\text{SH})$</td>
<td>757.1</td>
<td>1142.2</td>
<td>702.0</td>
</tr>
<tr>
<td>$^8\text{Li}(NL1)$</td>
<td>791.0</td>
<td>1188.1</td>
<td>730.0</td>
</tr>
<tr>
<td>$^8\text{Li}(NLz)$</td>
<td>789.2</td>
<td>1186.0</td>
<td>729.1</td>
</tr>
<tr>
<td>$^8\text{Li}(TM1)$</td>
<td>768.1</td>
<td>1157.3</td>
<td>710.9</td>
</tr>
<tr>
<td>$^8\text{Li}(TM2)$</td>
<td>760.7</td>
<td>1146.9</td>
<td>704.9</td>
</tr>
<tr>
<td>$^8\text{B}(NL-\text{SH})$</td>
<td>775.8</td>
<td>1050.2</td>
<td>714.5</td>
</tr>
<tr>
<td>$^8\text{B}(NL1)$</td>
<td>810.9</td>
<td>1090.7</td>
<td>743.5</td>
</tr>
<tr>
<td>$^8\text{B}(NLz)$</td>
<td>809.1</td>
<td>1088.4</td>
<td>742.1</td>
</tr>
<tr>
<td>$^8\text{B}(TM1)$</td>
<td>786.9</td>
<td>1063.1</td>
<td>723.5</td>
</tr>
<tr>
<td>$^8\text{B}(TM2)$</td>
<td>778.2</td>
<td>1171.6</td>
<td>716.6</td>
</tr>
</tbody>
</table>

5 Summary

In summary, we studied the RMS radii of $^8\text{Li}$ and $^8\text{B}$ in the framework of the Glauber model with densities obtained from Gaussian, HO-type distribution and RMF-theory. We studied in detail the relationship between densities and $\sigma_R$. Comparing the results obtained with the RMF-theory using different parameters showed that the $\sigma_R$ can be reproduced well with a combination of the Glauber model and the RMF-theory(TM1, TM2) at high energy. We found that $R_m$ and $\Delta r$ of $^8\text{B}$ are larger than those of $^8\text{Li}$, but no decision is yet possible whether $^8\text{Li}$ is a halo or a skin-nucleus yet. In July 2009 further studies at low energies have been started at HIMAC. We hope to be able to present in the future results for its charge and neutron density distribution by using a modified Glauber Model[9].

Acknowledgments

The authors want to thank Prof. Ren Zhong-Zhou (NJU) for providing the RMF-theory code, Associate Researcher Fang De-Qing (SINAP) for his help with this subject. We would like to acknowledge the financial support provided by the One Hundred Person Project of the Chinese Academy of Sciences (26010701), the Knowledge Innovation Project of the Chinese Academy of Sciences (KJCX2-SW-N13, KJCX3-SYW-N2) and the National Natural Science Foundation of China (10675156).

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