Medium-Modified Fragmentation Function due to Multiple Parton Scattering

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Outline

- Introduction
- Modified fragmentation function in “Brick”
- Modified fragmentation function in DIS
- Summary
Introduction:
Medium-Modified DGLAP
Medium Induced Gluon Radiation

Hard parton travel through QCD medium

Suffer multiple parton scattering

Induced gluon radiation

Reduce the energy and virtuality

The accepted explanation for Jet Quenching observed in RHIC
Modified FF due to Double Parton Scattering

\[ \tilde{D}_{q\to h}(z_h, \mu^2) = D_{q\to h}(z_h, \mu^2) + \Delta D_{q\to h}(z_h, \mu^2) \]

**Modification to FF:**

\[
\Delta D_{q\to h}(z_h, Q^2) = \int_0^Q \frac{d l_T^2}{l_T^2} \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \left[ \Delta \gamma_{qg}(z, x_B, x_L, l_T^2) D_{q\to h}(z_h/z, Q^2) \right. \\
\left. + \Delta \gamma_{gg}(z, x_B, x_L, l_T^2) D_{g\to h}(z_h/z, Q^2) \right]
\]

X. F. Guo and X. N. Wang  
B. W. Zhang and X. N. Wang  
Modified FF due to Double Parton Scattering

\[ \tilde{D}_{q\rightarrow h}(z_h, \mu^2) = D_{q\rightarrow h}(z_h, \mu^2) + \Delta D_{q\rightarrow h}(z_h, \mu^2) \]

Modification to FF:

\[ \Delta D_{q\rightarrow h}(z_h, Q^2) = \int_0^Q \frac{d\Delta^2}{d\Delta^2} \frac{\alpha_s}{2\pi z_h} \left[ \Delta \gamma_{q\rightarrow qg}(z, x_B, x_L, l_T^2) D_{q\rightarrow h}(z_h/z, Q^2) + \Delta \gamma_{q\rightarrow qg}(z, x_B, x_L, l_T^2) D_{g\rightarrow h}(z_h/z, Q^2) \right] \]

Modification to splitting function for quark:

\[ \Delta \gamma_{q\rightarrow qg}(z, x_B, x_L, l_T^2) = \frac{C_A}{l_T^2} \left[ \frac{1+z^2}{1-z_+} T_{qg}(x_B, x_L) + \delta(1-z) T_{qg}^A(x_B, x_L) \right] \frac{2\pi\alpha_s}{N_c f_q A(x_B)} \]

\[ \Delta \gamma_{q\rightarrow qg}(z, x_B, x_L, l_T^2) = \Delta \gamma_{q\rightarrow qg}(1-z, x_B, x_L, l_T^2) \]

X. F. Guo and X. N. Wang

B. W. Zhang and X. N. Wang
**Modified FF due to Double Parton Scattering**

\[
\tilde{D}_{q\to h}(z_h, \mu^2) = D_{q\to h}(z_h, \mu^2) + \Delta D_{q\to h}(z_h, \mu^2)
\]

**Modification to FF:**

\[
\Delta D_{q\to h}(z_h, Q^2) = \int^Q_0 \frac{dQ^2}{Q^2} \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \left[ \Delta y_{q\to qg}(z, x_B, x_L, l_T^2) D_{q\to h}(z_h, z, Q^2) + \Delta y_{q\to qg}(z, x_B, x_L, l_T^2) D_{g\to h}(z_h, z, Q^2) \right]
\]

**Modification to splitting function for quark:**

\[
\Delta y_{q\to qg}(z, x_B, x_L, l_T^2) = \frac{C_A}{l_T^2} \left[ \frac{1 + z^2}{(1 - z)_+} T_{qg}^A(x_B, x_L) + \delta(1 - z) \Delta^{(q)} T_{qg}^A(x_B, x_L) \right] \frac{2\pi\alpha_s}{N_c f_q^A(x_B)}
\]

\[
\Delta y_{q\to qg}(z, x_B, x_L, l_T^2) = \Delta y_{q\to qg}(1 - z, x_B, x_L, l_T^2)
\]

**Quark-gluon correlation distribution:**

\[
\frac{2\pi\alpha_s}{N_c f_q^A(x_B)} T_{qg}^A(x_B, x_L) \approx \int 4dy^- \hat{q}(y) \sin^2 \left( \frac{l_T^2 y^-}{4q^- z(1 - z)} \right)
\]

The generalized transport parameter

\[X. F. Guo and X. N. Wang\]


\n
\[B. W. Zhang and X. N. Wang\]

Modification to splitting function for gluon:

\[ \Delta y_{g \to q\bar{q}} = \frac{1}{2l_r^2} \left[ z^2 + (1 - z)^2 \right] \left[ 1 - \frac{N_c}{C_F} z(1 - z) \right] \times \int_{0}^{0} dy^{-} \hat{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right) \]

\[ \Delta y_{g \to g} = \frac{2C_A N_c \left( 1 - z + z^2 \right)^3}{l_r^2 C_F z(1 - z)} \times \int_{0}^{0} dy^{-} \hat{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right) - \delta(1 - z) \frac{\Delta_g(l_r^2)}{l_r^2} \]

B. W. Zhang and X. N. Wang
Modification to splitting function for gluon:

\[
\Delta \gamma_{g \rightarrow q\bar{q}} = \frac{1}{2l_T^2} \left[ z^2 + (1 - z)^2 \right] \left[ 1 - \frac{N_c}{C_F} z(1 - z) \right] \times \int_0^L dy^+ \hat{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right)
\]

\[
\Delta \gamma_{g \rightarrow g\bar{g}} = \frac{2C_A N_c}{l_T^2 C_F} \left( 1 - z + z^2 \right)^3 \times \int_0^L dy^+ \hat{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right) - \delta(1 - z) \frac{\Delta g(l_T^2)}{l_T^2}
\]

Medium-modified splitting function:

\[
\tilde{\gamma}_{a \rightarrow bc}(z, l_T^2) = \gamma_{a \rightarrow bc}(z) + \Delta \gamma_{a \rightarrow bc}(z, l_T^2)
\]
Modification to splitting function for gluon:

\[
\Delta \gamma_{g \to qg} = \frac{1}{2l_T^2} \left[ z^2 + (1 - z)^2 \right] \left[ 1 - \frac{N_c}{C_F} z(1 - z) \right] \times \int_0^L 4dy^\prime \hat{q}(y) \sin^2 \left( x_L P^+ y^\prime / 2 \right)
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Medium-modified splitting function:

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\tilde{\gamma}_{a \to bc} (z, l_T^2) = \gamma_{a \to bc} (z) + \Delta \gamma_{a \to bc} (z, l_T^2)
\]

Resum Multiple scattering and induced gluon radiations.

medium-modified DGLAP (mDGLAP)

\[
\frac{\partial \tilde{D}_q^h (z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s (\mu^2)}{2\pi} \int_0^1 \frac{dz}{z} \left[ \tilde{\gamma}_{q \to qg} (z, \mu^2) \tilde{D}_q^h \left( \frac{z_h}{z}, \mu^2 \right) + \tilde{\gamma}_{q \to gg} (z, \mu^2) \tilde{D}_g^h \left( \frac{z_h}{z}, \mu^2 \right) \right]
\]

\[
\frac{\partial \tilde{D}_g^h (z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s (\mu^2)}{2\pi} \int_0^1 \frac{dz}{z} \left[ \sum_{q=1}^{2n_f} \tilde{\gamma}_{g \to qg} (z, \mu^2) \tilde{D}_q^h \left( \frac{z_h}{z}, \mu^2 \right) + \tilde{\gamma}_{g \to gg} (z, \mu^2) \tilde{D}_g^h \left( \frac{z_h}{z}, \mu^2 \right) \right]
\]

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Modification to splitting function for gluon:

\[
\Delta \gamma_{g \rightarrow qg} = \frac{1}{2l_T^2} \left[ z^2 + (1 - z)^2 \right] \left[ 1 - \frac{N_c}{C_F} z(1-z) \right] \int_0^L 4dy^- \tilde{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right)
\]

\[
\Delta \gamma_{g \rightarrow gg} = \frac{2C_A N_c}{l_T^2 C_F} \left( 1 - z + z^2 \right)^3 \int_0^L 4dy^- \tilde{q}(y) \sin^2 \left( x_L P^+ y^- / 2 \right) - \delta(1-z) \frac{\Delta_g(l_T^2)}{l_T^2}
\]

Medium-modified splitting function:

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\tilde{\gamma}_{a \rightarrow bc}(z, l_T^2) = \gamma_{a \rightarrow bc}(z) + \Delta \gamma_{a \rightarrow bc}(z, l_T^2)
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medium-modified DGLAP (mDGLAP)

\[
\frac{\partial \tilde{D}_q^h(z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[ \tilde{\gamma}_{q \rightarrow qg}(z, \mu^2) \tilde{D}_q^h \left( \frac{z_h}{z}, \mu^2 \right) + \tilde{\gamma}_{q \rightarrow qg}(z, \mu^2) \tilde{D}_g^h \left( \frac{z_h}{z}, \mu^2 \right) \right]
\]

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\]
$\tilde{D}(z)$ in Uniform Finite Medium (the “Brick” Problem)
Test of This Method: Momentum Sum Rule

\[ D_q^h(z, Q_0^2) = \delta(1 - z) \]
\[ D_g^h(z, Q_0^2) = \delta(1 - z) \]
\[ D_s^h(z, Q_0^2) = \delta(1 - z) \]

The violation of the sum rule:

\[ \varepsilon = 1 - \sum \int_0^1 dzz\tilde{D}_z^h(z, Q^2) \]

The sum-rule is satisfied well.

the numerical error increase with \( \hat{q} \)
Medium Induced Energy Loss of a Valence Quark

\[ D_{q}^h(z, Q^2) = \delta(1 - z), \]
\[ D_{\bar{q}}^h(z, Q^2) = D_{g}^h(z, Q^2) = 0 \]

\[ D_{q}^v(z, Q^2) = D_{q}^g(z, Q^2) - D_{\bar{q}}^g(z, Q^2) \]

\[ \frac{\Delta E}{E} = \frac{\Delta E_{\text{medium}} - \Delta E_{\text{vacuum}}}{E} = \int_{0}^{1} dzz \left[ D_{q}^v(z, Q^2) - \tilde{D}_{q}^v(z, Q^2) \right] \]

\[ \frac{\Delta E}{E} \sim L^2 \]

\[ \frac{\Delta E}{E} \rightarrow 1 \]

\[ x_L = \frac{l_T^2}{2p^+Ez(1 - z)} < 1 \]
\[ \Rightarrow E > \frac{l_T^2}{2p^+z(1 - z)} \]
Modified Fragmentation Functions V.S. $\hat{q}$

The suppression of $\tilde{D}(z)$ increase with $\hat{q}$ at large $z$.

$\hat{q}$ is suppressed with $\vec{q}$ at large $z$.

Increase $\hat{q}$

- Increase gluon radiation and pair production

Enhancement

Suppression

$C_g : C_q = 9/4$

- The suppression for gluon is larger than quark.
The suppression of $\tilde{D}(z)$ increase with $Q^2$ at large $z$. 

Power suppressed for $Q^2$, so most modification comes from lower $Q^2$. 

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**Modified Fragmentation Functions V.S. $Q^2$**

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**Modified Fragmentation Functions V.S. $Q^2$**

The suppression of $\tilde{D}(z)$ increase with $Q^2$ at large $z$. 

Power suppressed for $Q^2$, so most modification comes from lower $Q^2$. 

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The suppression of $\tilde{D}(z)$ is less at large $E$.

\[ x_L = \frac{l_T^2}{2 p^+ Ez(1-z)} < 1 \]

\[ \Rightarrow z(1-z) > \frac{l_T^2}{2 p^+ E} \]

Increase $E$

Decrease the phase space for gluon radiation
\[ \tilde{D}(z) \text{ in } e + A \rightarrow e + h + X \]

From Ideal “Brick” to
Realistic Nuclear Medium
Path Integration in DIS

\[ \hat{q}(y) = \hat{q}_0 \frac{\rho(y,b)}{\rho(0,0)} \]

\[
\frac{2\pi\alpha_s}{N_c f_q^A(x_B)} T_{qs}^A(x_B, x_L) \approx \int dy^- \hat{q}(y) \text{Sin}^2 \left( \frac{l_T^2 y^-}{4 q^- z (1 - z)} \right)
\]

\[
= \frac{4\hat{q}_0}{A \rho(0,0)} \int_0^\infty d^2 b \int_{-\infty}^{\infty} dy_0 \rho(y_0, b) \int_{y_0}^{\infty} dy \rho(y, b) \text{Sin}^2 \left[ \frac{l_T^2 y^-}{4 q^- z (1 - z)} \right]
\]
Modification Factor in DIS

The multiplicity ratio:

\[
R^h_M = \frac{\left( \frac{N^h(z,\nu)}{N^e(\nu)} \right)_A}{\left( \frac{N^h(z,\nu)}{N^e(\nu)} \right)_D} = \frac{\left( \frac{\sum e_j^2 q_j(x_B, Q^2) D^h_j(z, Q^2)}{\sum e_j^2 q_j^4(x_B, Q^2)} \right)_A}{\left( \frac{\sum e_j^2 q_j(x_B, Q^2) D^h_j(z, Q^2)}{\sum e_j^2 q_j^4(x_B, Q^2)} \right)_D}
\]

\[
\langle \Delta q_T^2 \rangle = 0.016 A^{1/3} \ GeV^2 \text{ in Drell-Yan} \\
\sim \hat{q} = 0.03 \ GeV^2 / \text{fm}
\]

Our combined result:
\[
\hat{q} = 0.07 \ GeV^2 / \text{fm}
\]

Same order!
Summary and Outlook

- Multiple parton scattering $\rightarrow$ mDGLAP
- The properties of Medium modified Fragmentation function in uniform finite medium.
- Modification factor in DIS.
- $R_{AA}$ in Heavy Ion Collisions is available soon.